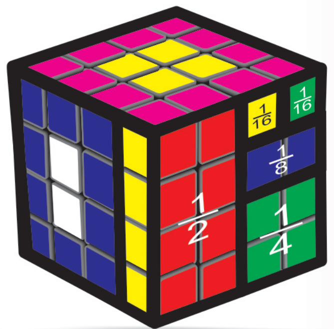
$a(b+c)=ab+ac$



State Council of Educational Reasearch and Training
Telangana, Hyderabad

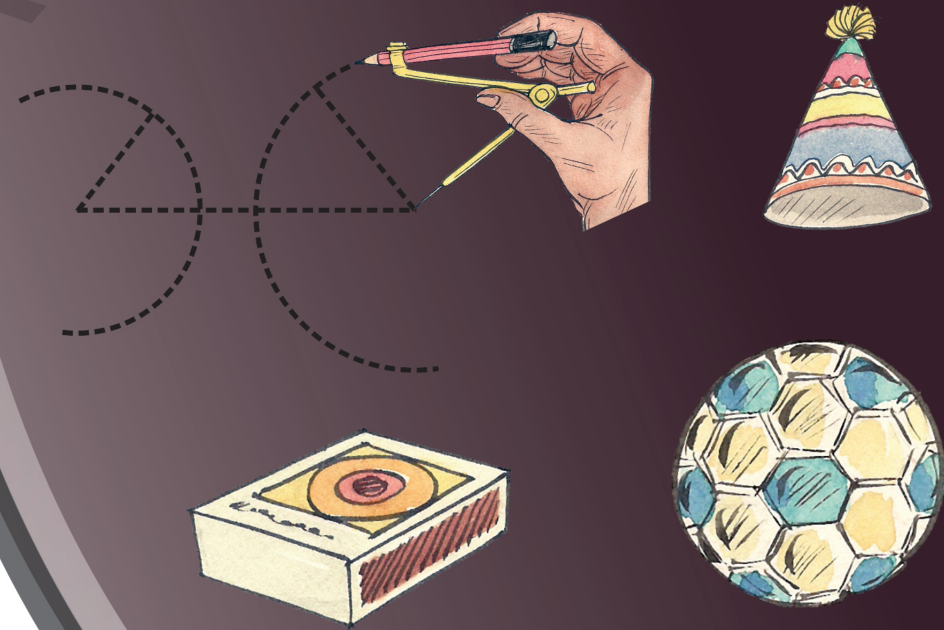
MATHEMATICS

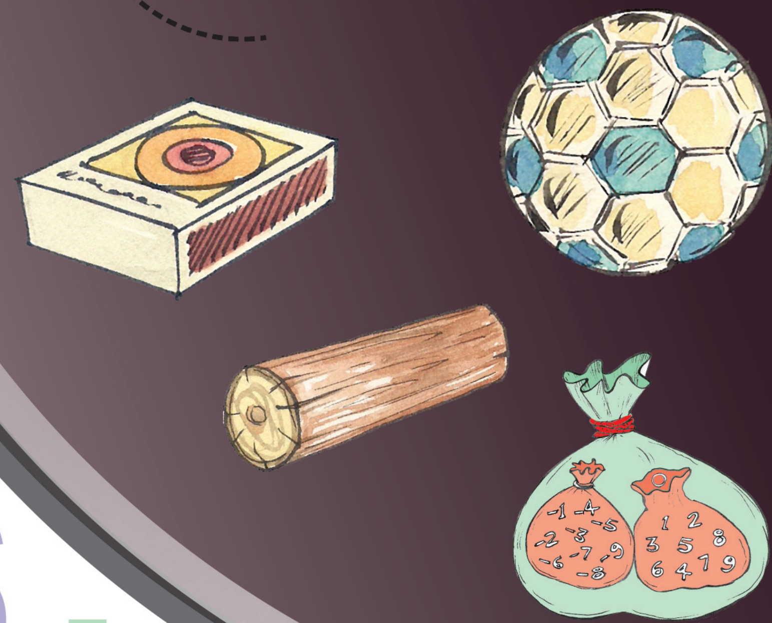
CLASS VII

FREE

MATHEMATICS

Class VII





Published by
The Government of Telangana
Hyderabad

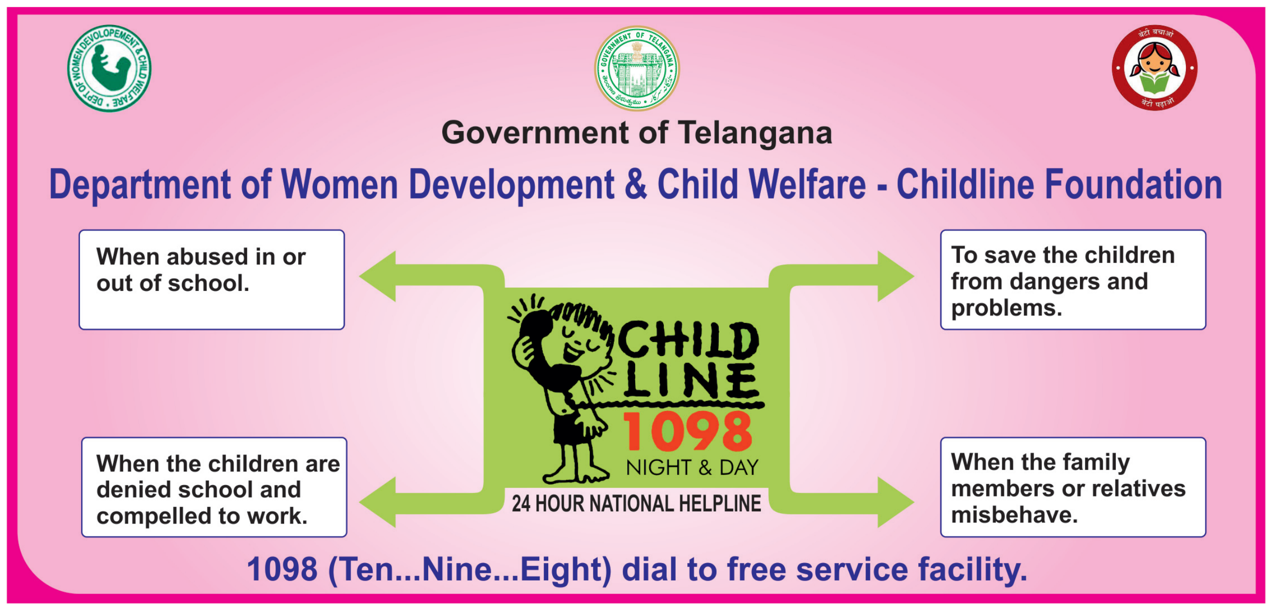
Free Distribution by T.S. Government

Free Distribution by T.S. Government

Graph

CHILDREN! THESE INSTRUCTIONS FOR YOU...

- For each and every conceptual understanding, a real life context with appropriate illustrations are given in the textbook. Try to understand the concept through keen reading of context along with observation of illustration.
- While understanding the concepts through activities, some doubts may arise. Clarify those doubts by through discussion with your friends and teachers, understand the mathematical concepts without any doubts.
- "Do this/Do these" exercises are given to test yourself, how far the concept has been understood. If you are facing any difficulty in solving problems in these exercises, you can clarify them by discussing with your teacher.
- The problems given in "Try this/try these", can be solved by reasoning, thinking creatively and extensively. When you face difficulty in solving these problems, you can take the help of your friends and teachers.
- The activities or discussion points given "Think & disicuss" have been given for extensive understanding of the concept by thinking critically. These activities should be solved by discussions with your fellow students and teachers.
- Different typs of problems with different concepts discussed in the chapter are given in an "Exercise" given at the end of the concept/chapter. Try to solve these problems by yourself at home or leisure time in school.
- The purpose of "Do this"/do these", and "Try this/try these" exercises is to solve problems in the presence of teacher only in the class itself.
- Where ever the "project works" are given in the textbook, you should conduct them in groups. But the reports of project works should be submitted indivedually.
- Try to solve the problems given as homework on the day itself. Clarify your doubts and make corrections also on the day itself by discussions with your teachers.
- Try to collect more problems or make new problems on the concepts learnt and show them to your teachers and fellow students.
- Try to collect more puzzles, games and interesting things related to mathematical concepts and share with your friends and teachers.
- Do not confine mathematical conceptual understanding to only classroom. But, try to relate them with your suroundings outside the classroom.
- Student must solve problems, give reasons and make proofs, be able to communicate mathematically, connect concepts to understand more concepts & solve problems and able to represent in mathematics learning.
- Whenever you face difficulty in achieving above competencies/skills/standards, you may take the help of your teachers.



Mathematics
VII class

TEXTBOOK DEVELOPMENT & PUBLISHING COMMITTEE

Chief Production Officer : **Smt.B. Seshu Kumari**
Director, SCERT, Hyderabad.

Executive Chief Organiser : **Sri. B. Sudhakar,**
Director, Govt. Text Book Press, Hyderabad.

Organising Incharge : **Dr. Nannuru Upender Reddy**
Prof. Curriculum & Text Book Department,
SCERT, Hyderabad.

Asst. Organising Incharge : **Sri. K. Yadagiri**
Lecturer, SCERT, Hyderabad.

Published by:

The Government of Telangana, Hyderabad

Respect the Law
Get the Rights

Grow by Education
Behave Humbly

First Published 2012
New Impressions 2013, 2014, 2015, 2016, 2017, 2018, 2019

This Book has been printed on 70 G.S.M. SS Maplitho
Title Page 200 G.S.M. White Art Card

Free distribution by T.S. Government 2019-20

Printed in India
at the Telangana Govt. Text Book Press,
Mint Compound, Hyderabad,
Telangana.

— o —

Textbook Development Committee

Members

Dr. P. Ramesh, Lecturer, Govt. IASE, Nellore
Sri. M. Ramanjaneyulu, Lecturer, DIET, Vikarabad, Ranga Reddy
Sri. T.V. Rama Kumar, HM, ZPPHS, Mulumudi, Nellore
Sri. P. Ashok, HM, ZPHS, Kumari, Adilabad
Sri. P. Anthony Reddy, HM, St. Peter's High School, R.N.Peta, Nellore
Sri. S. Prasada Babu, PGT, APTWR School, Chandrashekarapuram, Nellore
Sri. Kakulavaram Rajender Reddy, SA, UPS Thimmapur, Chandampet, Nalgonda
Sri. G. V. B. Suryanarayana Raju, SA, Municipal High School, Kaspa, Vizianagaram
Sri. S. Narasimha Murthy, SA, ZPHS, Mudivarthipalem, Nellore
Sri. P. Suresh Kumar, SA, GHS, Vijayanagar Colony, Hyderabad
Sri. K.V. Sunder Reddy, SA, ZPHS, Thakkasila, Alampur Mdl., Mababoobnagar
Sri. G. Venkateshwarlu, SA, ZPHS, Vemulakota, Prakasham
Sri. Ch. Ramesh, SA, UPS, Nagaram (M), Guntur.
Sri. P.D.L. Ganapathi Sharma, SA, GHS, Jamisthanpur, Manikeshwar Nagar, Hyderabad

Co-ordinators

Sri. K. Bramhaiah, Professor, SCERT, Hyderabad
Sri. Kakulavaram Rajender Reddy, SA, UPS Thimmapur, Chandampet, Nalgonda

Editors

Smt.B. Seshu Kumari, Director, SCERT, Hyderabad.
Sri. K. Bramhaiah, Professor, SCERT, Hyderabad
Sri. P. Adinarayana, Retd., Lecturer, New Science College, Ameerpet, Hyderabad

Chairperson for Position Paper and
Mathematics Curriculum and Textbook Development

Professor V. Kannan, Dept. of Mathematics and Statistics, University of Hyderabad

Chief Advisor

Dr. H. K. Dewan , Education Advisor, Vidya Bhavan Society, Udaipur, Rajasthan.

Academic Support Group Members

Smt. Namrita Batra, Vidyabhavan Society Resource Centre, Udaipur, Rajasthan
Sri. Inder Mohan Singh, Vidyabhavan Society Resource Centre, Udaipur, Rajasthan
Sri. Yashwanth Kumar Dave, Vidyabhavan Society Resource Centre, Udaipur, Rajasthan
Smt. Padma Priya Sherali, Community Mathematics Centre, Rishi Vally School, Chittoor
Kumari. M. Archana, Dept. of Mathematics & Statistics, University of Hyderabad
Sri. Sharan Gopal, Dept. of Mathematics & Statistics, University of Hyderabad
Sri. P. Chiranjeevi, Dept. of Mathematics & Statistics, University of Hyderabad
Sri. Abbaraju Kishore, Teacher, MPUPS, Chemallamudi, Guntur

Illustration & Design Team

Sri. Prashanth Soni, Artist, Vidyabhavan Society Resource Centre, Udaipur, Rajasthan
Sri. Shakir Ahammed, Operator, Vidyabhavan Society Resource Centre, Udaipur, Rajasthan
Sri. R. Madhusudhana Rao, Computer Operator, SCERT, A.P., Hyderabad

COVER PAGE DESIGNING

Sri. K. Sudhakara Chary, HM, UPS Neelikurthy, Mdl.Maripeda, Dist. Warangal

FOREWORD

State Curriculum Frame Work (SCF-2011) recommends that childrens' life at schools must be linked to their life outside the school. The Right To Education Act (RTE-2009) perceives that every child who enters the school should acquire the necessary skills prescribed at each level upto the age of 14 years. Academic standards were developed in each subject area accordingly to maintain the quality in education. The syllabi and text books developed on the basis of National Curriculum Frame work 2005 and SCF-2011 signify an attempt to implement this basic idea.

Children after completion of Primary Education enter into the Upper Primary stage. This stage is a crucial link for the children to continue their secondary education. We recognise that, given space, time and freedom, children generate new knowledge by exploring the information passed on to them by the adults. Inculcating creativity and initiating enquiry is possible if we perceive and treat children as participants in learning and not as passive receivers. The children at this stage possess characteristics like curiosity, interest, questioning, reasoning, insisting proof, accepting the challenges etc., Therefore the need for conceptualizing mathematics teaching that allows children to explore concepts as well as develop their own ways of solving problems in a joyful way.

We have begun the process of developing a programme which helps children understand the abstract nature of mathematics while developing in them the ability to construct own concepts. The concepts from the major areas of Mathematics like Number System, Arithmetic, Algebra, Geometry, Mensuration and Statistics are provided at the upper primary stage. Teaching of the topics related to these areas will develop the skills prescribed in academic standards such as problem solving, logical thinking, expressing the facts in mathematical language, representing data in various forms, using mathematics in daily life situations.

The textbooks attempt to enhance this endeavor by giving higher priority and space to opportunities for contemplation and wondering, discussion in small groups and activities required for hands on experience in the form of 'Do This' , 'Try This' and 'Projects'. Teachers support is needed in setting of the situations in the classroom. We also tried to include a variety of examples and opportunities for children to set problems. The book attempts to engage the mind of a child actively and provides opportunities to use concepts and develop their own structures rather than struggling with unnecessarily complicated terms and numbers. The chapters are arranged in such a way that they help the Teachers to evaluate every area of learning to comperehend the learning progress of children and in accordance with Continuous Comprehensive Evaluation (CCE).

The team associated in developing the textbooks consists of many teachers who are experienced and brought with them view points of the child and the school. We also had people who have done research in learning mathematics and those who have been writing textbooks for many years. The team tried to make an effort to remove fear of mathematics from the minds of children through their presentation of topics.

I wish to thank the national experts, university teachers, research scholars, NGOs, academicians, writers, graphic designers and printers who are instrumental to bring out this textbook in present form. I hope the teachers will make earnest effort to implement the syllabus in its true spirit and to achieve academic standards at the stage.

The process of developing materials is a continuous one and we hope to make this book better. As an organization committed to systematic reform and continuous improvement in quality of its products, SCERT, welcomes comments and suggestions which will enable us to undertake further revision and refinement.

B. Seshu kumari

DIRECTOR

SCERT, Hyderabad

Place: Hyderabad

Date: 28 January 2012

Mathematics
VII class

OUR NATIONAL ANTHEM

- Rabindranath Tagore

Jana-gana-mana-adhinayaka, jaya he
Bharata-bhagya-vidhata.
Punjab-Sindh-Gujarat-Maratha
Dravida-Utkala-Banga
Vindhya-Himachala-Yamuna-Ganga
Uchchala-Jaladhi-taranga.
Tava shubha name jage,
Tava shubha asisa mage,
Gahe tava jaya gatha,
Jana-gana-mangala-dayaka jaya he
Bharata-bhagya-vidhata.
Jaya he, jaya he, jaya he,
Jaya jaya jaya, jaya he!

PLEDGE

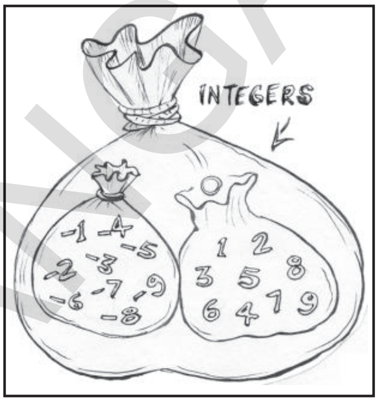
- Pydimarri Venkata Subba Rao

"India is my country. All Indians are my brothers and sisters.
I love my country, and I am proud of its rich and varied heritage.
I shall always strive to be worthy of it.
I shall give my parents, teachers and all elders respect,
and treat everyone with courtesy. I shall be kind to animals
To my country and my people, I pledge my devotion.
In their well-being and prosperity alone lies my happiness."

INTEGERS

1

1.0 Introduction



We start learning numbers like 1,2,3,4.... for counting objects around us. These numbers are called counting numbers or natural numbers. Let us think about these.

(i) Which is the smallest natural number?

(ii) Write five natural numbers between 100 and 10000.

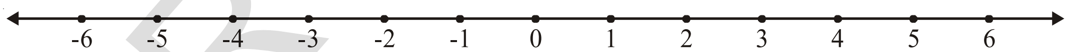
(iii) Can you find where the sequence of natural numbers ends?

(iv) What is the difference between any two consecutive natural numbers?

By including '0' to the collection of natural numbers, we get a new collection of numbers called whole numbers i.e., 0, 1, 2, 3, 4, ...

In class VI we also learnt about negative numbers. If we put whole number and negative numbers together we get a bigger collection of numbers called integers. In this chapter, we will learn more about integers, their operations and properties.

Let us now represent some integers on a number line.



(i) Which is the biggest integer represented on the above number line?

(ii) Which is the smallest integer?

(iii) Is 1 bigger than –3? Why?

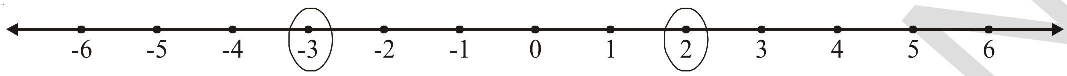
(iv) Is –6 bigger than –3? Why?

(v) Arrange 4, 6, –2, 0 and –5 in ascending order.

(vi) Compare the difference between (0 and 1) and (0 and –1) on the number line.

Exercise - 1

1. Some integers are marked on the number line. Which is the biggest and which is the smallest?



2. Write the integers between the pairs of integers given below. Also, choose the biggest and smallest intergers from them.

   (i) –5, –10   (ii) 3, –2   (iii) –8, 5

3. Write the following integers in ascending order (smallest to biggest).

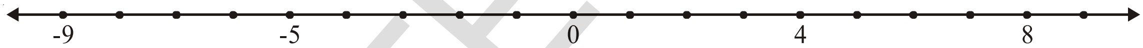
   (i) –5, 2, 1, –8   (ii) –4, –3, –5, 2   (iii) –10, –15, –7

4. Write the following integers in descending order (biggest to smallest).

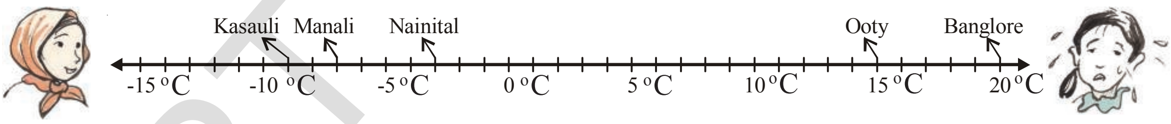
   (i) –2, –3, –5   (ii) –8, –2, –1   (iii) 5, 8, –2

5. Represent 6, –4, 0 and 4 on a number line.

6. Fill the missing integers on the number line given below



7. The temperatures (in degrees celsius/centigrade) of 5 cities in India on a particular day are shown on the number line below.



   (i) Write the temperatures of the cities marked on it?

   (ii) Which city has the highest temperature?

   (iii) Which city has the lowest temperature?

   (iv) Which cities have temperature less than 0ºC?

   (v) Name the cities with temperature more than 0ºC?

1.1 Operations of integers

We have done addition and subtraction of integers in class VI. First we will review our understanding of the same and then learn about multiplication and division of integers.

1.1.1 Addition of integers

Observe the additions given below.

$4 + 3 = 7$

$4 + 2 = 6$

$4 + 1 = 5$

$4 + 0 = 4$

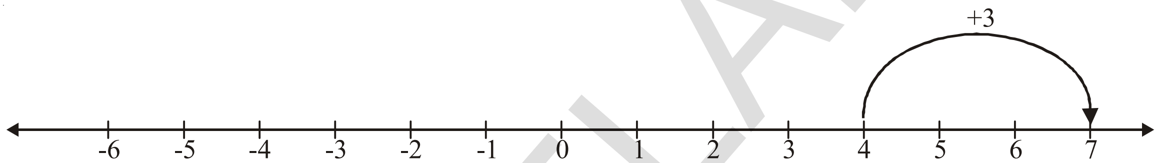
$4 + (-1) = 3$

$4 + (-2) = 2$

$4 + (-3) = 1$

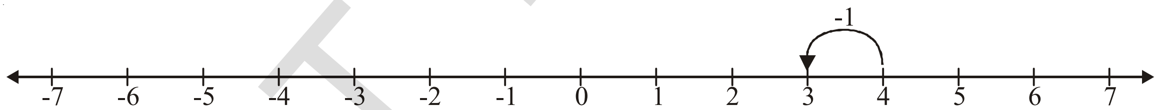
Do you find a pattern in the answers? You will find that when the number being added is decreased by one (3, 2, 1, 0, –1, –2, –3) then the value of the sum also decreases by 1.

On the number line, when you add 3 to 4 you move right on the number line:



Similarly can you now show the additions of 2 and 1 to 4 on the number line drawn above? You will find that in each case you have moved right on the number line.

Now, let us see what is happening when we add –1 to 4. From the above pattern we find that the answer is 3. Thus, we have moved one step left on the number line:



You can now similarly show addition of –2 and –3 to 4 on the number line drawn above? You will find that in each case you are moving left on the number line.

Thus, each time you add a positive integer you move right on the number line. On the other hand, each time you add a negative number you move left on the number line.

Try This

1.

$9 + 7$	=	16	$9 + 1$	=	
$9 + 6$	=	15	$9 + 0$	=	
$9 + 5$	=		$9 + (-1)$	=	
$9 + 4$	=		$9 + (-2)$	=	
$9 + 3$	=		$9 + (-3)$	=	
$9 + 2$	=				

(i) Now represent 9 + 2, 9 + (–1) and 9 + (–3) on the number line.

(ii) When you added a positive integer, in which direction did you move on the number line?

(iii) When you added a negative integer, in which direction did you move on the number line?

2. Sangeetha said that each time you add two integers, the value of the sum is greater than the numbers. Is Sangeetha right? Give reasons for your answer.

Exercise - 2

1. Represent the following additions on a number line.

   (i) 5 + 7 (ii) 5 + 2 (iii) 5 + (–2) (iv) 5 + (–7)

2. Compute the following.

   (i) 7 + 4 (ii) 8 + (–3) (iii) 11 + 3

   (iv) 14 + (–6) (v) 9 + (–7) (vi) 14 + (–10)

   (vii) 13 + (–15) (viii) 4 + (–4) (ix) 10 + (–2)

   (x) 100 + (–80) (xi) 225 + (–145)

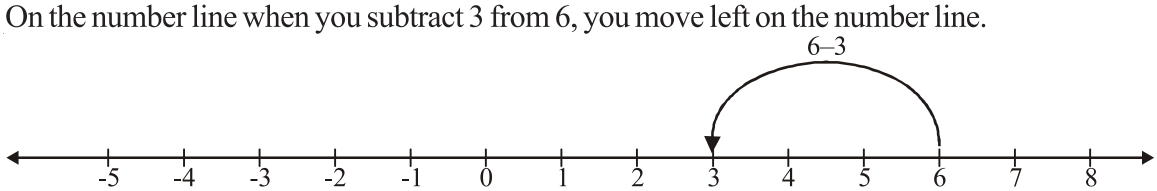
1.1.2 Subtraction of integers

Now let us study the subtractions given below.

6 – 3 = 3
6 – 2 = 4
6 – 1 = 5
6 – 0 = 6
6 – (–1) = 7
6 – (–2) = 8
6 – (–3) = 9
6 – (–4) = 10

Do you find a pattern in the answers? You will find that when the number being subtracted from 6 is decreased by one (3, 2, 1, 0, –1, –2, –3, –4) the value of the difference increased by 1.

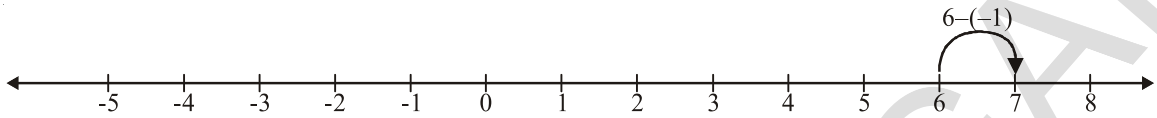
On the number line when you subtract 3 from 6, you move left on the number line.



You can now, similarly show subtraction of 2, 1 from 6 on the number line. You will find that in each case you have moved left on the number line.

Now, let us see what is happening when we subtract –1 from 6. As seen from the above pattern we find 6– (–1) = 7.

Thus, we have moved one step right on the number line .



You can now, similarly show subtraction of –2, –3, –4 from 6? You will find that in each case you are moving right on the number line.

Thus, each time you subtract a positive integer, you move left on the number line.

And each time you subtract a negative integer, you move right on the number line.

Try This

Complete the pattern.

1. 8 – 6 = 2
   8 – 5 = 3
   8 – 4 =
   8 – 3 =
   8 – 2 =
   8 – 1 =
   8 – 0 =
   8 – (–1) =
   8 – (–2) =
   8 – (–3) =
   8 – (–4) =

   (i) Now show 8 –6, 8–1, 8–0, 8–(–2), 8–(–4) on the number line.

   (ii) When you subtract a positive integer in which direction do you move on the number line?

   (iii) When you subtract a negative integer, in which direction do you move on the number line?

2. Richa felt that each time you subtract an integer from another integer, the value of the difference is less than the given two numbers. Is Richa right? Give reasons for your answer.

Exercise - 3

1. Represent the following subtractions on the number line.

   (i) $7 - 2$ (ii) $8 - (-7)$ (iii) $3 - 7$

   (iv) $15 - 14$ (v) $5 - (-8)$ (vi) $(-2) - (-1)$

2. Compute the following.

   (i) $17 - (-14)$ (ii) $13 - (-8)$ (iii) $19 - (-5)$

   (iv) $15 - 28$ (v) $25 - 33$ (vi) $80 - (-50)$

   (vii) $150 - 75$ (viii) $32 - (-18)$

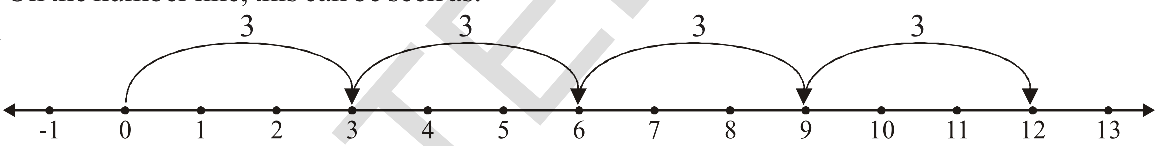
3. Express '-6' as the sum of a negative integer and a whole number.

1.1.3 Multiplication of integers

Now, let us multiply integers.

We know that $3 + 3 + 3 + 3 = 4 \times 3$ (4 times 3)

On the number line, this can be seen as.

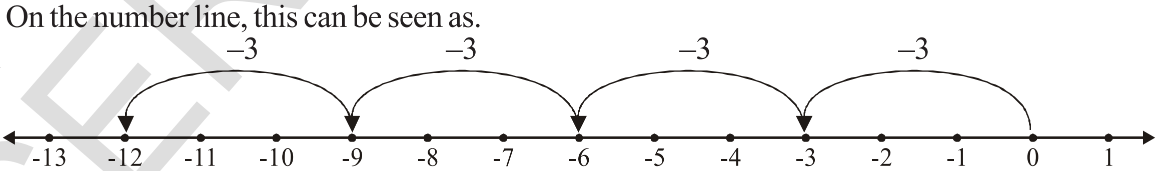


Thus, 4×3 means 4 jumps each of 3 steps from zero towards right on the number line and therefore $4 \times 3 = 12$.

Now let us discuss $4 \times (-3)$ i.e., 4 times (-3)

$4 \times (-3) = (-3) + (-3) + (-3) + (-3) = -12$

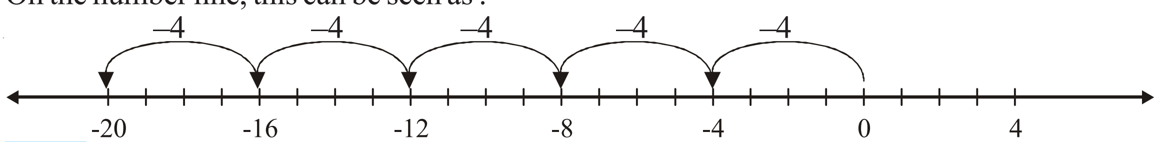
On the number line, this can be seen as.



Thus, $4 \times (-3)$ means 4 jumps each of 3 steps from zero towards left on the number line and therefore $4 \times (-3) = -12$

Similarly, $5 \times (-4) = (-4) + (-4) + (-4) + (-4) + (-4) = -20$

On the number line, this can be seen as :



Thus, 5×-4 means 5 jumps each of 4 steps from zero towards left on the number line and therefore $5 \times -4 = -20$

Similarly, $2 \times -5 = (-5) + (-5) = -10$

$3 \times -6 = (-6) + (-6) + (-6) = -18$

$4 \times -8 = (-8) + (-8) + (-8) + (-8) = -32$

Do This

1. Compute the following.

(i) 2×-6 (ii) 5×-4 (iii) 9×-4

Now, let us multiply -4×3

Study the following pattern-

$4 \times 3 = 12$

$3 \times 3 = 9$

$2 \times 3 = 6$

$1 \times 3 = 3$

$0 \times 3 = 0$

$-1 \times 3 = -3$

$-2 \times 3 = -6$

$-3 \times 3 = -9$

$-4 \times 3 = -12$

You see that as the multiplier decreases by 1, the product decreases by 3.

Thus, based on this pattern $-4 \times 3 = -12$.

We already know that $4 \times -3 = -12$

Thus, $-3 \times 4 = 3 \times -4 = -12$

Observe the symbol of the product as the negative sign differ in the multiplication.

Using this pattern we can say that

$4 \times -5 = -4 \times 5 = -20$

$2 \times -5 = -2 \times 5 = -10$

$3 \times -2 =$

$8 \times -4 =$

$6 \times -5 =$

From the above examples you would have noticed that product of positive integer and a negative integer is always a negative integer.

1.1.3(a) Multiplication of two negative integers

Now, what if we were to multiply –3 and –4.

Study the following pattern.

$-3 \times 4 = -12$

$-3 \times 3 = -9$

$-3 \times 2 = -6$

$-3 \times 1 = -3$

$-3 \times 0 = 0$

$-3 \times -1 = 3$

$-3 \times -2 = 6$

$-3 \times -3 = 9$

$-3 \times -4 = 12$

Do you observe any a pattern? You will see that as we multiply –3 by 4, 3, 2, 1, 0, –1, –2, –3, –4 the product increases by 3.

Now let us multiply –4 and –3. Study the following products and fill the blanks.

$-4 \times 4 = -16$

$-4 \times 3 = -12$

$-4 \times 2 = -8$

$-4 \times 1 = -4$

$-4 \times 0 = 0$

$-4 \times -1 =$ ______

$-4 \times -2 =$ ______

$-4 \times -3 =$ ______

You will see that as we multiply –4 by 4, 3, 2, 1, 0, –1, –2, –3, the product increases by 4.

According to the two patterns given above, $-3 \times -4 = -4 \times -3 = 12$

You have also observed that.

–3 × –1 = 3	–4 × –1 = 4
–3 × –2 = 6	–4 × –2 = 8
–3 × –3 = 9	–4 × –3 = 12

Thus, every time if we multiply two negative integers, the product is always a positive integer.

Activity 1

Fill the grid by multiplying each number in the first column with each number in the first row.

×	3	2	1	0	–1	–2	–3
3	9	6	3	0	–3	–6	–9
2	6	4	2	0			
1							
0							
–1	–3	–2	–1	0	1	2	3
–2							
–3							

(i) Is the product of two positive integers always a positive integer?

(ii) Is the product of two negative integers always a positive integer?

(iii) Is the product of a negative and positive integer always a negative integer?

1.1.3(b) Multiplication of more than two negative integers

We observed that the product of two negative integers is a positive integer. What will be the product of three negative integers? Four negative integers? and so on .....

Let us study the following examples.

(i) $(-2) \times (-3) = 6$

(ii) $(-2) \times (-3) \times (-4) = [(-2) \times (-3)] \times (-4) = 6 \times (-4) = -24$

(iii) $(-2) \times (-3) \times (-4) \times (-5) = [(-2) \times (-3) \times (-4)] \times (-5) = (-24) \times (-5) = 120$

(iv) $[(-2) \times (-3) \times (-4) \times (-5) \times (-6)] = 120 \times (-6) = -720$

From the above products we can infer that.

(i) The product of two negative integers is a positive integer.

(ii) The product of three negative integers is negative integer.

(iii) The product of four negative integers is a positive integer.

(iv) The product of five negative integers is a negative integer.

Will the product of six negative integers be positive or negative? State reasons.

Try This

(a) $(-1) \times (-1) =$ ——

(b) $(-1) \times (-1) \times (-1) =$ ——

(c) $(-1) \times (-1) \times (-1) \times (-1) =$ ——

(d) $(-1) \times (-1) \times (-1) \times (-1) \times (-1) =$ ——

We further see that in (a) and (c) above, the number of negative integers that are multiplied are even (two and four respectively) and the products are positive integers. The number of negative integers that are multiplied in (b) and (d) are odd and the products are negative integers.

Thus, we find that if the number of negative integers being multiplied is even, then the product is a positive integer. And if the number of negative integers being multiplied is odd, the product is a negative integer.

Exercise - 4

1. Fill in the blanks.

(i) $(-100) \times (-6) =$ ....

(ii) $(-3) \times$ .... $= 3$

(iii) $100 \times (-6) =$ ....

(iv) $(-20) \times (-10) =$ ....

(v) $15 \times (-3) =$ ....

2. Find each of the following products.

(i) $3 \times (-1)$	(ii) $(-1) \times 225$
(iii) $(-21) \times (-30)$	(iv) $(-316) \times (-1)$
(v) $(-15) \times 0 \times (-18)$	(vi) $(-12) \times (-11) \times (10)$
(vii) $9 \times (-3) \times (-6)$	(viii) $(-18) \times (-5) \times (-4)$
(ix) $(-1) \times (-2) \times (-3) \times 4$	(x) $(-3) \times (-6) \times (-2) \times (-1)$

3. A certain freezing process requires that the room temperature be lowered from 40ºC at the rate of 5ºC every hour. What will be the room temperature 10 hours after the process begins?

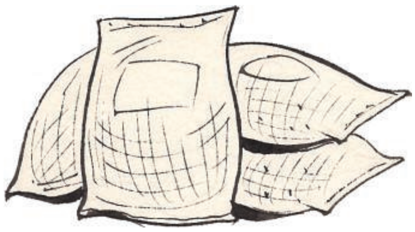
4. In a class test containing 10 questions, '3' marks are awarded for every correct answer and (–1) mark is for every incorrect answer and '0' for questions not attempted.

   (i) Gopi gets 5 correct and 5 incorrect answers. What is his score?

   (ii) Reshma gets 7 correct answers and 3 incorrect answers. What is her score?

   (iii) Rashmi gets 3 correct and 4 incorrect answers out of seven questions she attempts. What is her score?

5. A merchant on selling rice earns a profit of ₹10 per bag of basmati rice sold and a loss of ₹5 per bag of non-basmati rice.



   (i) He sells 3,000 bags of basmati rice and 5,000 bags of non-basmati rice in a month. What is his profit or loss in a month?

   (ii) What is the number of basmati rice bags he must sell to have neither profit nor loss, if the number of bags of non-basmati rice sold is 6,400.

6. Replace the blank with an integer to make it a true statement.

(i) $(-3) \times$ ______ $= 27$	(ii) $5 \times$ ______ $= -35$
(iii) ______ $\times (-8) = -56$	(iv) ______ $\times (-12) = 132$

1.1.4 Division of integers

We know that division is the inverse operation of multiplication. Let us study some examples for natural numbers.

We know that $3 \times 5 = 15$

Therefore, $15 \div 5 = 3$ or $15 \div 3 = 5$

Similarly, $4 \times 3 = 12$

Therefore, $12 \div 4 = 3$ or $12 \div 3 = 4$

Thus, we can say that for each multiplication statement of natural numbers there are two corresponding division statements.

Can we write a multiplication statement and its corresponding division statements for integers?

Study the following and complete the table.

Multiplication statement	Division statements		
$2 \times (-6) = (-12)$	$(-12) \div (-6) = 2$	,	$(-12) \div 2 = (-6)$
$(-4) \times 5 = (-20)$	$(-20) \div (5) = (-4)$	,	$(-20) \div (-4) = 5$
$(-8) \times (-9) = 72$	$72 \div (-8) = (-9)$	,	$72 \div (-9) = (-8)$
$(-3) \times (-7) =$ ________	______ $\div (-3) =$ ______	,	____________________
$(-8) \times 4 =$ ___________	____________________	,	____________________
$5 \times (-9) =$ ___________	____________________	,	____________________
$(-10) \times (-5) =$	____________________	,	____________________

We can infer from the above that when we divide a negative integer by a positive integer or a positive integer by a negative integer, we divide them as whole numbers and then negative (–) sign for the quotient. We thus, get a negative integer as the quotient.

Do This

1. Compute the following.

(i) $(-100) \div 5$ (ii) $(-81) \div 9$ (iii) $(-75) \div 5$ (iv) $(-32) \div 2$

(v) $125 \div (-25)$ (vi) $80 \div (-5)$ (vii) $64 \div (-16)$

Try This

Can we say that $(-48) \div 8 = 48 \div (-8)$?

Check whether-

(i) $90 \div (-45)$ and $(-90) \div 45$ (ii) $(-136) \div 4$ and $136 \div (-4)$

We also observe that

$(-12) \div (-6) = 2$; $(-20) \div (-4) = 5$; $(-32) \div (-8) = 4$; $(-45) \div (-9) = 5$

So, we can say that when we divide a negative integer by a negative integer, we get a positive number as the quotient.

Do This

1. Compute the following.

(i) $-36 \div (-4)$ (ii) $(-201) \div (-3)$ (iii) $(-325) \div (-13)$

1.2 Properties of integers

In class VI we have learnt the properties of whole numbers. Here we will learn the properties of integers.

1.2.1 Properties of integers under addition

(i) Closure property

Study the following.

Statement	Conclusion
$5 + 8 = 13$	The sum is a whole number
$6 + 3 =$	
$13 + 5 =$	
$10 + 2 =$	
$2 + 6 = 8$	The sum is a whole number

Is the sum of two whole numbers always a whole number? You will find this to be true. Thus, we say that whole numbers follow the closure property of addition.

Do integers satisfy closure property of addition ? Study the following additions and complete the blanks.

Statement	Conclusion
$6 + 3 = 9$	The sum is an integer
$-10 + 2 =$	
$-3 + 0 =$	
$-5 + 6 = 1$	
$(-2) + (-3) = -5$	
$7 + (-6) =$	The sum is an integer

Is the sum of two integers always an integer?

Can you give an example of a pair of integers whose sum is not an integer? You will not be able to find such a pair. **Therefore, integers are also closed under addition.**

In general, for any two integers a and b, a + b is also an integer.

(ii) Commutative property

Study the following and fill in the blanks.

Statement 1	Statement 2	Conclusion
$4 + 3 = 7$	$3 + 4 = 7$	$4 + 3 = 3 + 4 = 7$
$3 + 5 =$	$5 + 3 =$	
$3 + 1 =$	$1 + 3 =$	

Similarly, add as many pairs of whole numbers, as you wish. Did you find any pair of whole numbers for which the sum is different, when the order is changed. You will not find such a pair. Thus, we say that the addition of whole numbers is commutative.

Is addition of integers commutative? Study the following and fill in the blanks.

Statement 1	Statement 2	Conclusion
$5 + (-6) = -1$	$(-6) + 5 = -1$	$5 + (-6) = (-6) + 5 = -1$
$-9 + 2 =$	$2 + (-9) =$	
$-4 + (-5) =$	$(-5) + (-4) =$	

Did you find any pair of integers for which the sum is different when the order is changed? You would have not. **Therefore, addition is commutative for integers.**

In general, for any two integers a and b, a + b = b + a

(iii) Associative property

Let us study the following examples-

(i) $(2 + 3) + 4$ $\quad$ $2 + (3 + 4)$
$= 5 + 4$ $\quad$ $= 2 + 7$
$= 9$ $\quad$ $= 9$

(ii) $(-2 + 3) + 5$ $\quad$ $-2 + (3 + 5)$
$= 1 + 5$ $\quad$ $= -2 + 8$
$= 6$ $\quad$ $= 6$

(iii) $(-2 + 3) + (-5)$ $\quad$ $(-2) + [3 + (-5)]$
$= 1 + (-5)$ $\quad$ $= (-2) + (-2)$
$= -4$ $\quad$ $= -4$

(iv) $[(-2) + (-3)] + (-5)$ $\quad$ $-2 + [(-3) + (-5)]$
$= -5 + (-5)$ $\quad$ $= -2 + (-8)$
$= -10$ $\quad$ $= -10$

Is the sum in each case equal? You will find this to be true.

Therefore, integers follow the associative property under addition.

Try This

1. (i) $(2 + 5) + 4 = 2 + (5 + 4)$

   (ii) $(2 + 0) + 4 = 2 + (0 + 4)$

Does the associative property hold for whole numbers? Take two more examples and write your answer.

In general, for any three integers a, b and c, $(a + b) + c = a + (b + c)$

(iv) Additive identity

Carefully study the following.

$-2 + 0 = -2$

$5 + 0 = 5$

$8 + 0 =$

$-10 + 0 =$

On adding zero to integers, do you get the same integer? Yes.

Therefore, '0' is the additive identity for integers.

In general, for any integer a, $a+0 = 0 + a= a$

Try This

1. Add the following

   (i) $2 + 0 =$

   (ii) $0 + 3 =$

   (iii) $5 + 0 =$

2. Similarly, add zero to as many whole numbers as possible.

   Is zero the additive identity for whole numbers?

(v) Additive Inverse

What should be added to 3 to get its additive identity '0'?

Study the following-

$3 + (-3) = 0$

$7 + (-7) = 0$

$(-10) + 10 = 0$

Check whether we get similar pairs for other integers.

In each pair given above, one integer is called the additive inverse of the other integer.

In general, for any integer 'a' there exists an integer (–a) such that a + (–a) = 0. Both the integers are called additive inverse of each other.

1.2.2 Properties of integers under multiplication

(i) Closure property

Study the following and complete the table

Statement	Conclusion
$9 \times 8 = 72$	The product is an integer
$10 \times 0 =$	
$-15 \times 2 =$	
$-15 \times 3 = -45$	
$-11 \times -8 =$	
$10 \times 10 =$	
$5 \times -3 =$	

Is it possible to find a pair of integers whose product is not an integer? You will not find this to be possible.

Note: Do you remember fractions and decimals are not Integers.

Therefore, integers follow the closure property of multiplication.

In general, if a and b are two integers, a × b is also an integer.

Try This

1. Multiply the following
   (i) 2×3 = ________
   (ii) 5×4 = ________
   (iii) 3×6 = ________
2. Similarly, multiply any two whole numbers of your choice.
   Is the product of two whole numbers always a whole number?

(ii) Commutative property

We know that multiplication is commutative for whole numbers. Is it also commutative for integers?

Statement 1	Statement 2	Conclusion
$5 \times (-2) = -10$;	$(-2) \times 5 = -10$	$5 \times (-2) = (-2) \times 5 = -10$
$(-3) \times 6 =$	$6 \times (-3) =$	
$-20 \times 10 =$	$10 \times (-20) =$	

Therefore, multiplication of integers follows the commutative property.

In general, for any two integers a and b, a × b = b × a

(iii) Associative property

Consider the multiplication of 2, –3, –4 grouped as follows.

$[2 \times (-3)] \times (-4)$ and $2 \times [(-3) \times (-4)]$

We see that-

$[2 \times (-3)] \times (-4)$ and $2 \times [(-3) \times (-4)]$

$= (-6) \times (-4)$ $\quad$ $= 2 \times 12$

$= 24$ $\quad$ $= 24$

In first case 2, –3 are grouped together and in the second –3, –4 are grouped together. In both cases the product is the same.

Thus, $[2 \times (-3)] \times [(-4)] = 2 \times [(-3) \times (-4)]$

Does the grouping of integers affect the product of integers? No, it does not.

The product of three integers does not depend upon the grouping of integers. Therefore, the multiplication of integers is associative.

In general, for any integers, a, b and c, (a × b) × c = a × (b × c)

Do This

1. Is $[(-5) \times 2)] \times 3 = (-5) \times [(2 \times 3)]$?

2. Is $[(-2) \times 6] \times 4 = (-2) \times [(6 \times 4)]$?

Try This

$(5 \times 2) \times 3 = 5 \times (2 \times 3)$

Is the associative property true for whole numbers? Take many more examples and verify.

(iv) Distributive property

We know that, $9 \times (10 + 2) = (9 \times 10) + (9 \times 2)$

Thus, multiplication distributes over addition is true for whole numbers.

Let us see, is this true for integers-

(i) $-2 \times (1 + 3) = [(-2) \text{ x } 1] + [(-2) \times 3]$

$-2 \times 4 = -2 + (-6)$

$-8 = -8$

(ii) $-1 \times [3 + (-5)] = [(-1) \times 3] + [(-1) \times (-5)]$

$-1 \times (-2) = -3 + (+5)$

$2 = 2$

Verify $-3 \times (-4+2) = [(-3) \times (-4)] + [-3 \times (2)]$

You will find that in each case, the left hand side is equal to the right hand side.

Thus, multiplication distributes over addition of integers too.

In general, for any integers a, b and c, $a \times (b + c) = a \times b + a \times c$

(v) Multiplicative identity

$2 \times 1 = 2$

$-5 \times 1 = -5$

$-3 \times 1 =$ ______

$-8 \times 1 =$ ______

$1 \times -5 =$ ______

1 is the multiplicative identity of integers

You will find that multiplying an integer by 1 does not change the integer. Thus, 1 is called the multiplicative identity for integers.

In general, for any integer 'a', $a \times 1 = 1 \times a = a$

(vi) Multiplication by zero

We know that any whole number when multiplied by zero gives zero. What happens in case of integers? Study the following-

$(-3) \times 0 = 0$

$0 \times (-8) =$ ______

$9 \times 0 =$ ______

This shows that the product of an integer and zero is zero.

In general for any integer a, $a \times 0 = 0 \times a = 0$

Exercise 5

1. Verify the following.
   (i) $18 \times [7 + (-3)] = [18 \times 7] + [18 \times (-3)]$
   (ii) $(-21) \times [(-4) + (-6)] = [(-21) \times (-4)] + [(-21) \times (-6)]$
2. (i) For any integer a, what is $(-1) \times a$ equal to?
   (ii) Determine the integer whose product with (-1) is 5
3. Find the product, using suitable properties.
   (i) $26 \times (-48) + (-48) \times (-36)$
   (ii) $8 \times 53 \times (-125)$
   (iii) $15 \times (-25) \times (-4) \times (-10)$
   (iv) $(-41) \times 102$
   (v) $625 \times (-35) + (-625) \times 65$
   (vi) $7 \times (50 - 2)$
   (vii) $(-17) \times (-29)$
   (viii) $(-57) \times (-19) + 57$

1.2.3 Properties of integers under subtraction

(i) Closure under subtraction

Do we always get an integer, when subtracting an integer from an integer?

Do the following.

$9 - 7 \quad = \quad$ ______

$7 - 10 \quad = \quad$ ______

$2 - 3 \quad = \quad$ ______

$-2 - 3 \quad = \quad$ ______

$-2 - (-5) = \quad$ ______

$0 - 4 \quad = \quad$ ______

What did you find? Can we say that integers follow the closure property for subtraction?

Therefore, for any integers a and b, a – b is also an integer.

(ii) Commutativity under subtraction

Let us take an example. Consider the integers 6 and –4

$$6 - (-4) = 6 + 4 = 10 \quad \text{and}$$

$$-4 - (6) = -4 - 6 = -10$$

Therefore, $\quad 6 - (-4) \neq -4 - (6)$

Thus, subtraction is not commutative for integers.

Try This

Take atleast 5 different pairs of integers and see if subtraction is commutative.

1.2.4 Properties of integers under division

(i) Closure Property

Study the following table and complete it.

Statement	Inference	Statement	Inference
$(-8) \div (-4) = 2$	Result is an integer	$(-8) \div 4 = \frac{-8}{4} = -2$	
$(-4) \div (-8) = \frac{-4}{-8} = \frac{1}{2}$	Result is not an integer	$4 \div (-8) = \frac{4}{-8} = \frac{-1}{2}$	

What can you infer? You will infer that integers are not closed under division.

Try This

Take atleast five pairs of integers and check whether they are closed under division.

(ii) Commutative Property

We know that division is not commutative for whole numbers. Let us check it for integers also.

You can see from the table given above that $(-8) \div (-4) \neq (-4) \div (-8)$.

Is $(-9) \div 3$ equal to $3 \div (-9)$?

Is $(-30) \div (6)$ equal to $(-6) \div (-30)$?

Thus, we can say that **division of integers is not commutative.**

Try This

Take atleast 5 pairs of integers and observe whether the division of integers is commutative or not?

(iii) Division by Zero

Like whole numbers, any integer divided by zero is meaningless and zero divided by a non-zero integer is equal to zero.

For any integer a, $a \div 0$ is not defined but $0 \div a = 0$ for $a \neq 0$.

(iv) Division by 1

When we divide a whole number by 1 it gives the same whole number. Let us check whether this is true for negative integers also.

Observe the following-

$(-8) \div 1 = (-8)$ $(11) \div 1 = +11$ $(-13) \div 1 =$ _____ $(-25) \div 1 =$ _______

Thus, a negative integer or a positive integer divided by 1 gives the same integer as quotient.

In general, for any integer a, $a \div 1 = a$.

What happens when we divide any integer by (-1)? Complete the following table-

$(-8) \div (-1) = 8$ $11 \div (-1) = -11$ $13 \div (-1) =$ _____ $(-25) \div (-1) =$ _____

We can say that if any integer is divided by (-1) it does not give the same integer, but gives its additive identity.

Try This

1. For any integer a, is

   (i) $a \div 1 = 1$?

   (ii) $a \div (-1) = -a$?

   Take different values of 'a' and check.

(iii) Associative property

Is $[(-16) \div 4] \div (-2) = (-16) \div [4 \div (-2)]$?

$[(-16) \div 4] \div (-2) = (-4) \div (-2) = 2$

$(-16) \div [4 \div (-2)] = (-16) \div (-2) = 8$

Therefore, $[(-16) \div 4] \div (-2) \neq (-16) \div [4 \div (-2)]$

Thus, division of integers is not associative.

Try This

Take atleast five more examples and check whether division is associative for integers.

Exercise - 6

1. Fill the following blanks.

   (i) $-25 \div$ .... $=$ 25

   (ii) .... $\div 1 = -49$

   (iii) $50 \div 0 =$ ....

   (iv) $0 \div 1 =$ ....

1.3 Some practical problems using negative numbers

Example 1 : In a test (+5) marks are given for every correct answer and (–2) marks are given for every incorrect answer. (i) Radhika answered all the questions and scored 30 marks through 10 correct answers. (ii) Jaya also answered all the questions and scored (–12) marks through 4 correct answers. How many incorrect answers had both Radhika and Jaya attempted?

Solution : (i) Marks given for one correct answer $= 5$

So marks given for 10 correct answers $= 5 \times 10 = 50$

Radhika's score $= 30$

Marks obtained for incorrect answers $= 30 - 50 = -20$

Marks given for one incorrect answer $= (-2)$

Therefore, number of incorrect answers $= (-20) \div (-2) = 10$

(ii) Marks given for 4 correct answers $= 5 \times 4 = 20$

Jaya's score $= -12$

Marks obtained for incorrect answers $= -12 - 20 = -32$

Marks given for one incorrect answer $= (-2)$

Therefore number of incorrect answers $= (-32) \div (-2) = 16$

Example 2 : A shopkeeper earns a profit of ₹ 1 by selling one pen and incurs a loss of 40 paise per pencil while selling pencils of her old stock.

(i) In a particular month she incurs a loss of ₹ 5. In this period, she sold 45 pens. How many pencils did she sell in this period?

(ii) In the next month she earns neither profit nor loss. If she sold 70 pens, how many pencils did she sell?

Solution : (i) Profit earned by selling one pen ₹ 1

Profit earned by selling 45 pens = ₹ 45, which we denote by 45

Total loss given = ₹ 5, which we denote by -5.

Profit earned + Loss incurred = Total loss

Therefore, Loss incurred = Total loss-Profit earned

$= -5 - (45) = (-50) = -$ ₹ $50 = -5000$ paise

Loss incurred by selling one pencil = 40 paise which we write as -40 paise

So, number of pencils sold $= (-5000) \div (-40) = 125$ pencils.

(ii) In the next month there is neither profit nor loss.

So, profit earned + loss incurred $= 0$

i.e., Profit earned = – Loss incurred.

Now, profit earned by selling 70 pens = ₹ 70

Hence, loss incurred by selling pencils = $-$₹ 70 or -7000 paise.

Total number of pencils sold $= (-7000) \div (-40) = 175$ pencils.

Exercise - 7

1. In a class test containing 15 questions, 4 marks are given for every correct answer and (-2) marks are given for every incorrect answer. (i) Bharathi attempts all questions but only 9 answers are correct. What is her total score? (ii) One of her friends Hema attempts only 5 questions and all are correct. What will be her total score?

2. A cement company earns a profit of ₹ 9 per bag of white cement sold and a loss of ₹ 5 per bag of grey cement sold.

(i) The company sells 7000 bags of white cement and 6000 bags of grey cement in a month. What is its profit or loss?

(ii) What is the number of white cement bags it must sell to have neither profit nor loss, if the number of grey bags sold is 5400.

3. The temperature at 12 noon was 10^o C above zero. If it decreases at the rate of 2^oC per hour until midnight, at what time would the temperature be 8^oC below zero? What would be the temperature at midnight?

4. In a class test (+3) marks are given for every correct answer and (–2) marks are given for every incorrect answer and no marks for not attempting any question. (i) Radhika scored 20 marks. If she has got 12 correct answers, how many questions has she attempted incorrectly? (ii) Mohini scores (–5) marks in this test, though she has got 7 correct answers. How many questions has she attempted incorrectly?

5. An elevator descends into a mine shaft at the rate of 6 meters per minute. If the descent starts from 10 m above the ground level, how long will it take to reach – 350 m.

Looking Back

1. **N** (natural numbers) = 1, 2, 3, 4, 5 . . .

   W (whole numbers) = 0, 1, 2, 3, 4, 5 . . .

   Z (Integers) = ..., –4, –3, –2, –1, 0, 1, 2, 3, 4 . . .

   also 0, ±1, ±2, ±3 (set of integers also represented as I.)

2. (i) Each time you add a positive integer, you move right on the number line.

   (ii) Each time you add a negative integer, you move left on the number line.

3. (ii) Each time you subtract a positivie integer, you move left on the number line.

   (iii) Each time you subtract a negative integer, you move right on the number line.

4. (i) Each time you multiply a negative integer by a positive integer or a positive integer by a negative integer, the product is a negative integer.

(ii) Each time you multiply two negative integers, the product is a positive integer.

(iii) Product of even number of negative integers is positive (+ve), product of odd number of negative integers is negative (–ve).

5. (i) Each time you divide a negative integer by a positive integer or a positive integer by a negative integer the quotient is negative integer.

(ii) Each time you divide negative integer by a negative integer the quotient is positive integer.

(iii) When you multiply or divide two integers of same sign the result is always positive; if they are of opposite signs the result is negative.

6. The following are the propeties satisfied by addition and subtraction of integers-

(i) Integers are closed for addition and subtraction both. i.e., $a + b$ and $a–b$ are integers, where a and b are any integers.

(ii) Addition is commutative for integers, i.e., $a + b = b + a$, for all integers a and b.

(iii) Addition is associative for integers, i.e., $(a+b) + c = a + (b + c)$, for all integers a, b, and c.

(iv) Integer 0 is the identity under addition, i.e., $a + 0 = 0 + a = a$, for every integer a.

7. Integers show some properties under multiplication.

(i) Integers are closed under multiplication. i.e., $a \times b$ is an integer for any two integers a and b.

(ii) Multiplication is commutative for integers. i.e., $a \times b = b \times a$ for any integers a and b.

(iii) The integer 1 is the identity under multiplication, i.e., $1 \times a = a \times 1 = a$, for any integer a.

(iv) Multiplication is associative for integers, i.e., $(a\times b) \times c = a \times (b\times c)$ for any three integers a, b, and c.

8. In integers multiplication distributes over addition. i.e., $a \times (b+c) = a \times b + a \times c$ for any three integers a, b and c. This is called distributive property.

9. The properties of commutativity and associativity under addition and multiplication and the distributive property help us make our calculations easier.

10. For any integer a, we have

(i) $a \div 0$ is not defined or meaningless

(ii) $0 \div a = 0$ (for $a \neq 0$)

(iii) $a \div 1 = a$

FRACTIONS, DECIMALS AND RATIONAL NUMBERS

2

2.0 Introduction

We come across many examples in our day-to-day life where we use fractions. Just try to recall them. We have learnt how to represent proper and improper fractions and their addition and subtraction in the previous class. Let us review what we have already learnt and then go further to multiplication and division of fractional numbers as well as of decimal fractions. We will conclude by an introduction to a bigger set of numbers called rational numbers.

The shaded portion of the figures given below have been represented using fractions. Are the representations correct?

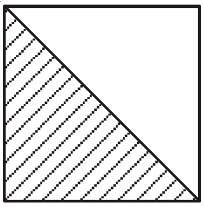


Figure 1

$\frac{1}{2}$

Y/N

Reason ....

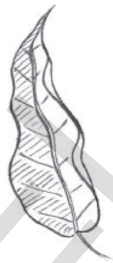


Figure 2

$\frac{1}{2}$

Y/N

Reason ....

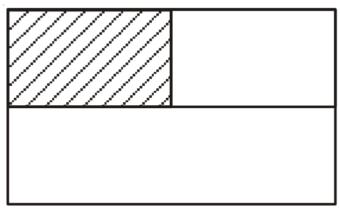


Figure 3

$\frac{1}{3}$

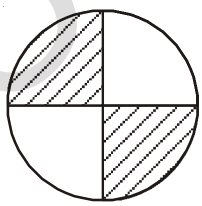
Y/N

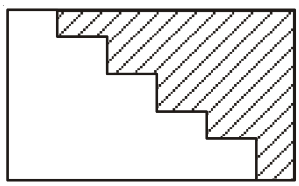
Reason ....

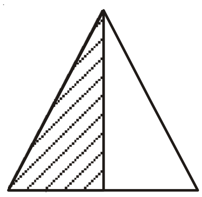
While considering the above you must have checked if parts of each figure are equal or not?.

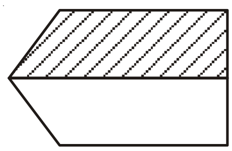
Make 5 more such examples and give them to your friends to verify.

Here is Neha's representation of $\frac{1}{2}$ in different figures.









Do you think that the shaded portions correctly represent $\frac{1}{2}$? Then what fractions are represented by unshaded portions?

Try This

Represent $\frac{3}{4}, \frac{1}{4}$ in different ways using different figures. Justify your representation.

Share, and check it with your friends.

Proper and Improper fractions

You have learnt about proper and improper fractions. A proper fraction is a fraction that represents a part of a whole. Give five examples of proper fractions.

Is $\frac{3}{2}$ a proper fraction? How do you check if it is a proper fraction or not?

What are the properties of improper fractions? One of them is that in improper fractions the numerator is more than or equal to the denominator. What else do we know about these fractions.

We can see that all improper fractions can be written as mixed fractions. The improper fraction $\frac{3}{2}$ can be written as $1\frac{1}{2}$. This is a mixed fraction. This contains an integral part and a fractional part. The fractional part should be a proper fraction.

Do This

1. Write five examples, each of proper, improper and mixed fractions?

Try This

Represent $2\frac{1}{4}$ pictorially. How many units are needed for this.

Comparison of fractions

Do you remember how to compare like fractions, for e.g. $\frac{1}{5}$ and $\frac{3}{5}$? $\frac{3}{5}$ is bigger than $\frac{1}{5}$. Why?

Can you recall how to compare two unlike fractions, for e.g. $\frac{5}{7}$ and $\frac{3}{4}$?

We convert these into like fractions and then compare them.

$$\frac{5}{7} \times \frac{4}{4} = \frac{20}{28} \quad \text{and} \quad \frac{3}{4} \times \frac{7}{7} = \frac{21}{28}$$

Since $\frac{5}{7} = \frac{20}{28}$ and $\frac{3}{4} = \frac{21}{28}$

Thus, $\frac{5}{7} < \frac{3}{4}$

Do These

1. Write five equivalent fractions for (i) $\frac{3}{5}$ (ii) $\frac{4}{7}$.
2. Which is bigger $\frac{5}{8}$ or $\frac{3}{5}$?
3. Determine if the following pairs are equal by writing each in their simplest form.

   (i) $\frac{3}{8}$ and $\frac{375}{1000}$ (ii) $\frac{18}{54}$ and $\frac{23}{69}$

   (iii) $\frac{6}{10}$ and $\frac{600}{1000}$ (iv) $\frac{17}{27}$ and $\frac{25}{45}$

You have already learnt about addition and subtraction of fractions in class VI. Let us solve some problems now.

Example 1 : Razia completes $\frac{3}{7}$ part of her homework while Rekha completed $\frac{4}{9}$ of it. Who has completed the least part?

Solution : To find this we have to compare $\frac{3}{7}$ and $\frac{4}{9}$.

Converting them to like fractions we have $\frac{3}{7} = \frac{27}{63}, \frac{4}{9} = \frac{28}{63}$

$\frac{27}{63} < \frac{28}{63}$

Thus, $\frac{27}{63} \langle \frac{28}{63}$ and so $\frac{3}{7} < \frac{4}{9}$

Razia has completed a least part of her homework than Rekha.

Example 2 : Shankar's family consumed $3\frac{1}{2}$ kg sugar in the first 15 days of a month. For the next 15 days they consumed $3\frac{3}{4}$ kg sugar. How much sugar did they consume for the whole month?

Solution : The total weight of the sugar for the whole month

$$= \left(3\frac{1}{2} + 3\frac{3}{4}\right) \text{ kg}$$

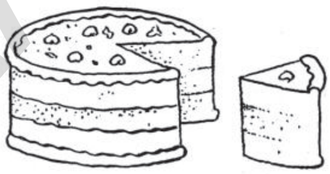
$$= \left(\frac{7}{2} + \frac{15}{4}\right) \text{ kg} = \left(\frac{14}{4} + \frac{15}{4}\right)$$

$$= \frac{29}{4} \text{ kg} = 7\frac{1}{4} \text{ kg.}$$

Example 3 : At Ahmed's birthday party, $\frac{5}{7}$ part of the total cake was distributed. Find how much cake is left?

Solution : Total cake is one = 1 or $\frac{1}{1}$

Cake distributed $= \frac{5}{7}$



The cake left $= 1 - \frac{5}{7}$

$$= \frac{7}{7} - \frac{5}{7} = \frac{2}{7}$$

Thus, $\frac{2}{7}$ part of the total cake is left now.

Exercise - 1

1. Compute and express the result as a mixed fraction?

(i) $2 + \frac{3}{4}$ (ii) $\frac{7}{9} + \frac{1}{3}$ (iii) $1 - \frac{4}{7}$

(iv) $2\frac{2}{3} + \frac{1}{2}$ (v) $\frac{5}{8} - \frac{1}{6}$ (vi) $2\frac{2}{3} + 3\frac{1}{2}$

2. Arrange the following in ascending order.

(i) $\frac{5}{8}, \frac{5}{6}, \frac{1}{2}$ (ii) $\frac{2}{5}, \frac{1}{3}, \frac{3}{10}$

3. Check in the following square, whether in this square the sum of the numbers in each row and in each column and along the diagonals is the same.

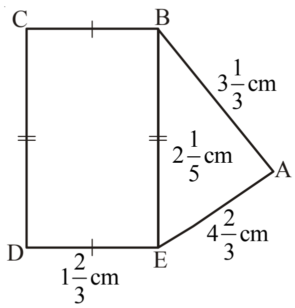
$\frac{6}{13}$	$\frac{13}{13}$	$\frac{2}{13}$
$\frac{3}{13}$	$\frac{7}{13}$	$\frac{11}{13}$
$\frac{12}{13}$	$\frac{1}{13}$	$\frac{8}{13}$

4. A rectangular sheet of paper is $5\frac{2}{3}$ cm long and $3\frac{1}{5}$ cm wide. Find its perimeter.

5. The recipe requires $3\frac{1}{4}$ cups of flour. Radha has $1\frac{3}{8}$ cups of flour. How many more cups of flour does she need?

6. Abdul is preparing for his final exam. He has completed $\frac{5}{12}$ part of his course content. Find out how much course content is left?

7. Find the perimeters of (i) ΔABE (ii) the rectangle BCDE in this figure. Which figure has greater perimetre and by how much?



2.1 Multiplication of fractions

2.1.1 Multiplication of a fraction by a whole number

When we multiply whole numbers we know we are repeatedly adding a number. For example 5 x 4 means adding 5 groups of 4 each or 5 times 4.

Thus, when we say $2\times\frac{1}{4}$ it means adding $\frac{1}{4}$ twice or 2 times $\frac{1}{4}$.

Let us represent this pictorially. Look at Figure 1. Each shaded part is $\frac{1}{4}$ part of a square. The two shaded parts together will represent $2\times\frac{1}{4}=\frac{1}{4}+\frac{1}{4}=\frac{2}{4}$.

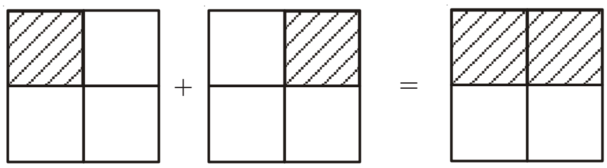


Figure 1 Figure 2

Let us now find $3 \times \frac{1}{2}$. This means three times $\frac{1}{2}$ or three halves.

We have $$3 \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$$

Do This

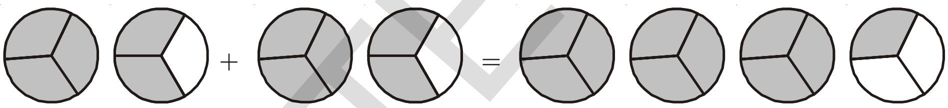
1. Find (i) $4 \times \frac{2}{7}$ (ii) $4 \times \frac{3}{5}$ (iii) $7 \times \frac{1}{3}$

The fractions that we have considered till now, i.e., $\frac{1}{2}, \frac{2}{3}, \frac{2}{7}$ and $\frac{3}{5}$ are proper fractions.

Let us consider some improper fractions like $\frac{5}{3}$ .and how to multiply $2 \times \frac{5}{3}$

$$2 \times \frac{5}{3} = \frac{5}{3} + \frac{5}{3} = \frac{10}{3} = 3\frac{1}{3}$$

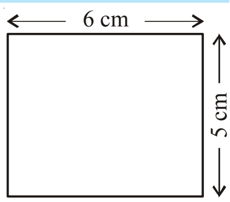
Pictorially



$\frac{5}{3}$ + $\frac{5}{3}$ = $\frac{10}{3}$

Do This

1. Find (i) $5 \times \frac{3}{2}$ (ii) $4 \times \frac{7}{5}$ (iii) $7 \times \frac{8}{3}$



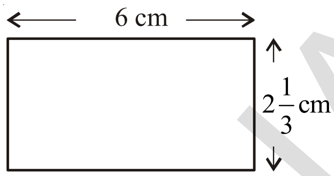
We know the area of a rectangle is equal to length × breadth. If the length and breadth of a rectangle are 6 cm and 5 cm respectively, then what will be its area? Obviously the area would be $6 \times 5 = 30$ cm^2.

If the length and breadth of a rectangle are 6 cm, $2\frac{1}{3}$ cm respectively, what would be the area of that rectangle.

Area of a rectangle is the product of its length and breadth. To multiply a mixed fraction with a whole number, first convert the mixed fractions into an improper fraction and then multiply.

Therefore, area of a rectangle $= 6 \times 2\frac{1}{3}$

$= 6 \times \frac{7}{3} = \frac{42}{3}$ cm^2 = 14cm^2



You might have realised by now that to multiply a whole number with a proper or an improper fraction, we multiply the whole number with the numerator of the fraction, keeping the denominator the same.

Do These

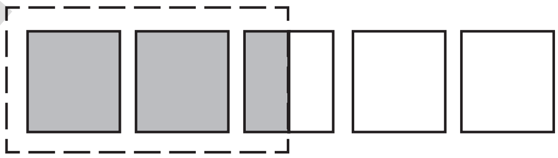
1. Find the following.

(i) $3 \times 2\frac{2}{7}$ (ii) $5 \times 2\frac{1}{3}$ (iii) $8 \times 4\frac{1}{7}$ (iv) $4 \times 1\frac{2}{9}$ (v) $5 \times 1\frac{1}{3}$

2. Represent pictorially $2 \times \frac{1}{5} = \frac{2}{5}$

Consider $\frac{1}{2} \times 5$. How do you understand it?

$\frac{1}{2} \times 5$ means half of 5, which is $\frac{5}{2}$ or $2\frac{1}{2}$



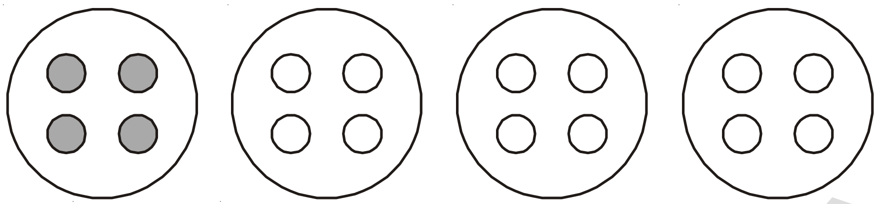
Thus, $\frac{1}{2}$ of $5 = \frac{1}{2} \times 5 = \frac{5}{2}$

Similarly $= \frac{1}{2}$ of $3 = \frac{1}{2} \times 3 = \frac{3}{2}$ or $1\frac{1}{2}$

Here onwards 'of' represents multiplication.

So what would $\frac{1}{4}$ of 16 mean? It tells us that the whole (16) is to be divided into 4 equal parts and one part out of that has to be taken. When we make 4 equal parts of 16, each part will be 4. So $\frac{1}{4}$ of 16 is 4.

This can be illustrated with marbles as :



$\frac{1}{4}$ of 16 or $\frac{1}{4} \times 16 = \frac{16}{4} = 4$

Similarly, $\frac{1}{2}$ of $16 = \frac{1}{2} \times 16 = \frac{16}{2} = 8$.

Example 4 : Nazia has 20 marbles. Reshma has $\frac{1}{5}$ of the number of marbles that Nazia has. How many marbles does Reshma have?

Solution : Reshma has $\frac{1}{5} \times 20 = 4$ marbles.

Example 5 : In a family of four persons 15 chapaties were consumed in a day. $\frac{1}{5}$ of the chapaties were consumed by the mother and $\frac{3}{5}$ were consumed by the children and the remaining were eaten by the father.

(i) How many chapaties were eaten by the mother?

(ii) How many chapaties were eaten by the children?

(iii) What fraction of the total chapaties has been eaten by the father?

Solution : Total number of chapaties = 15

(i) Number of chapaties eaten by mother $\frac{1}{5} \times 15 = 3$ chapaties

(ii) $\frac{3}{5}$ of the total number is eaten by children, $\frac{3}{5} \times 15 = 9$ chapaties

(iii) The chapaties left for father = 15 – 3 – 9 = 3 chapaties

Fraction of chapathies eaten by father $= \frac{3}{15} = \frac{1}{5}$

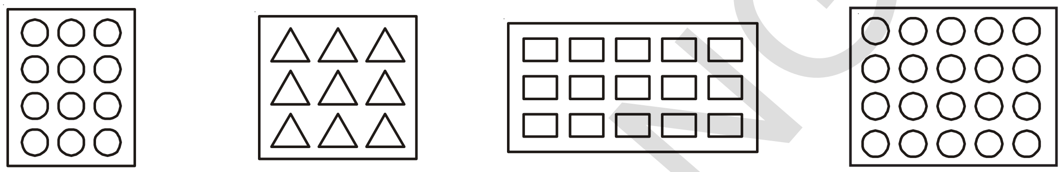
Exercise - 2

1. Multiply the following. Write the answer as a mixed fraction.

   (i) $\frac{3}{6} \times 10$ (ii) $\frac{1}{3} \times 4$ (iii) $\frac{6}{7} \times 2$ (iv) $\frac{2}{9} \times 5$ (v) $15 \times \frac{2}{5}$

2. Shade : (i) $\frac{1}{2}$ of the circles in box (a) (ii) $\frac{2}{3}$ of the triangles in box (b)

   (iii) $\frac{3}{5}$ of the rectangles in box (c) (iv) $\frac{3}{4}$ of the circles in box (d)



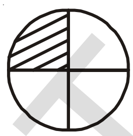
(a) (b) (c) (d)

3. Find (i) $\frac{1}{3}$ of 12 (ii) $\frac{2}{5}$ of 15

2.1.2 Multiplication of a fraction with a fraction

What does $\frac{1}{2} \times \frac{1}{4}$ mean? From the above we can understand that it means $\frac{1}{2}$ of $\frac{1}{4}$.

Consider $\frac{1}{4}$ -



How will we find $\frac{1}{2}$ of this shaded part? We can divide this one-fourth $\left(\frac{1}{4}\right)$ shaded part into two equal parts (Figure 1). Each of these two parts represents $\frac{1}{2}$ of $\frac{1}{4}$.

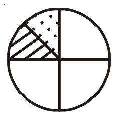


Figure 1

Let us call one of these parts as part 'A'. What fraction is 'A' of the whole circle? If we divide the remaining parts of the circle into two equal parts each, we get a total of eight equal parts. 'A' is one of these parts.

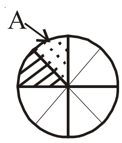
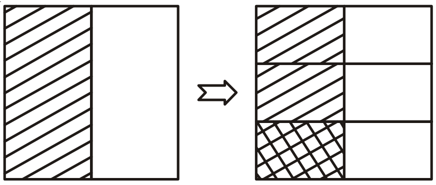


Figure 2

So, 'A' is $\frac{1}{8}$ of the whole. Thus, $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

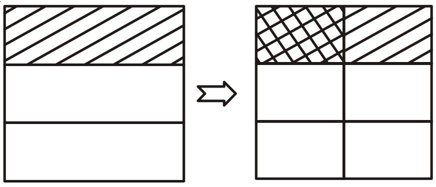
Find $\frac{1}{3} \times \frac{1}{2}$ and $\frac{1}{2} \times \frac{1}{3}$.

$\frac{1}{3}$ of $\frac{1}{2}$ is



$= \frac{1}{6}$ thus $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

$\frac{1}{2}$ of $\frac{1}{3}$ is



$= \frac{1}{6}$ thus $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

We can see that $\frac{1}{3} \times \frac{1}{2} = \frac{1}{2} \times \frac{1}{3}$

Do These

1. Fill in these boxes:

(i) $\frac{1}{5} \times \frac{1}{7} = \frac{1 \times 1}{5 \times 7} = \boxed{}$ (ii) $\frac{1}{2} \times \frac{1}{6} = \boxed{} = \boxed{}$

2. Find $\frac{1}{2} \times \frac{1}{5}$ and $\frac{1}{5} \times \frac{1}{2}$ using diagram, check whether $\frac{1}{2} \times \frac{1}{5} = \frac{1}{5} \times \frac{1}{2}$

Consider one more example $\frac{2}{3}$ of $\frac{2}{5}$. We have shown $\frac{2}{5}$ in Figure 1 and $\frac{2}{3} \times \frac{2}{5}$ in Figure 2.

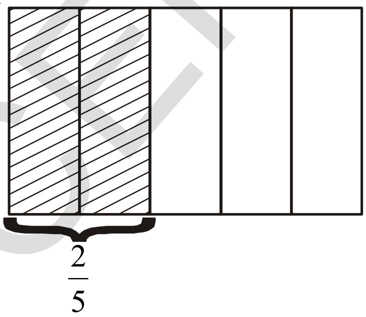


Figure 1

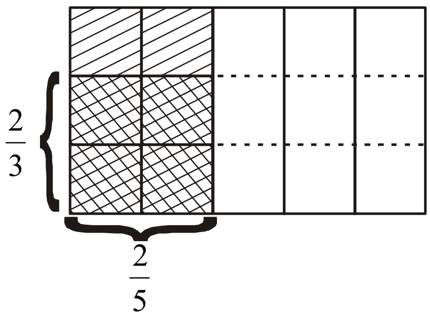


Figure 2

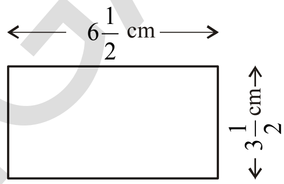
The cross hatched portion in figure (2) represents $\frac{2}{3}$ of $\frac{2}{5}$ or $\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$

To find the $\frac{2}{3}$ of $\frac{2}{5}$, we have made three equal parts of $\frac{2}{5}$ and then selected 2 out of the 3 parts.

This represent 4 parts out of a total 15 parts so $\frac{2}{3}$ of $\frac{2}{5} = \frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$.

Here, we can observe that Product of two fractions = $\dfrac{\textbf{Product of Numerators}}{\textbf{Product of Denominators}}$.

Now, what will be the area of the rectangle be if its length and breadth are $6\frac{1}{2}$ cm and $3\frac{1}{2}$ cm respectively.



The area = $6\frac{1}{2} \times 3\frac{1}{2} = \frac{13}{2} \times \frac{7}{2}$ cm^2. $= \frac{91}{4} = 22\frac{3}{4}$ cm^2.

Example 6 : Narendra reads $\frac{1}{4}$ of a short novel in 1 hour. What part of the book will he have read in $2\frac{1}{2}$ hours?

Solution : The part of the novel read by Narendra in 1 hour = $\frac{1}{4}$

So the part of the novel read by him in $2\frac{1}{2}$ hours $= 2\frac{1}{2} \times \frac{1}{4} = \frac{5}{2} \times \frac{1}{4} = \frac{5}{8}$

So Narendra would read $\frac{5}{8}$ part of the novel in $2\frac{1}{2}$ hours.

Example 7 : A swimming pool is filled $\frac{3}{10}$ part in half an hour. How much will it be filled in $1\frac{1}{2}$ hour?

Solution : The part of the pool filled in half an hour = $\frac{3}{10}$.

So, the part of pool which is filled in $1\frac{1}{2}$ hour is 3 times the pool filled in half an hour.

$= 3 \times \frac{3}{10} = \frac{9}{10}$ Thus, $\frac{9}{10}$ part of the pool will be filled in $1\frac{1}{2}$ hours.

Try This

You have seen that the product of two natural numbers is one or more than one or bigger than each of the two natural numbers. For example, $3 \times 4 = 12$, $12 > 4$ and $12 > 3$. What happens to the value of the product when we multiply two proper fractions?

Fill the following table and conclude your observations.

Eg: $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$	$\frac{8}{15} < \frac{2}{3}, \frac{8}{15} < \frac{4}{5}$	Product is less than each of the fractions
$\frac{1}{5} \times \frac{2}{7} = ------$		
$\frac{3}{5} \times \frac{\square}{2} = \frac{21}{10}$		
$\frac{5}{\square} \times \frac{4}{3} = \frac{20}{6}$		

Exercise - 3

1. Find each of the following products.

   (i) $\frac{5}{6} \times \frac{7}{11}$ (ii) $6 \times \frac{1}{5}$ (iii) $2\frac{1}{3} \times 3\frac{1}{5}$

2. Multiply and reduce to lowest form.

   (i) $\frac{2}{3} \times 5\frac{1}{5}$ (ii) $\frac{2}{7} \times \frac{1}{3}$ (iii) $\frac{9}{3} \times \frac{5}{5}$

3. Which one is greater?

   (i) $\frac{2}{5}$ of $\frac{4}{7}$ or $\frac{3}{4}$ of $\frac{1}{2}$ (ii) $\frac{1}{2}$ of $\frac{4}{7}$ or $\frac{2}{3}$ of $\frac{3}{7}$

4. Rehana works $2\frac{1}{2}$ hours each day on her embroidery. She completes the work in 7 days. How many hours did she take to complete her work?

5. A truck runs 8 km using 1 litre of petrol. How much distance will it cover using $10\frac{2}{3}$ litres of petrol?

6. Raja walks $1\frac{1}{2}$ meters in 1 second. How much distance will he walk in 15 minutes?

7. Provide the number in the box $\square$ to make the statement true.

(i) $\frac{2}{3} \times \square = \frac{20}{21}$. (ii) $\frac{5}{7} \times \frac{\square}{5} = \frac{3}{\square}$

2.2 Division of fractions

Imagine you have 15 meters length of cloth and you want to make pieces of $1\frac{1}{2}$ metres length each from it. How many $1\frac{1}{2}$ meter pieces will you get? Here we will successively subtract $1\frac{1}{2}$ meters from 15 meters and see how many times we can do this, till we have no cloth left.

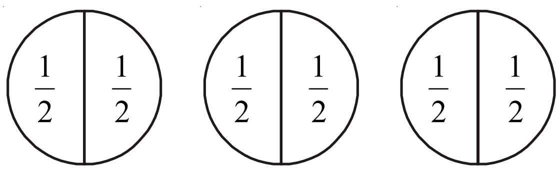
Look at one more example. A paper strip of length $\frac{21}{2}$ cm has to be cut into smaller strips of length $\frac{3}{2}$ cm each. How many pieces would we get? Clearly, we can cut $\frac{3}{2}$ cm each time or divide $\frac{21}{2}$ by $\frac{3}{2}$ i.e., $\frac{21}{2} \div \frac{3}{2}$.

Let us recall division with whole numbers. In $15 \div 3$, we find out how many 3's are there in 15. The answer to this is 5. Similarly, to find the number of 2's in 18, we divide 18 by 2 or $18 \div 2$. The answer to this is 9. Now correlate the same process in dividing whole numbers by fractions and fractions by fractions.

2.2.1 Division of whole number by a fraction

Let us find $3 \div \frac{1}{2}$.

Kiran says we have to find how many halves $\left(\frac{1}{2}\right)$ are there in 3. To find the number of $\frac{1}{2}$ in 3, we draw the following.



The figure above suggests that there are 6 halves in 3.

We can therefore say $3 \div \frac{1}{2} = 6$

Think about $2 \div \frac{1}{3}$

This means finding how many one-thirds $\left(\frac{1}{3}\right)$ are there two wholes. How would you find these?

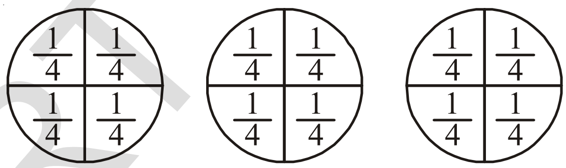
We can see that there are 6 one-thirds in two wholes or $2 \div \frac{1}{3} = 6$

Do This

Find (i) $2 \div \frac{1}{4}$ (ii) $7 \div \frac{1}{2}$ (iii) $3 \div \frac{1}{5}$

2.2.1a) Reciprocal of a fraction

Now consider $3 \div \frac{1}{4}$. This means the number of $\frac{1}{4}$ parts obtained, when each of the three wholes, are divided into $\frac{1}{4}$ equal parts



The number of one-fourths is 12, or $3 \div \frac{1}{4} = 12$

We also see that, $3 \div \frac{1}{4} = 3 \times \frac{4}{1} = 12$.

This suggests that $3 \div \frac{1}{4} = 3 \times \frac{4}{1}$

Also examine $2 \div \frac{1}{3}$.

We already found that $2 \div \frac{1}{3} = 6$

As inthe above example $2 \div \frac{1}{3} = 2 \times \frac{3}{1} = 6$

Similarly, $4 \div \frac{1}{4} = 16$ and $4 \times \frac{4}{1} = 16$.

The number $\frac{3}{1}$ can be obtained by interchanging the numerator and denominator of $\frac{1}{3}$ or by inverting $\frac{1}{3}$. Similarly, $\frac{4}{1}$ is obtained by inverting $\frac{1}{4}$.

Observe these products and fill in the blanks:

$7 \times \frac{1}{7} = 1$	$\frac{2}{3} \times \frac{3}{2} = \frac{2 \times 3}{3 \times 2} = \frac{6}{6} = 1$
$\frac{1}{9} \times 9 =$	$\frac{2}{7} \times$ .... $= 1$
$\frac{5}{4} \times \frac{4}{5} =$	.... $\times \frac{5}{9} = 1$

Multiply five more such pairs.

Any two non-zero numbers whose product is 1, are called reciprocals of one another. So the reciprocal of $\frac{4}{7}$ is $\frac{7}{4}$ and the reciprocal of $\frac{7}{4}$ is $\frac{4}{7}$.

What is the reciprocal of $\frac{5}{9}$ and $\frac{2}{5}$?

Try This

1. Will the reciprocal of a proper fraction be a proper fraction?
2. Will the reciprocal of an improper fraction be an improper fraction?

Therefore, we can say that,

$1 \div \frac{1}{2} = 1 \times$ reciprocal of $\frac{1}{2} = 1 \times \frac{2}{1}$.

$3 \div \frac{1}{4} = 3 \times$ reciprocal of $\frac{1}{4} = 3 \times \frac{4}{1}$.

$3 \div \frac{1}{2} =$ .... = ...

Raju applied this inverting procedure to mixed fractions and said that the reciprocal of $1\frac{1}{2}$ is $1\frac{2}{1}$. Is he right? Check.

So, $2 \div \frac{3}{4} = 2 \times$ reciprocal of $\frac{3}{4} = 2 \times \frac{4}{3}$.

$5 \div \frac{2}{4} = 5 \times$ .... $= 5 \times$......................

Thus dividing by a fraction is equivalent to multiplying the number by the reciprocal of that fraction.

Do This

Find (i) $9 \div \frac{2}{5}$ (ii) $3 \div \frac{4}{7}$ (iii) $2 \div \frac{8}{9}$

For dividing a whole number by a mixed fraction, first convert the mixed fraction into an improper fraction and then solve it.

Example $4 \div 3\frac{2}{5} = 4 \div \frac{17}{5} = 4 \times \frac{5}{17} = \frac{20}{17}$ Find , $11 \div 3\frac{1}{3} = 11 \div \frac{10}{3} = ?$

Do This

Find (i) $7 \div 5\frac{1}{3}$ (ii) $5 \div 2\frac{4}{7}$

2.2.2 Division of a fraction by a whole number

What will $\frac{3}{4} \div 3$ be equal to?

Based on our earlier observations we have : $\frac{3}{4} \div 3 = \frac{3}{4} \div \frac{3}{1} = \frac{3}{4} \times \frac{1}{3} = \frac{3}{12} = \frac{1}{4}$

So, $\frac{2}{3} \div 5 = \frac{2}{3} \times \frac{1}{5} = ?$ What is $\frac{5}{7} \div 6$ and $\frac{2}{7} \div 8$?

For dividing mixed fractions by whole numbers, we convert the mixed fractions into improper fractions. For example

$2\frac{1}{3} \div 5 = \frac{7}{3} \div 5 = \frac{7}{3} \times \frac{1}{5} = \frac{7}{5}$. Find $4\frac{2}{5} \div 3 =$ .... = ...; $2\frac{3}{5} \div 2 =$ .... = ...

2.2.3 Division of a fraction by another fraction

We can now find $\frac{1}{4} \div \frac{5}{6}$.

$\frac{1}{4} \div \frac{5}{6} = \frac{1}{4} \times$ reciprocal of $\frac{5}{6} = \frac{1}{4} \times \frac{6}{5} = \frac{6}{20} = \frac{3}{10}$.

Similarly, $\frac{8}{5} \div \frac{2}{3} = \frac{8}{5} \times$ reciprocal of $\frac{2}{3} =$ ....=........ and $\frac{1}{2} \div \frac{3}{4} =$............=.........

Do This

Find (i) $\frac{3}{5} \div \frac{1}{2}$ (ii) $\frac{1}{2} \div \frac{3}{5}$ (iii) $2\frac{1}{2} \div \frac{3}{5}$ (iv) $5\frac{1}{6} \div \frac{9}{2}$

Example 8 : An empty swimming pool is to be filled up to $\frac{9}{10}$ of its capacity. A pump takes half an hour to fill $\frac{3}{10}$ of the pool, how long will it take to fill $\frac{9}{10}$ of the pool?

Solution : We need to find how many $\frac{3}{10}$'s are there in $\frac{9}{10}$, solve the division problem $\frac{9}{10} \div \frac{3}{10}$.

$\frac{9}{10} \times \frac{10}{3} = 3$ Thus, it would take 3 half an hours, or $1\frac{1}{2}$ hours to fill the pool to its $\frac{9}{10}$.

Exercise 4

1. Find the reciprocal of each of the following fractions.

   (i) $\frac{5}{8}$ (ii) $\frac{8}{7}$ (iii) $\frac{13}{7}$ (iv) $\frac{3}{4}$

2. Find

   (i) $18 \div \frac{3}{4}$ (ii) $8 \div \frac{7}{3}$ (iii) $3 \div 2\frac{1}{3}$ (iv) $5 \div 3\frac{4}{7}$

3. Find

   (i) $\frac{2}{5} \div 3$ (ii) $\frac{7}{8} \div 5$ (iii) $\frac{4}{9} \div \frac{4}{5}$

4. Deepak can paint $\frac{2}{5}$ of a house in one day. If he continues working at this rate, how many days will he take to paint the whole house?

2.3 Decimal numbers or Fractional decimals

In class VI we have learnt about decimal numbers and their addition and subtraction. Let us review our understanding and then learn about multiplication and division.

Let us write 12714 in its expanded form:

$$12714 = 1\times 10000 + 2 \times 1000 + 7 \times \text{......} + 1 \times \text{......} + 4 \times 1$$

What will the expended form of 12714.2 be?

You will find that on moving from right to left, the value increase in multiples of 10.

so on $\xleftarrow{\times 10}$ 10 thousands = 10,000 $\xleftarrow{\times 10}$ 10 hundreds = 1,000 $\xleftarrow{\times 10}$ 10 tens = 100 $\xleftarrow{\times 10}$ 10 units = 10 $\xleftarrow{\times 10}$ 1 unit

Now, what happens when we move from left to right? You will find that the value gets, divided by 10. Now think, if the unit is divided by 10, what will happen? Remember you have learnt that

$$1 \div 10 = \frac{1}{10} = 0.1$$

1 unit $\xrightarrow{\div 10}$ 10^{th} part or $\frac{1}{10}$ or 0.1 $\xrightarrow{\div 10}$ 100^{th} part or $\frac{1}{100}$ or 0.01 $\xrightarrow{\div 10}$ 1000^{th} part or $\frac{1}{1000}$ or 0.001 $\xrightarrow{\div 10}$ so on

Thus, the expanded form of 12714.2 is-

$$12714.2 = 1\times 10000 + 2 \times 1000 + 7 \times ......+ 1 \times ...... + 4 \times 1 + 2 \times \frac{1}{10}$$

Now find the place value of all the digits of 3.42. You might have noticed that a dot (.) or a decimal point separates whole part of the number from the fractional part. The part right side of the decimal point is called the decimal part of the number as it represents a part of 1. The part left to the decimal point is called the integral part of the number.

In the number 3.42-

	3 is at units place	4 is at the first place after the decimal point	2 is at the second place after the decimal point
Place value	$3 \times 1 = 3$	$4 \times \frac{1}{10} = \frac{4}{10}$ or 0.4	$2 \times \frac{1}{100} = \frac{2}{100}$ or 0.02

Try This

1. Look at the following table and fill up the blank spaces.

Hundreds (100)	Tens (10)	Units (1)	Tenth $\left(\frac{1}{10}\right)$	Hundredths $\left(\frac{1}{100}\right)$	Thousandths $\left(\frac{1}{1000}\right)$	Number
5	4	7	8	2	9	547.829
0	7	2	1	7	7	______
3	2	__	__	5	4	327.154
6	__	4	__	2	__	614.326
2	__	6	5	__	2	236.512

2. Write thefollowing numbers in their expanded form.

(i) 30.807 (ii) 968.038 (iii) 8370.705

To convert money, length, weight, etc from one unit to the other we often use decimals. For e.g. 5 paise = ₹ $\frac{5}{100}$ = ₹0.05; 220 g = $\frac{220}{1000}$ kg = 0.220 kg ; 5 cm = $\frac{5}{100}$ m = 0.05 m

Do This

Find (i) 50 paise = ₹ ______ (ii) 22 g = ______ kg (iii) 80 cm = m

2.3.1 Comparison of decimal numbers

Who has more money?

Abhishek and Neha have ₹ 375.50 and ₹375.75 respectively in their kiddy bank. To find who has more money, we first compare the digits on the left of the decimal point. Since both the children have ₹ 375 we compare the digits to the right of the decimal point starting from the tenth place. We find that Abhishek has 7 tenths and Neha has 5 tenths, 7 tenths >5 tenths, therefore, Abhishek has more money than Neha, i.e., 375.75 > 375.50.

Now compare quickly, which of the following pair of numbers is greater ?

(i) 37.65 and 37.60 (ii) 1.775 with 19.780 (iii) 364.10 and 363.10

Let us see how to add or subtract decimals.

(i) 221.85 + 37.10 (ii) 39.70 - 6.85

$$\begin{array}{r} 221.85 \\ +37.10 \\ \hline 258.95 \\ \hline \end{array} \qquad \begin{array}{r} 39.70 \\ -\ 06.85 \\ \hline 32.85 \\ \hline \end{array}$$

While adding or subtracting decimal numbers, the digits in the same places must be added or subtracted, i.e., while writing numbers one below the other, see that decimal points must come one below the other. Decimal places may be made equal by placing zeroes on the right side of the decimal number.

Do This

Find (i) 0.25 + 5.30 (ii) 29.75 - 25.97.

Example 9 : The equal sides of an isosceles triangle are 3.5 cm each and the other side is 2.5 cm. What is the perimeter of the triangle?

Solution : The sides of isosceles triangle are 3.5 cm, 3.5 cm and 2.5 cm. Therefore, the perimeter of the given triangle is = sum of lengths of three sides = 3.5 cm + 3.5 cm + 2.5 cm = 9.5 cm

Exercise - 5

1 cm	= 10 mm
1 m	= 100cm
1 km	= 1000m
1 kg	= 1000gm

1. Which one is greater?
   (i) 0.7 or 0.07 (ii) 7 or 8.5
   (iii) 1.47 or 1.51 (iv) 6 or 0.66
2. Express the following as rupees using decimals.
   (i) 9 paise (ii) 77 rupees 7 paise (iii) 235 paise
3. (i) Express 10 cm in metre and kilometre.
   (ii) Express 45 mm in centimeter, meter and kilometer.

4. Express the following in kilograms.

(i) 190 g (ii) 247 g (iii) 44 kg 80 gm

5. Write the following decimal numbers in expanded form.

(i) 55.5 (ii) 5.55 (iii) 303.03

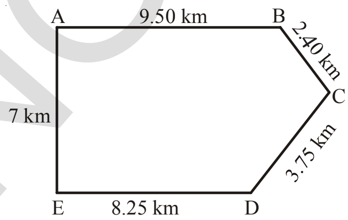
(iv) 30.303 (v) 1234.56

6. Write the place value of **3** in the following decimal numbers.

(i) 3.46 (ii) 32.46 (iii) 7.43

(iv) 90.30 (v) 794.037

7. Aruna and Radha start their journey from two different places. A and E. Aruna chose the path from A to B then to C, while Radha chose the path from E to D then to C.



Find who travelled more and by how much?

8. Upendra went to the market to buy vegetables. He brought 2 kg 250 gm tomatoes, 2 kg 500 gm potatoes, 750 gm lady fingers and 125 gm green chillies. How much weight did Upendra carry back to his house?

2.4 Multiplication of decimal numbers

Rajendra of class 7 went with his mother to the bazar to buy vegetables. There they purchased 2.5 kg potatoes at the rate of ₹ 8.50 per kg. How much money do they need to pay?

We come across various situations in day-to-day life where we need to know how to multiply two decimals. Let us now learn the multiplication of two decimal numbers.

Let us first multiply- 0.1×0.1

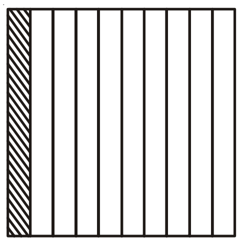


Figure 1

0.1 means one part of 10 parts. This is represented as $\frac{1}{10}$ using fractions and pictorially in Fig.1.

Thus, $0.1 \times 0.1 = \frac{1}{10} \times \frac{1}{10}$ which means $\frac{1}{10}$ of $\frac{1}{10}$. So here we are finding the 10th part of $\frac{1}{10}$. Thus, we divide $\frac{1}{10}$ into 10 equal parts and take one part. This is represented by one square in Figure 2. How many squares are there in Figure 2? There are 100 squares. So one square represents one out of 100 or 0.01. So we can conclude that

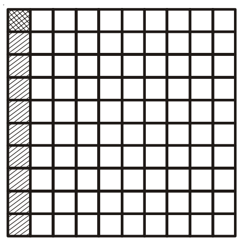


Figure 1

$$0.1 \times 0.1 = \frac{1}{10} \times \frac{1}{10} = \frac{1}{100} = 0.01$$

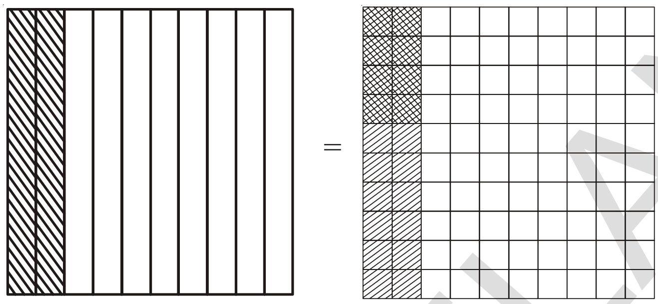
Let us now find 0.4×0.2

$$0.4 \times 0.2 = \frac{4}{10} \times \frac{2}{10} \text{ or } \frac{4}{10} \text{ of } \frac{2}{10}$$

Pictorially

$\frac{2}{10}$

$\frac{4}{10}$ of



$= \frac{8}{100}$

Since there are 8 double shaded squares out of 100, they represent 0.08.

While finding 0.1×0.1 and 0.4×0.2, you might have noticed that we first multiplied them as Whole numbers ignoring the decimal point. In 0.1×0.1, we found 01×01 or 1×1. Similarly in 0.4×0.2 we found 04×02 or 4×2. The products obtained are 1 and 8 respectively.

We then counted the total number of digits to the right of the decimal point in the numbers being multiplied. In both 0.1×0.1 and 0.4×0.2, the total number of digits to the right of the decimal point in the numbers being multiplied is 2 each. Thus, in each of their products we put the decimal point by counting two places from right to left.

For any decimal number which has no integral number part, we generally place a zero on the left side of decimal point to give prominance to decimal point.

Thus, $0.1 \times 0.1 = .01$ or 0.01

$0.4 \times 0.2 = .08$ or 0.08

If we had multiplied 0.5×0.05 then we would have put the decimal point in the product by counting three places from right to left i.e. $0.5 \times 0.05 = 0.025$.

Let us now find 1.2×2.5

Multiply 12 and 25. We get 300. In both 1.2 and 2.5, there is 1 digit to the right of the decimal point. So, count $1 + 1 = 2$ digits. From the rightmost digit (i.e., 0) in 300, move two places towards left. We get 3.00 or 3. Thus, $1.2 \times 2.5 = 3$

While multiplying 2.5 and 1.25 you will first multiply 25 and 125. For placing the decimal in the product obtained, we will count $1 + 2 = 3$ (Why?). Thus, $2.5 \times 1.25 = 3.225$.

Do These

1. Find (i) 1.7×3 (ii) 2.0×1.5 (iii) 2.3×4.35
2. Arrange the products obtained in (1) in descending order.

Example 10 : The length of a rectangle is 7.1 cm and its breadth is 2.5 cm. What is the area of the rectangle?

Solution : Length of the rectangle = 7.1 cm

Breadth of the rectangle = 2.5 cm

Therefore, area of the rectangle = $7.1 \times 2.5 = 17.75$ cm^2

2.4.1 Multiplication of decimal number by 10, 100, 1000 etc.,

Reshma observed that $3.2 = \frac{32}{10}$ whereas $2.35 = \frac{235}{100}$. Thus, she found that depending on the position of the decimal point, the decimal number can be converted to a fraction with denominator 10 or 100etc., She wondered what would happen if a decimal number is multiplied by 10, 100 or 1000 etc.,

Let us see if we can find a pattern in multiplying numbers by 10 or 100 or 1000.

Have a look at the table given below and fill in the blanks :

$1.76 \times 10 = \frac{176}{100} \times 10 = 17.6$	$2.35 \times 10 =$	$12.356 \times 10 =$ ....
$1.76 \times 100 = \frac{176}{100} \times 100 = 176$ or 176.0	$2.35 \times 100 =$	$12.356 \times 100 =$ ....
$1.76 \times 1000 = \frac{176}{100} \times 1000 = 1760$ or 1760.0	$2.35 \times 1000 =$	$12.356 \times 1000 =$ ....
$0.5 \times 10 = \frac{5}{10} \times 10 = 5$; $0.5 \times 100 =$ .... ; $0.5 \times 1000 =$		

Look at your answers. Could you find any pattern? The decimal point in the products shifts to the right by as many zeroes as in 10, 100, 1000 etc,.

2.4.2 Division of decimal numbers

Gopal was preparing a design to decorate his classroom. He needed a few coloured strips of paper of length 1.6 cm each. He had a strip of coloured paper of length 9.6 cm . How many pieces of the required length will he get out of this strip? He thought it would be $\frac{9.6}{1.6}$ cm. Is he correct?

Both 9.6 and 1.6 are decimal numbers. So we need to know the division of decimal numbers too!

2.4.2 (a) Division by numbers like 10, 100, 1000 etc.,

Let us now divide a decimal number by 10, 100 and 1000.

Consider 31.5 $\div$ 10.

$$31.5 \div 10 = \frac{315}{10} \div 10 = \frac{315}{10} \times \frac{1}{10} = \frac{315}{100} = 3.15$$

Similarly, $31.5 \div 100 = \frac{315}{10} \div 100 = \frac{315}{10} \times \frac{1}{100} = \frac{315}{1000} = 0.315$

Is there a pattern while dividing numbers by 10, 100 or 1000? This may help us in dividing numbers by 10, 100 or 1000 in a shorter way.

Observe the pattern in the table, given bellow and complete it.

29.5 ÷ 10 = 2.95	132.7 ÷ 10 = ...	1.5 ÷ 10 = ...	17.36 ÷ 10 = ...
29.5 ÷ 100 = 0.295	132.7 ÷ 10 = ...	1.5 ÷ 100 = ...	17.36 ÷ 100 = ...
29.5 ÷ 1000 = 0.0295	132.7 ÷ 1000 = ...	1.5 ÷ 1000 = ...	17.36 ÷ 1000 = ...

2.4.2 (b) Division of a decimal number by a whole number

Let us find $\frac{6.4}{2}$. Remember we also write it as $6.4 \div 2$.

So, $6.4 \div 2 = \frac{64}{10} \div 2 = \frac{64}{10} \times \frac{1}{2}$ (as learnt in fractions)

$$= \frac{64 \times 1}{10 \times 2} = \frac{1 \times 64}{10 \times 2} = \frac{1}{10} \times \frac{64}{2} = \frac{1}{10} \times 32 = \frac{32}{10} = 3.2$$

Now, let us calculate $12.96 \div 4 = \frac{1296}{100} \div 4 = \frac{1296}{100} \times \frac{1}{4} = \frac{1}{100} \times \frac{1296}{4} = \frac{1}{100} \times 324 = 3.24$

Do This

1. Find (i) $35.7 \div 3$ (ii) $25.5 \div 3$

Example 11 : Find the average of 4.2, 3.8 and 7.6.

Solution : The average of 4.2, 3.8 and 7.6 is $\frac{4.2+3.8+7.6}{3} = \frac{15.6}{3} = 5.2$

2.4.2 (c) Division of a decimal number by another decimal number

Let us find how we divide a decimal number by another decimal number, For example $35.5 \div 0.5$.

$$= \frac{355}{10} \div \frac{5}{10} = \frac{355}{10} \times \frac{10}{5} = 71$$

Thus $35.5 \div 0.5 = 71$.

Example 12 : A truck covers a distance of 92.5 km in 2.5 hours. If the truck is travelling at the same speed through out the journey what is the distance covered by it in 1 hour?

Solution : Distance travelled by the truck $= 92.5$ km.

Time required to travel this distance $= 2.5$ hours.

So distance travelled by it in 1 hour $= \frac{92.5}{2.5} = \frac{925}{25} = 37$ km.

Exercise - 6

1. Solve the following.

(i) 0.3×6 (ii) 7×2.7 (iii) 2.71×5

(iv) 19.7×4 (v) 0.05×7 (vi) 210.01×5

(vii) 2×0.86

2. Find the area of a rectangle whose length is 6.2 cm and breadth is 4 cm.

3. Solve the following.

(i)	21.3×10	(ii)	36.8×10	(iii)	53.7×10
(iv)	168.07×10	(v)	131.1×100	(vi)	156.1×100
(vii)	3.62×100	(viii)	43.07×100	(ix)	0.5×10
(x)	0.08×10	(xi)	0.9×100	(xii)	0.03×1000

4. A motor bike covers a distance of 62.5 km.consuming one litre of petrol. How much distance does it cover for 10 litres of petrol?

5. Solve the following.

(i)	1.5×0.3	(ii)	0.1×47.5	(iii)	0.2×210.8
(iv)	4.3×3.4	(v)	0.5×0.05	(vi)	11.2×0.10
(vii)	1.07×0.02	(viii)	10.05×1.05	(ix)	101.01×0.01
(x)	70.01×1.1				

6. Solve the following.

(i)	$2.3 \div 100$	(ii)	$0.45 \div 5$	(iii)	$44.3 \div 10$
(iv)	$127.1 \div 1000$	(v)	$7 \div 3.5$	(vi)	$88.5 \div 0.15$
(vii)	$0.4 \div 20$				

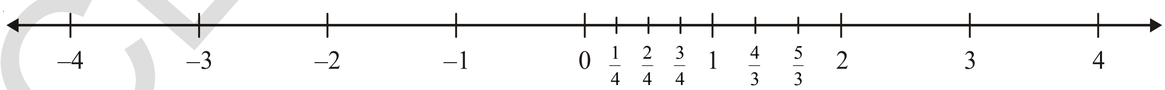
7. A side of a regular polygon is 3.5 cm in length. The perimeter of the polygon is 17.5 cm. How many sides does the polygon have?

8. A rain fall of 0.896 cm. was recorded in 7 hours, what was the average amount of rain per hour?

2.5 Introduction to Rational numbers

2.5.1 Positive fractional numbers:

We have learnt about integers and fractions. Let us see how the number line looks when both are marked on it.



We have $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}$ ..... between 0 and 1 on the number line. All these are numbers that are less than one. We call them as proper fractions and say that all proper fractions lie between 0 and 1.

Similarly, we know $\frac{4}{3}$ and $\frac{5}{3}$ would lie between 1 and 2. We can recall them as improper fractions. All these are called positive fractional numbers.

Do These

1. Write 5 more fractions between (i) 0 and 1 and (ii) 1 and 2.
2. Where does $4\frac{3}{5}$ lie on the number line?

On the left side of 0 we have integers –1, –2, –3 ....

Do the numbers increase or decrease as we move further left on the number line?

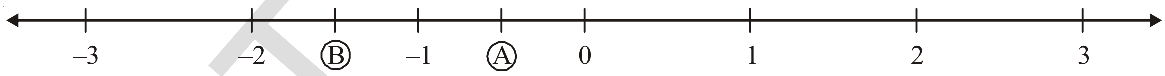
You know that number decreases as we move further left. The farther the number is from 0 on the left the smaller it is.

Do These

1. Find the greatest and the smallest numbers among the follwing groups?
   (i) 2, –2, –3, 4, 0, –5
   (ii) –3, –7, –8, 0, –5, –2
2. Write the following numbers in ascending order.
   (i) –5, –75, 3 – 2, 4, $\frac{3}{2}$ (ii) $\frac{2}{3}, \frac{3}{2}$, 0, –1, –2, 5

2.5.2 Negative fractional numbers

Consider the point A shown on the line.



It lies on the number line between 0 and –1. Is it more than 0 or less than 0?

Is it $\frac{1}{2}$? We cannot say it is $\frac{1}{2}$ as it is less than zero.

We write A as $-\frac{1}{2}$ since it is $\frac{1}{2}$ less than zero.

Similarly, B the mid point of –1 and –2 is. $-\frac{3}{2}$

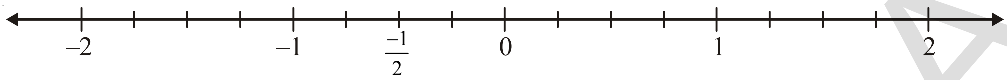
Neha found an easy way to represent $\frac{-9}{4}$. She first wrote it in a mixed fraction $\frac{-9}{4} = -2\frac{1}{4}$ and then represented it between –2 and –3.

You can see that negative fractional numbers like $-\frac{1}{2}, -\frac{3}{2}, -\frac{9}{4}$ give us points in between any two negative integers or between zero and a negative integer.

Do These

1. On the number line given below represent the following numbers.

(i) $-\frac{7}{2}$ (ii) $\frac{3}{2}$ (iii) $\frac{7}{4}$ (iv) $\frac{-7}{4}$ (v) $\frac{-1}{4}$ (vi) $\frac{1}{4}$



2. Consider the following numbers on a number line.

$27, -\frac{7}{8}, \quad \frac{11}{943}, \quad \frac{54}{17}, \quad -68, \quad -3, \quad -\frac{9}{6}, \quad \frac{7}{2}$

(i) Which of these are to the left of

(a) 0 (b) –2 (c) 4 (d) 2

(ii) Which of these would be to the right of

(a) 0 (b) –5 (c) $3\frac{1}{2}$ (d) $\frac{-5}{2}$

2.5.3 Rational Numbers

We know 0, 1, 2, 3, 4, 5 are whole numbers. We also know that –7, –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5 is a bigger collection of numbers called integers.

Rani says "All whole numbers are integers but the converse is not true." Do you agree with her? Rani is right as negative numbers like –6, –5, –4, –3, –2, –1 etc are integers but not whole numbers. Thus, all whole numbers are integers and all integers are not whole numbers.

We further know that positive fractional numbers like $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{5}{6}, \frac{11}{5}, \frac{8}{8}$ are ratios of whole numbers. All fractional numbers can in general be written as $\frac{w_1}{w_2}$ with the condition w_1 and w_2 are whole numbers and that w_2 is not equal to zero.

Try This

Write 5 fractional numbers and identify w_1 and w_2 in each of these.

Rational numbers are a bigger collection of numbers, which includes all integers, all positive fractional numbers and all negative fractional numbers.

The numbers $\frac{-7}{3}, \frac{-5}{2}, \frac{-7}{7}, \frac{-2}{7}, 0, \frac{1}{4}, \frac{4}{4}, \frac{17}{5}, \frac{6}{1}$ **etc. are all rational numbers.**

In all these we have a ratio of two integers, thus the numbers in the form of $\frac{p}{q}$**, where p and** q **are integers except that** q **is not equal to zero are called as rational numbers. The set of rational numbers is denoted by Q**

Try These

(i) Take any 5 integers and make all possible rational numbers with them.

(ii) Consider any 5 rational numbers. Find out which integers constitute them?

2.5.4 Comparing rational numbers

We know that $\frac{3}{4}$ and $\frac{9}{12}$ are equivalent fractional numbers. We also know that when we compare fractional numbers we convert each of them to equivalent fractional numbers and then compare the ones with a common denominator.

For example, to compare $\frac{3}{4}$ and $\frac{5}{7}$.

We write equivalent fractional numbers for both

$\frac{3}{4} = \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}, \frac{21}{28}$ and

$\frac{5}{7} = \frac{10}{14}, \frac{15}{21}, \frac{20}{28}$ ....

We can compare $\frac{21}{28}$ with $\frac{20}{28}$ as they have same denominators.

$\frac{21}{28}$ is bigger than $\frac{20}{28}$

Therefore, $\frac{3}{4} > \frac{5}{7}$

Try These

1. Write three more equivalent fractions of $\frac{3}{4}$ and mark them on the number line.

   What do you observe?

2. Do all equivalent fractions of $\frac{6}{7}$ represent the same point on the number line?

Now compare $\frac{-1}{2}$ and $\frac{-2}{3}$

We write equivalent fractions for both

$$\frac{-1}{2} = \frac{-2}{4}, \frac{-3}{6}, \frac{-4}{8} \quad \ldots\ldots \ \ldots\ldots$$

$$\frac{-2}{3} = \frac{-4}{6}, \frac{-6}{9} \quad \ldots\ldots \ \ldots\ldots$$

We can compare $\frac{-3}{6}$ and $\frac{-4}{6}$ as they have same denominators. $\frac{-4}{6} < \frac{-3}{6}$

$$\therefore \frac{-2}{3} < \frac{-1}{2}$$

Try These

1. Are $\frac{-1}{2}$ and $\frac{-3}{6}$ represent same point on the number line?

2. Are $\frac{-2}{3}$ and $\frac{-4}{6}$ equivalent

Eg: When we place them on the number line we find that they occupy the same point. Thus We can say that $\frac{-1}{2}$ and $\frac{-2}{4}$ are equivalent rationals.

Do These

1. Write 5 equivalent rational numbers to (i) $\frac{5}{2}$ (ii) $\frac{-7}{9}$ (iii) $-\frac{3}{7}$

2. Identify the equivalent rational numbers in each question:

   (i) $\frac{-1}{2}, \frac{-3}{4}, \frac{-2}{4}, \frac{-4}{8}$

   (ii) $\frac{1}{4}, \frac{3}{4}, \frac{5}{3}, \frac{10}{6}, \frac{2}{4}, \frac{20}{12}$

We can say that to get equivalent rational numbers we multiply or divide the integer in the numerator and in the denominator by the same number.

For example,

For $\frac{1}{5}$ we would have $\frac{1\times 2}{5\times 2}=\frac{2}{10}$ as one equivalent number another is $\frac{1\times 3}{5\times 3}=\frac{3}{15}$.

For $\frac{-2}{7}$ we would have $\frac{-2\times 2}{7\times 2}=\frac{-4}{14}$ as one and $-\frac{2\times 3}{7\times 3}=\frac{-6}{21}$ as another.

We can go on to build more such equivalent rational numbers, just by multiplying with $\frac{1}{1}=\frac{2}{2}=\frac{3}{3}=\frac{4}{4}\ldots$

Exercise - 7

1. Write any three equivalent rational numbers to each of the following

   (i) $\frac{2}{3}$ (ii) $-\frac{3}{8}$

2. What is the equivalent rational number for $\frac{-15}{36}$ with (i) denominator 12 (ii) numerator -75?

3. Mark the following rational numbers on the number line.

   (i) $\frac{1}{2}$ (ii) $\frac{3}{4}$ (iii) $\frac{3}{2}$ (iv) $\frac{10}{3}$

3. Find whether the following statements are true or false.

(i) Every integer is a rational number and vice versa ( )

(ii) In a rational number of the form $\frac{p}{q}$, q must be a non zero integer. ( )

(iii) Every decimal number can be represented as a rational number. ( )

(iv) $\frac{5}{7}, \frac{6}{7}, \frac{7}{7}$ are equivalent rational numbers. ( )

(v) Equivalent rational numbers of a positive rational numbers are all positive ( )

Looking back

1. We have learnt that for addition and subtraction of fractions; the fractions should be like fractions.

2. We have also learnt how to multiply fractions i.e, $\frac{\text{Product of numerators}}{\text{Product of denominators}}$

3. "of" can be used to represent multiplication. For example, $\frac{1}{3}$ of $6 = \frac{1}{\cancel{3}} \times \cancel{6}^{2} = 2$.

4. The product of two proper fractions is less than each of the fractions that are multiplied. The product of a proper and improper fraction is less than the improper fraction and greater than the proper fraction. The product of two improper fractions is greater than each of the fractions.

5. A reciprocal of a fraction is obtained by inverting the numerator and denominator.

6. We have seen how to divide two fractions.

   (i) While dividing a whole number with a fraction, we multiply the whole number with the reciprocal of that fraction.

   (ii) While dividing a fraction by a whole number we multiply the fraction with the reciprocal of the whole number.

   (iii) While dividing one fraction by another fraction, we multuiply the first fraction with the reciprocal of the second. So $\frac{3}{4} \div \frac{5}{7} = \frac{3}{4} \times \frac{7}{5} = \frac{21}{20}$.

(iii) While dividing one fraction by another fraction, we multuiply the first fraction with the reciprocal of the second. So $\frac{3}{4} \div \frac{5}{7} = \frac{3}{4} \times \frac{7}{5} = \frac{21}{20}$.

7. We also learnt how to multiply two decimal numbers. While multiplying two decimal numbers, we first multiply them as whole numbers. We then count the total number of digits to the right of the decimal point in both the decimal numbers being multiplied. Lastly, we put the decimal point in the product by counting the digits from its rightmost place.

8. To multiply a decimal number by 10, 100, 1000 ... etc., we move the decimal point in the number to the right by as many places as there are zeros in the numbers 10, 100, 1000 . . .

9. We have learnt how to divide decimal numbers.

   (i) To divide a decimal number by a whole number, we first divide them as whole numbers. We then place the decimal point in the quotient as in the decimal number.

   Note that here we are considering only those divisions in which the remainder is zero.

   (ii) To divide a decimal number by 10, 100, 1000 or any other multiple of 10, we shift the decimal point in the decimal number to the left by as many places as there are zeros in 10, 100, 1000 etc.,

   (iii) While dividing two decimal numbers, first shift the decimal point to the right by equal number of places in both, to convert the divisor to a whole number.

10. Rational numbers are a bigger collection of numbers, which includes all integers, all positive fractional numbers and all negative fractional numbers. In all these we have a ratio of two integers, thus $\frac{p}{q}$ represents a rational number.

In this i) p, q are integers and

ii) $q \neq 0$

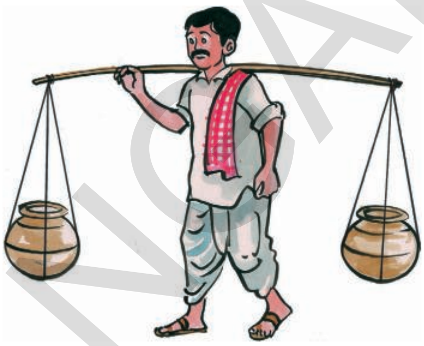
The set of rational numbers is denoted by Q

John Napier (Scotland)
1550-1617 AD
Found logarithms.
Introduced napier rods for multiplictions.
Also introduced System of decimal fractions.

SIMPLE EQUATIONS

3

3.0 Introduction



You have already come across simple equations like $4x = 44$, $2m = 10$ and their solutions in Class VI. You have seen how these equations help us in solving puzzles and daily life problems. Let us review what we have already learnt about simple equations and their solutions through the following exercise.

Exercise - 1

1. Write L.H.S and R.H.S of the following simple equations.

(i)	$2x = 10$	(ii)	$2x–3 = 9$
(iii)	$4z + 1 = 8$	(iv)	$5p + 3 = 2p + 9$
(v)	$14 = 27 – y$	(vi)	$2a – 3 = 5$
(vii)	$7m = 14$	(viii)	$8 = q + 5$

2. Solve the following equations by trial and error method.

(i)	$2 + y = 7$	(ii)	$a – 2 = 6$
(iii)	$5m = 15$	(iv)	$2n = 14$

3.1 Equation - Weighing balance

You have seen in class VI that an equation is compared with a weighing balance with equal weights on both sides.

What will happen if the left pan of a weighing balance holds 5 kg and the right pan holds 2 kg?

What will happen if the left pan of a weighing balance holds 3 kg and the right pan holds 7 kg?

What will happen if the left pan of a weighing balance holds 3 kg and the right pan holds 3 kg?

A weighing balance needs to have equal weights on both sides to be perfectly balanced.

The same principle will hold in an equality.

Consider the equality $12–2 = 6 + 4$

Here-

LHS = $12 – 2 = 10$ and

RHS = $6 + 4 = 10$

Since both sides are equal, the equality holds.

What will happen if we add 3 on both sides of an equation? Will the values of both sides remain equal? Will the values be equal if 10 is added? Try with some other number of your choice.

What will happen if we subtract 5 from both sides of the equation? Will both sides remain equal? Will the values be equal if 7 is subtracted? Try with some other numbers of your choice.

What will happen if we multiply both sides by 6? Will both sides remain equal? Will they be equal if 8 multiplied by 8? Try with some other numbers of your choice.

What will happen if we divide both sides of the equation by 5? Will both sides remain equal? Will they be equal if both sides are divided by 2?

You will find that answer is 'yes' in all cases. If the same number is added or subtracted on both sides or if both sides of the equality are either multiplied or divided by same number ; the equality remains unchanged. This principle of equality is going to help in solving equations ahead.

3.2 Solving equations

You have already learnt how to solve equations using the trial and error method. Now we will use the above principles of equality to solve equations in a much lesser time.

To solve equations we first need to separate the numerical terms from the terms containing variables/unknowns by taking them on the two different sides of the equality and then use the principles of equality.

Let us study the examples given below.

Example 1 : Solve $x + 3 = 7$

Given equation is

Solution : $x + 3 = 7$ .... (1)

The L.H.S of the equation is $x + 3$.

Total value of L.H.S. is 3 more than x

To find the value of 'x' we have to remove 3 from the LHS. Thus, we need to subtract 3 from the LHS. If 3 is subtracted from LHS it should also be subtracted from RHS, to balance the equality.

We have, $x + 3 = 7$

$$x + 3 - 3 = 7 - 3$$

$$x = 7 - 3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

$$x = 4$$

Thus, $x = 4$.

From (1) and (2) it is clear that removing '+3' from LHS is equivalent to 'subtracting 3' from the RHS. That means '+3' on LHS transforms as '–3' to RHS.

Check: let us substitute 4 for x in the given equation and find whether LHS = RHS.

LHS $= x + 3$

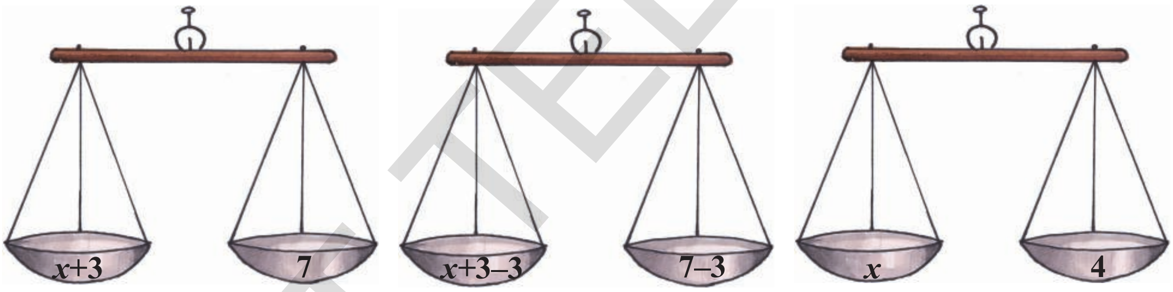
$= 4 + 3$ (substituting $x = 4$)

$= 7$

RHS $= 7$

LHS = RHS.

Let us also understand the above solution with a weighing balance:



Example 2 : Solve $y - 7 = 9$

Solution : $y - 7 = 9 \ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (1)

Here the L.H.S of the equation is $y - 7$

So to get the value of 'y' we have to add 7 on both sides of the equation.

Therefore, $y - 7 + 7 = 9 + 7$

$$y = 9 + 7 \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

$$y = 16$$

Thus, $y = 16$.

From (1) and (2) it is clear that '–7' on LHS transforms to RHS as '+7'.

Check : Substitute '16' for 'y' and check whether LHS = RHS.

Example 3 : Solve $5x = -30$

Solution : $5x = -30$ .... (1)

$$\frac{5x}{5} = \frac{-30}{5} \quad \text{(dividing both sides by 5)}$$

$$x = \frac{-30}{5} \text{ .... (2)}$$

$\therefore x = -6$

From (1) and (2) it is clear that multiplier '5' on LHS transforms to RHS as 'divisor'.

Check: Substitute $x = -6$ and check whether LHS = RHS.

Example 4 : Solve $\frac{z}{6} = -3$

Solution : $\frac{z}{6} = -3$ .... (1)

$$6\left(\frac{z}{6}\right) = 6 \times (-3) \quad \text{(multiplying both sides by 6)}$$

$$z = 6 \times (-3) \text{ .... (2)}$$

$\therefore \quad z = -18$

From (1) and (2) it is clear that divison '6' on LHS transforms to RHS as 'multipier 6'.

Check : Substitute $z = -18$ and check whether LHS = RHS.

Example 5 : Solve $3x + 5 = 5x - 11$

Solution : $3x + 5 = 5x - 11$

$3x + 5 - 5x = 5x - 11 - 5x$ (subtracting 5x from both sides)

$-2x + 5 = -11$

$-2x + 5 - 5 = -11 - 5$ (subtracting '5' from both sides)

$-2x = -16$

$$\frac{-2x}{-2} = \frac{-16}{-2} \quad \text{(Dividing both sides by '}-2\text{')}$$

$\therefore x = 8$

Check : Substituting x=8 in the given equation:

LHS = $3x + 5 = 3(8) + 5 = 24 + 5 = 29$

RHS = $5x - 11 = 5(8) - 11 = 40 - 11 = 29$

$\therefore$ LHS = RHS

Thus, in transforming terms from L.H.S. to R.H.S.

'+ quantity' becomes '– quantity'

'– quantity' becomes '+ quantity'

'× quantity' becomes ÷ quantity

'÷ quantity' becomes ' × quantity'

Example 6 : Solve $12 = x + 3$

Here if 12 is moved from LHS to RHS it becomes –12 and if x+3 is moved from RHS to LHS it becomes $-x - 3$.

i.e $-x - 3 = -12$

Multiplying both sides by –1

$-1 (-x - 3) = -1 (-12)$

$x + 3 = 12$

$x = 12 - 3$

$\therefore x = 9$

Therefore, if both LHS and RHS of an equation are moved (transposed) from one side to another side, the values of terms remain same.

Exercise - 2

1. Solve the following equations without transposing and check your result.

(i)	$x + 5 = 9$	(ii)	$y - 12 = -5$
(iii)	$3x + 4 = 19$	(iv)	$9z = 81$
(v)	$3x + 8 = 5x + 2$	(vi)	$5y + 10 = 4y - 10$

2. Solve the following equations by transposing the terms and check your result.

(i)	$2 + y = 7$	(ii)	$2a - 3 = 5$
(iii)	$10 - q = 6$	(iv)	$2t - 5 = 3$
(v)	$14 = 27 - x$	(vi)	$5 (x+4) = 35$
(vii)	$-3x = 15$	(viii)	$5x - 3 = 3x - 5$
(ix)	$3y + 4 = 5y - 4$	(x)	$3 (x - 3) = 5 (2x + 1)$

3.3 Usage of algebraic equations in solving day to day problems.

Let us look at the following examples:

(i) The total number of boys and girls in a class is 52. If the number of girls is 10 more than boys, find the number of boys?

(ii) The present age of Ramu's father is three times that of Ramu. After five years the sum of their ages will be 70 years. Find their present ages.

(iii) A purse contains ₹ 250 in the denomination of ₹10 and ₹ 50. If the number of ₹10 notes is one more than that of ₹50 notes find the number of notes of each denomination.

(iv) Length of a rectangle is 8 m less than twice its breadth. The perimeter of the rectangle is 56 m. Find its length and breadth.

Likes in all the problems given above, we can use simple equations to solve various problems of day to day life. The following steps can be followed in doing so

Step 1: Read the problem carefully.

Step 2 : Denote the unknown or the quantity to be found with some letters such as x, y, z, u, v, w, p, t.

Step 3: Write the problem in the form of an algebraic equation by making a relation among the quantities.

Step 4: Solve the equation.

Step 5: Check the solution.

Example 7: Total number of the boys and girls in a class is 52. If the number of girls is 10 more than that of boys, find the number of boys?

Solution : Let us assume the number of boys to be x.

The number of girls will be $x + 10$.

The total number of boys and girls $= x + (x + 10)$

$= x + x + 10$

$= 2x + 10$

According to the question, the total number of boys and girls is 52.

Therefore, $2x + 10 = 52$

Solving this equation, $2x + 10 = 52$

$2x = 52 - 10$ (transposing 10 from LHS to RHS)

$2x = 42$

$x = \dfrac{42}{2}$ (transposing 2 from LHS to RHS)

$\therefore\ x = 21$

Thus, the number of boys = 21

and the number of girls = 21 + 10 = 31

Check : 21 + 31 = 52 i.e. the total number of boys and girls is 52.

And 31 – 21=10 i.e. the number of girls is 10 more than the number of boys.

Example 8: The present age of Ramu's father is three times that of Ramu. After five years the sum of their ages would be 70 years. Find their present ages.

Solution : Let Ramu's present age = x years.

Then the present age of his father = $3x$ years.

After 5 years Ramu's age = $x+5$ years.

His father's age = $3x + 5$ years.

Sum of their ages after 5 years is = $(x + 5) + (3x + 5) = 4x + 10$ years.

According to the problem

Sum of their ages $4x + 10 = 70$

$4x = 70 - 10$

$4x = 60$

$$x = \frac{60}{4} = 15$$

Thus, Ramu's present age = 15 years.

So, present age of his father = 3 × 15 years = 45 years.

Check : 45 is three times of 15 i.e., at present Ramu's father is 3 times that of Ramu,

After 5 years Ramu's age = 15 + 5 = 20 years and his father's age = 45 + 5 = 50 years.

Sum of their ages 20 + 50 = 70 years.

Example 9 : A purse contains ₹250 in the denomination of ₹10 and ₹ 50. If the number of ₹10 notes is one more than that of ₹50 notes, find the number of notes of each denomination.

Solution : Let the number of ₹50 notes = x

Then the total value of ₹50 notes = $50x$

Number of ₹ 10 notes = $x + 1$

Then the total value of ₹ 10 notes $= 10\,(x+1)$

$\therefore$ Total value of money $= 50x + 10\,(x+1)$

$= 50x + 10x + 10$

$= 60x + 10$

Given, total value of the money that the purse contains is ₹250

Therefore, $60x + 10 = 250$

$60x = 250 - 10$

$60x = 240$

$$x = \frac{240}{60}$$

$\therefore x = 4$

Thus, the number of ₹50 notes = 4

Number of ₹10 notes = 4 + 1 = 5

Check : ₹10 notes (5) are one more than ₹50 notes (4).

Value of the money $= (50\times4) + (10\times5)$

$= 200 + 50$

$=$ ₹250

Example 10: Length of a rectangle is 8 m less than twice its breadth. If the perimeter of the rectangle is 56 m, find its length and breadth.

Solution : Let the breadth of the rectangle $= x$ m.

Twice the breadth $= 2x$ m.

Therefore, its length $= (2x - 8)$ m. (by problem)

Perimeter of the rectangle = 2 (length + breadth)

Thus, perimeter $= 2\,(2x - 8 + x)$ m.

$= 2\,(3x - 8)$ m.

$= (6x - 16)$ m.

Given, the perimeter of the rectangle is 56 m.

Therefore, $6x - 16 = 56$

$$6x = 56 + 16$$

$$6x = 72$$

$$x = \frac{72}{6}$$

$$\therefore\ x = 12$$

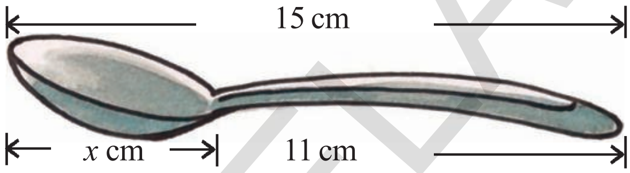
Breadth of the rectangle = 12 m.

Length of the rectangle = 2 × 12 – 8 = 16 m.

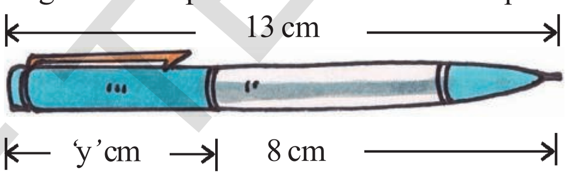
Check : Perimeter = 2 (16 + 12) = 2 × 28 = 56 m.

Exercise 3

1 Write the information given in the picture in the form of an equation. Also, find 'x' in the following figure.



2. Write the information given in the picture in the form of an equation. Also, find 'y' in the following figure.



3. If we add 7 to twice a number, we get 49. Find the number.

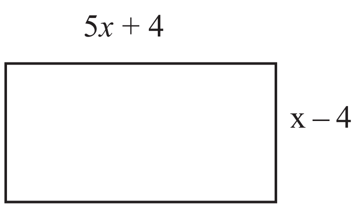
4. If we subtract 22 from three times a number, we get 68. Find the number.

5. Find the number when multiplied by 7 and then reduced by 3 is equal to 53.

6. Sum of two numbers is 95. If one exceeds the other by 3, find the numbers.

7. Sum of three consecutive integers is 24. Find the integers.

8. Find the length and breadth of the rectangle given below if its perimeter is 72m.



9. Length of a rectangle exceeds its breadth by 4 m. If the perimeter of the rectangle is 84 m, find its length and breadth.

10. After 15 years, Hema's age will become four times that of her present age. Find her present age.

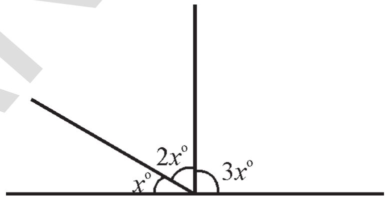
11. A sum of ₹.3000 is to be given in the form of 63 prizes. If the prize money is either ₹.100 or ₹.25. Find the number of prizes of each type.

12. A number is divided into two parts such that one part is 10 more than the other. If the two parts are in the ratio 5:3, find the number and the two parts.

13 . Suhana said, "multiplying my number by 5 and adding 8 to it gives the same answer as subtracting my number from 20".
Find Suhana's numbers.

14. The teacher tells the class that the highest marks obtained by a student in her class is twice the lowest marks plus 7. The highest mark is 87. What is the lowest mark?

15. In adjacent figure find the magnitude of each of the three angles formed?



(Hint: Sum of all angles at a point on a line is 180^o)

16. Solve the following riddle:

I am a number

Tell my identity.

Take me two times over

And add a thirty six.

To reach a century

You still need four.

Looking Back

- Simple equations help in solving various problems in daily life.
- For balancing an equation we
  (i) add the same number on both the sides or
  (ii) subtract the same number form both the sides or
  (iii) multiply both sides with the same number or
  (iv) divides both the sides by the same number, so that the equality remains undisturbed.
- An equation remains same if the LHS and the RHS are interchanged.

LINES AND ANGLES

4

4.0 Introduction

You have learnt some geometrical ideas in previous classes. Let us have fun trying some thing we have already done.

Exercise - 1

1. Name the figures drawn below.

(i) (ii) (iii) (iv)

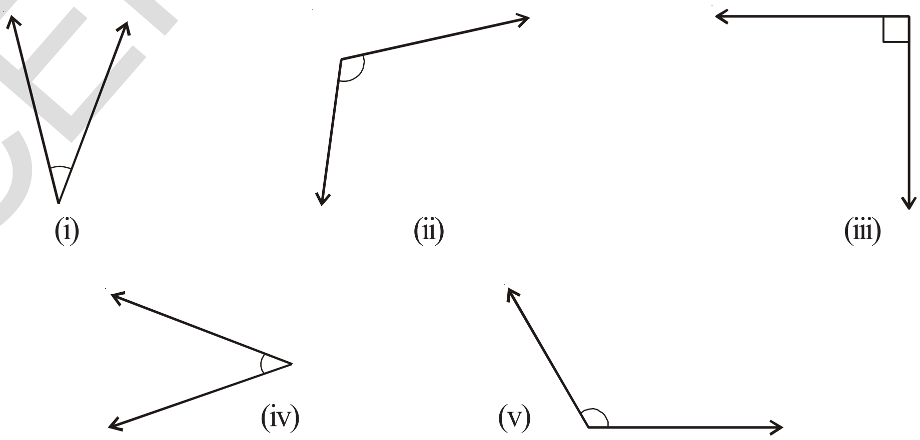
2. Draw the figures for the following.

(i) $\overrightarrow{OP}$ (ii) Point X (iii) $\overleftrightarrow{RS}$ (iv) $\overline{CD}$

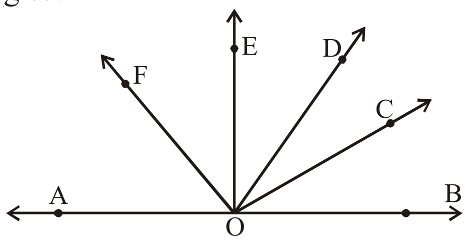
3. Name all the possible line segments in the figure.

4. Write any five examples of angles that you have observed arround.
Example : The angle formed when a scissor is opened.

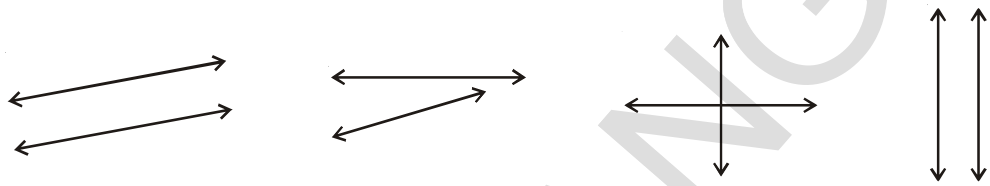
5. Identify the following given angles as acute, right or obtuse.



6. Name all the possible angles you can find in the following figure. Which are acute, right, obtuse and straight angles?

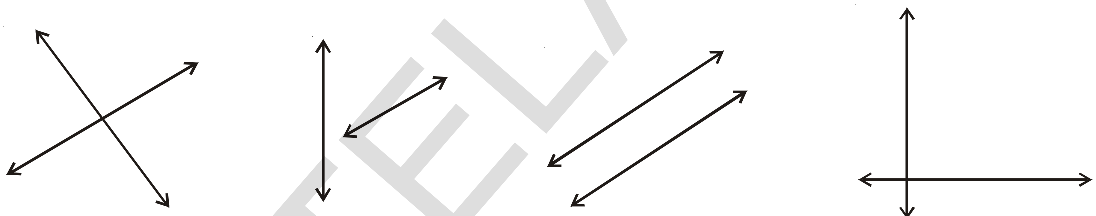


7. Which of the following pairs of lines are parallel? Why?



(i) (ii) (iii) (iv)

8. Which of the following lines are intersecting?



(i) (ii) (iii) (iv)

4.1 Learning about Pairs of Angles

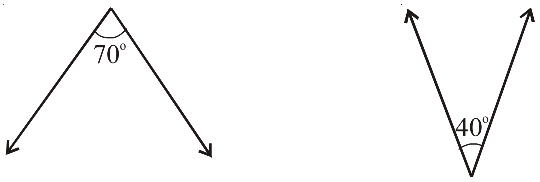
We have learnt how to identify some angles in the previous chapter. Now we will learn about some more angles as well as various pairs of angles.

4.1.1 Complementary Angles

When the sum of two angles is equal to 90°, the angles are called complementary angles.

These are complementary angles, as their sum is $30^0 + 60^0 = 90^0$.

We can also say that the complement of 30^0 is 60^0 and the complement of 60^0 is 30°.



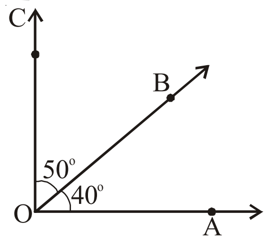
In the above figures, the sum of the two angles is $70^0 + 40^0 \neq 90^0$. Thus, these angles are not a pair of complementary angles.

Try This

Draw five pairs of complementary angles of your choice.

Do This

Draw an angle $\angle AOB = 40^0$. With the same vertex 'O' draw $\angle BOC = 50^0$, taking $\overrightarrow{OB}$ as initial ray as shown in the figure.

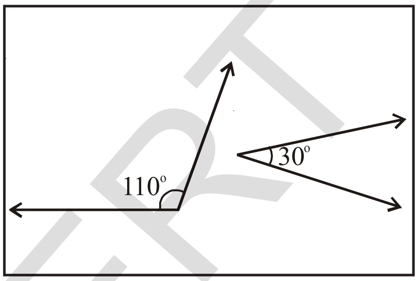


Since the sum of these angles is 90^o, they together form a right angle.

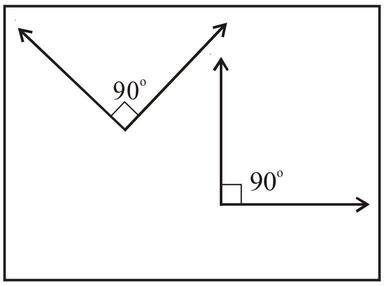
Take another pair 60^0 and 50^0 and join in the same way. Do they form complementary angles? Why? Why not?

Exercise - 2

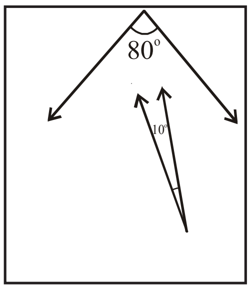
1. Which of the following pairs of angles are complementary?



(i)



(ii)



(iii)

2. Find the complementary angles of the following.

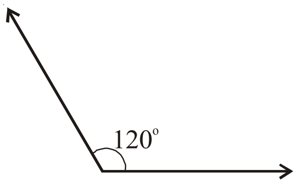
   (i) 25^0 (ii) 40^0 (iii) 89^0 (iv) 55^0

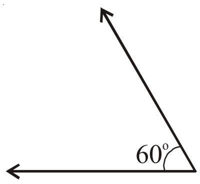
3. Two angles are complement to each other and are also equal. Find them.

4. Manasa says, "Each angle in any pair of complementary angles is always acute". Do you agree? Give reason.

4.1.2 Supplementary Angles

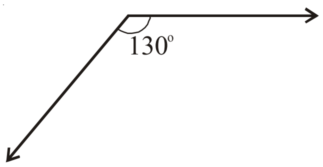
When the sum of two angles are equal to 180^0, then the angles are called supplementary angles.

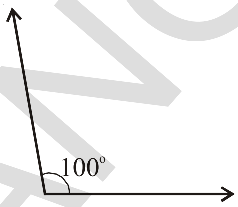




These are a pair of supplementary angles as their sum is $120^0 + 60^0 = 180^0$.

We say that the supplement of 120^0 is 60^0 and the supplement of 60^0 is 120^0.

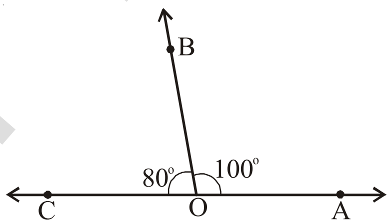




130^0 and 100^0 angles are not a pair of supplementary angles. Why?

Do This

Draw an angle $\angle AOB = 100^0$. With the same vertex O, draw $\angle BOC = 80^0$ such that $\overrightarrow{OB}$ is common to two angles.



You will observe that the above angles form a straight angle that is 180^0.

Thus, the angles 100^0 and 80^0 are supplementary to each other.

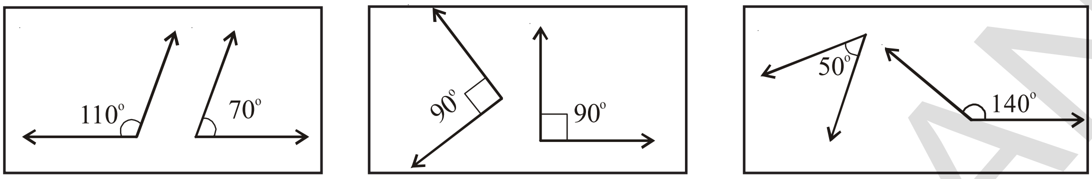
Are 130^0 and 70^0 supplementary angles? Why? Why not?

Try This

Write any five pairs of supplementary angles of your choice.

Exercise - 3

1. Which of the following pairs of angles are supplementary?



(i) (ii) (iii)

2. Find the supplementary angles of the given angles.

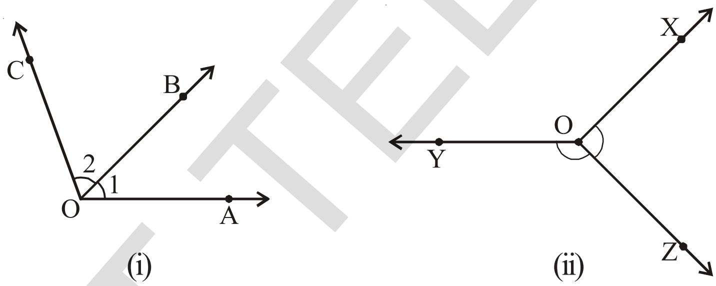
   (i) 105^0 (ii) 95^0 (iii) 150^0 (iv) 20^0

3. Two acute angles cannot form a pair of supplementary angles. Justify.

4. Two angles are equal and supplementary to each other. Find them.

4.1.3 Adjacent Angles

The angles having a common arm and a common 'vertex' are called as adjacent angles.

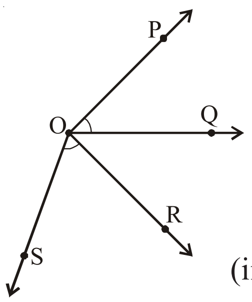


(i) (ii)

The angles $\angle AOB$ and $\angle BOC$ in Figure (i) are adjacent angles, as they have a common vertex 'O' and common arm $\overrightarrow{OB}$.

Are the angles in Figure (ii) adjacent angles? Which is the common vertex and which is the common arm?

Now, look at figure (iii).



(iii).

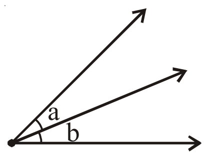
Are $\angle POQ$ and $\angle ROS$ adjacent angles. Why? Why not?

Which angles are adjacent to each other in the adjacent figure?

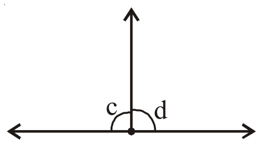
Why do you think they are adjacent angles?

Exercise - 4

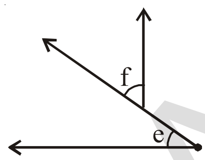
1. Which of the following are adjacent angles?



(i)

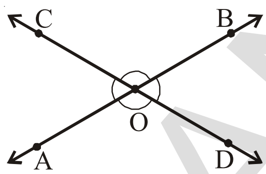


(ii)



(iii)

2. Name all pairs of adjacent angles in the figure. How many pairs of adjacent angles are formed? Why these angles are called adjacent angles?



3. Can two adjacent angles be supplementary? Draw figure.

4. Can two adjacent angles be complementary? Draw figure.

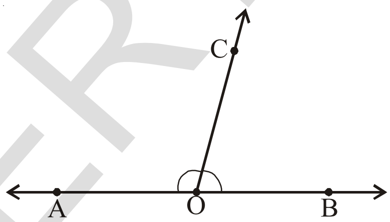
5. Give four examples of adjacent angles in daily life.

   Example : Angles between the spokes at the centre of a cycle wheel.

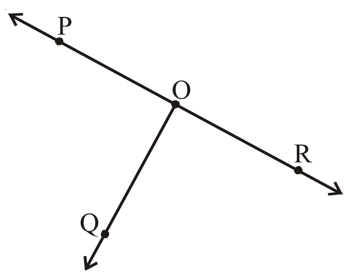
   (i) ____________________ (ii) ____________________

   (iii) ____________________ (iv) ____________________

4.1.3 (a) Linear Pair



(i)



(ii)

Look at Figure (i). $\angle AOC$ and $\angle BOC$ are adjacent angles. What is the sum of these angles?

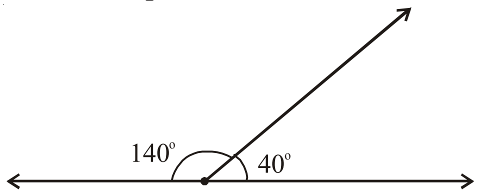
These angles together form a straight angle. Similarly, look at Figure (ii). Do $\angle POQ$ and $\angle ROQ$ together form a straight angle?

A pair of adjacent angles whose sum is a straight angle (or 180^0) is called a Linear Pair.

Do This

Two adjacent angles are 40° and 140°. Do they from a linear pair?

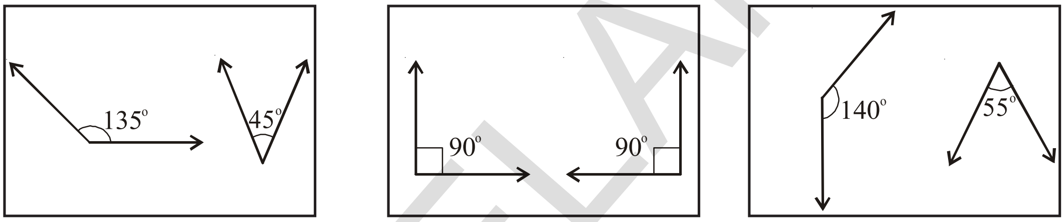
Draw a picture and check. Renu drew the picture like this.



Has she drawn correctly? Do these adjacent angles form a linear pair?

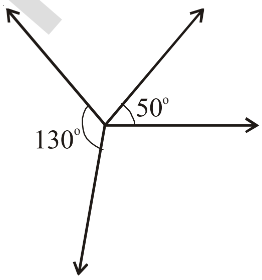
Exercise - 5

1. Draw the following pairs of angles as adjacent angles. Check whether they form linear pair.



(i) (ii) (iii)

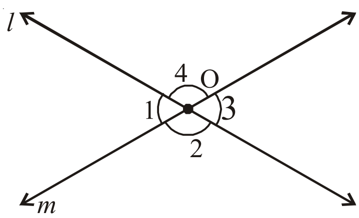
2. Niharika took two angles- 130^0 and 50^0 and tried to check whether they form a linear pair. She made the following picture.



Can we say that these two angles do not form a linear pair? If not, what is Niharika's mistake?

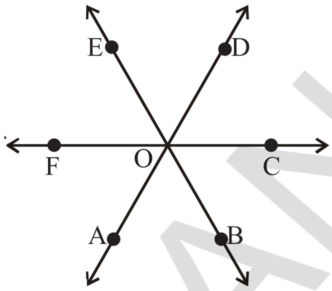
4.1.4 Vertically Opposite Angles

When two lines intersect, the angles that are formed opposite to each other at the point of intersection (vertex) are called vertically opposite angles.



In above figure two lines 'l' and 'm' intersect each other at 'O'. Angle $\angle 1$ is opposite to angle $\angle 3$ and the other pair of opposite angles is $\angle 2$ and $\angle 4$. Thus, $\angle 1$, $\angle 3$ and $\angle 2$, $\angle 4$ are the two pairs of vertically opposite angles.

Which are the vertically opposite angles in the adjacent figure.



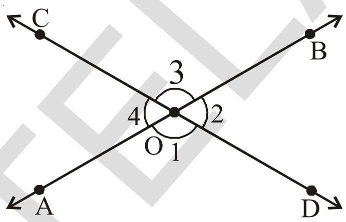
Do This

Draw two lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ such that they intersect at point 'O'.

Trace the figure given below on a tracing paper. Place the traced figure over the figure given below and rotate it such that $\angle BOD$ coincides $\angle AOC$.

Observe the angles $\angle AOD$ and $\angle BOC$ also $\angle AOC$ and $\angle BOD$.

You will notice that $\angle AOD = \angle BOC$ and $\angle AOC = \angle BOD$.

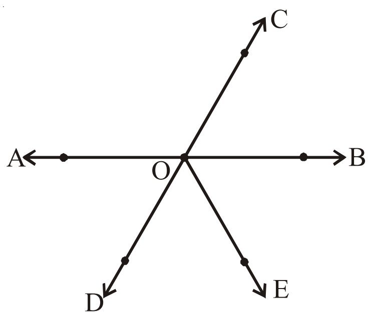


We can conclude that vertically opposite angles are equal.

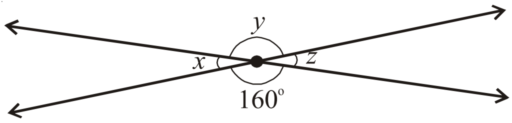
Note : Take two straws. Fix them at a point 'O' with a pin. Place them such that the straw on top covers the one below. Rotate the straws. You will find that they make vertically opposite angles.

Exercise - 6

1. Name two pairs of vertically opposite angles in the figure.



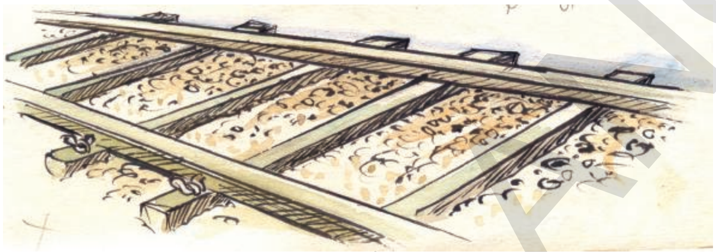
2. Find the measure of x, y and z without actually measuring them.



3. Give some examples of vertically opposite angles in your surroundings.

4.2 Transversal

You might have seen railway track. This figure is an example when two lines are intersected by a transversal.



A line which intersects two or more lines at distinct points is called a transversal.

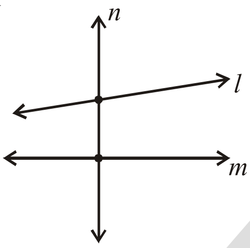


Figure (i)

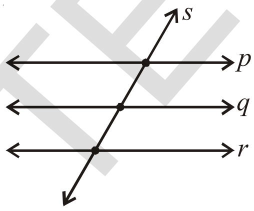


Figure (ii)

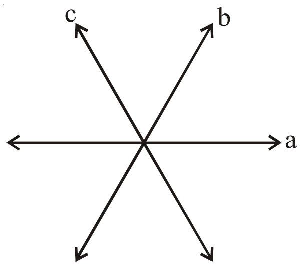


Figure (iii)

In Fig (i) two lines 'l' and 'm' are intersected by a line 'n', at two distinct points.

Therefore, 'n' is a transversal to 'l' and m''.

In Fig (ii) three lines 'p', 'q' and 'r' are intersected by a line 's', at three distinct points.

So, 's' is a transversal to 'p' 'q' and 'r'.

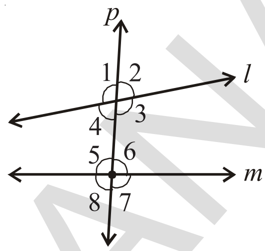
In Fig (iii) two lines 'a' and 'b' are intersected by a line 'c'. The point of intersection of 'c' is the same as that of 'a' and 'b'. The three lines are thus intersecting lines and none of them is a transversal to the other.

Try This

How many transversals can be drawn for two distinct lines?

4.2.1 Angles made by a transversal

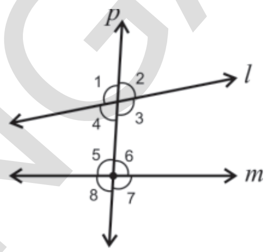
When a transversal cuts two lines, 8 angles are formed. This is because at each intersection 4 angles are formed. Observe the figure.



Here 'l' and 'm' are two lines intersected by the transversal 'p'. Eight angles $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$, $\angle 7$ and $\angle 8$ are formed.

Angles $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$, are lying inside 'l' and 'm'.

They are thus called interior angles. The angles $\angle 1$, $\angle 2$, $\angle 7$ and $\angle 8$ are on the outside of the lines 'l' and 'm'. They are thus called exterior angles.



Look at adjacent figure.

$\angle 1$, $\angle 2$, $\angle 7$ and $\angle 8$ are exterior angles.

$\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$ are interior angles.

We have learnt about vertically opposite angles and noted the fact that they are equal.

Renu looked at figure for vertically opposite angles, and said $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$.

Which are the other two pairs of vertically opposite angles?

She said that each exterior angle is paired with an vertically opposite angle which is in the interior. The angles in these pairs are equal. Do you agree with Renu?

Do This

1. Identify the transversal in Figure (i) and (ii).

   Identify the exterior and interior angles and fill the table given below:

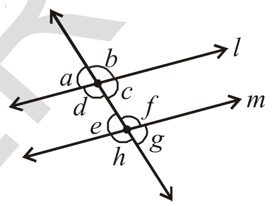


Figure (i)

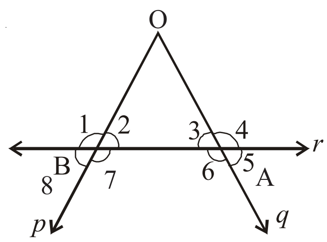
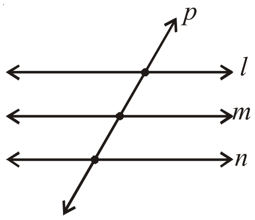
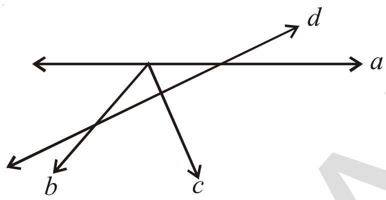


Figure (ii)

Figure	Transversal	Exterior angles	Interior angles
(i)			
(ii)			

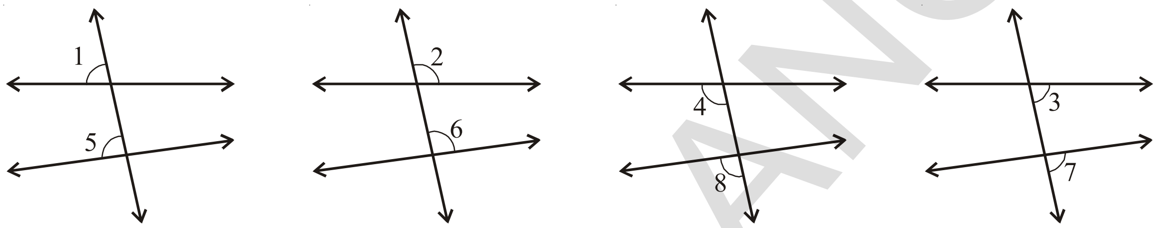
2. Consider the following lines. Which line is a transversal? Number and list all the angles formed. Which are the exterior angles and which are the interior angles?





4.2.1 (a) Corresponding Angles

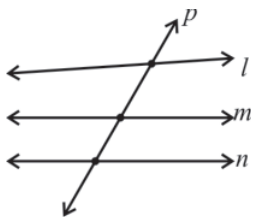
Look at figures (i), (ii), (iii) and (iv) below-



(i) (ii) (iii) (iv)

Consider the pairs of angles ($\angle 1$, $\angle 5$), ($\angle 2$, $\angle 6$), ($\angle 4$, $\angle 8$), ($\angle 3$, $\angle 7$). Is there something common among these pairs of angles? These angles lie on different vertices. They are on the same side of the transversal and in each pair one is an interior angle and the other is an exterior angle.

Each of the above pair of angles is called corresponding angles.



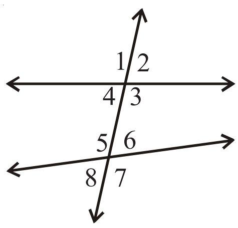
What happens when a line is transversal to three lines? Which are the corrosponding angles in this case? What is the number of exterior and interior angles in this case?

What would happen if number of lines intersected by the transversal becomes 4,5 and more?

Can you predict the number of exterior and the interior angles that are corrosponding to each other.

4.2.1 (b) Interior and Exterior Alternate Angles

Look at the adjacent figure. Find the angles which have the following three properties:

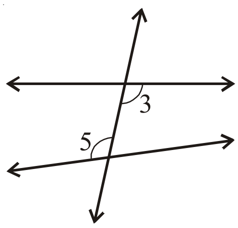


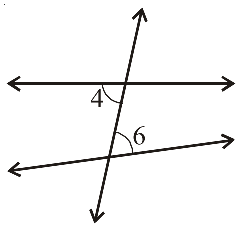
(i) Have different vertices.

(ii) Are on the either side of the transversal

(iii) Lie 'between' the two lines (i.e are interior angles).

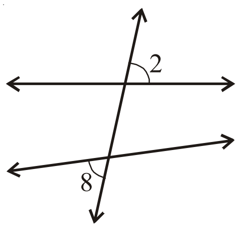
Such pairs of angles are called interior alternate angles.

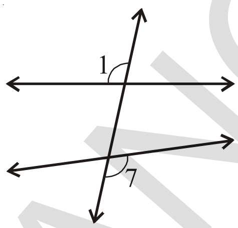




The pairs of angles (∠3, ∠5) and (∠4, ∠6) are the two pairs of interior alternate angles.

Similarly, you may find two pairs of exterior alternate angles.

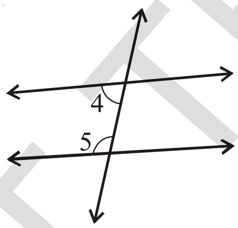


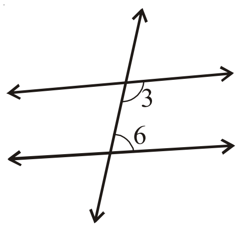


The pairs of angles (∠2, ∠8) and (∠1, ∠7) are called alternate exterior angles.

4.2.1 (c) Interior Angles on the same side of the transversal

Interior angles can be on the same side of the transversal too.

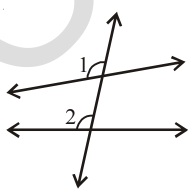




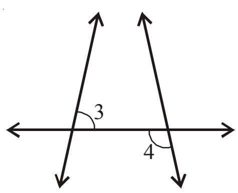
Angles ∠4, ∠5 and ∠3, ∠6 are the two pairs of interior angles on the same side of the transversal.

Do This

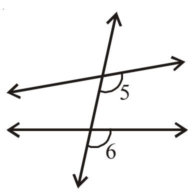
1. Name the pairs of angles in each figure by their property.



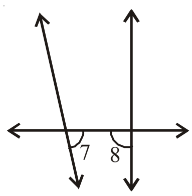
(i)



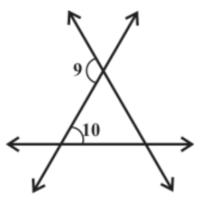
(ii)



(iii)



(iv)

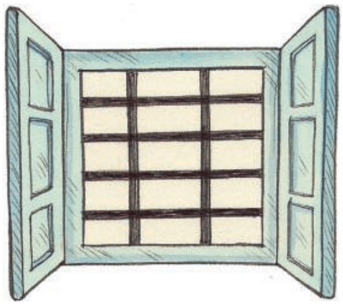


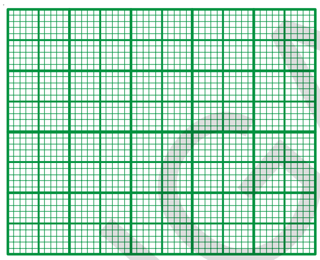
(v)

4.2.2 Transversal on parallel lines

You know that two coplanar lines which do not intersect are called parallel lines. Let us look at transversals on parallel lines and the properties of angles on them.

Look at the pictures of a window and a graph paper.





These give examples for parallel lines with a transversal.

Do This

Take a ruled sheet of paper. Draw two lines '*l*' and 'm' parallel to each other.

Draw a transversal 'p' on these lines.

Label the pairs of corresponding angles as shown in Figures (i), (ii), (iii) and (iv).

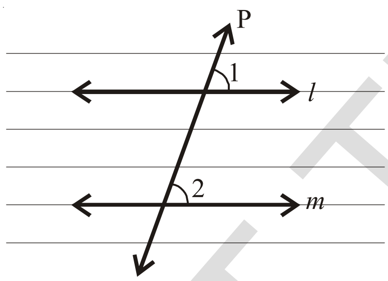


Figure (i)

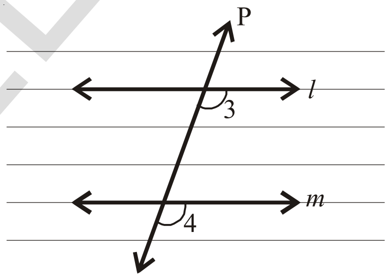


Figure (ii)

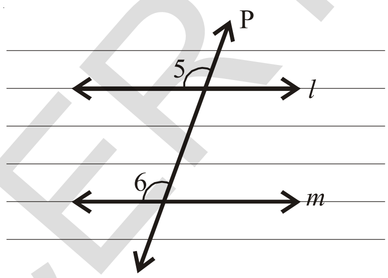


Figure (iii)

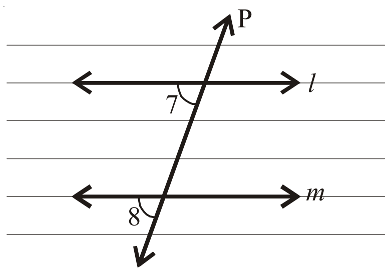


Figure (iv)

Place the tracing paper over Figure (i). Trace the lines '*l*', 'm' and 'p'. Slide the tracing paper along 'p', until the line '*l*' coincides with line 'm'. You find that $\angle 1$ on the traced figure coincides with $\angle 2$ of the original figure.

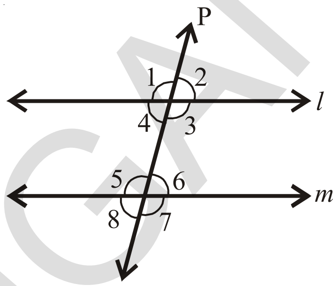
Thus $\angle 1 = \angle 2$

Are the remaining pairs of corrosponding angles equal? Check by tracing and sliding.

You will find that if a pair of parallel lines are intersected by a transversal then the angles in each pair of corresponding angles are equal.

We can use this 'corresponding angles' property to get another result.

In the adjacent figure '*l*' and '*m*' are a pair of parallel lines and 'p' is a transversal.



As all pairs of corresponding angles are equal,

$\angle 1 = \angle 5$

But $\angle 1 = \angle 3$ (vertically opposite angles)

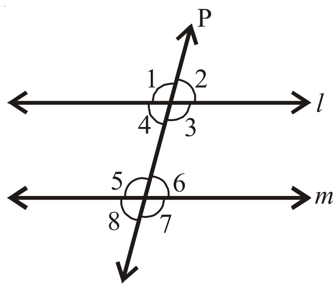
Thus, $\angle 3 = \angle 5$

Similarly, you can show that $\angle 4 = \angle 6$.

Therefore, if a pair of parallel lines are intersected by a transversal then the angles in each pair of alternate interior angles are equal.

Do you find the same result for exterior alternate angles? Try.

Now, we find one more interesting result about interior angles on the same side of the transversal.



In the adjacent figure '*l*' and '*m*' a pair of parallel lines intersected by a transversal '*p*'.

$\angle 3 = \angle 5$ (alternate interior angles)

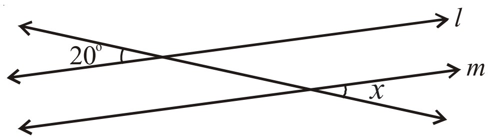
But $\angle 3 + \angle 4 = 180^0$ (Why?)

Thus, $\angle 4 + \angle 5 = 180^0$

Similarly $\angle 3 + \angle 6 = 180^0$ (Give reason)

Thus, if a pair of parallel lines are intersected by a transversal then the angles in each pair of interior angles on the same side of the transversal are supplementary.

Example 1 : In the figure given below, '*l*' and '*m*' are a pair of parallel lines. '*p*' is a transversal. Find '*x*'.



Solution : Given $l \parallel m$, p is a transversal.

$\angle x$ and 20^0 are a pair of exterior alternate angles, therefore are equal.

Thus, $\angle x = 20^0$.

Do This

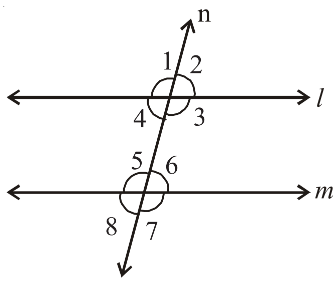


Figure (i)

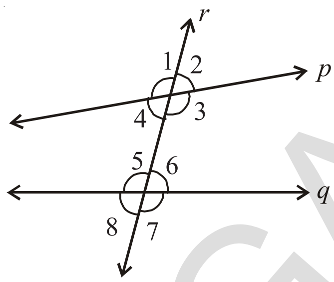


Figure (ii)

Trace the copy of figures (i) and (ii) in your note book. Measure the angles using a protractor and fill the table.

Table 1 : Fill the table with the measures of the corresponding angles.

Figure	Pairs of corresponding angles			
	1st pair	2nd pair	3rd pair	4th pair
(i)	$\angle 1 =$	$\angle 2 =$	$\angle 3 =$	$\angle 4 =$ ....
	$\angle 5 =$	$\angle 6 =$	$\angle 7 =$	$\angle 8 =$ ....
(ii)	$\angle 1 =$	$\angle 2 =$	$\angle 3 =$	$\angle 4 =$ ....
	$\angle 5 =$	$\angle 6 =$	$\angle 7 =$	$\angle 8 =$ ....

Find out in which figure the pairs of corresponding angles are equal?

What can you say about the lines '*l*' and '*m*' ?

What can you say about the lines '*p*' and '*q*' ?

Which pair of lines is parallel?

Thus, when a transversal intersects two lines and the pair of corresponding angles are equal then the lines are parallel.

Table 2 : Fill the table with the measures of the interior alterate angles.

Figure	Pairs of interior alternate angles	
	1st pair	2nd pair
(i)	$\angle 3$ = ... $\angle 5$ = ...	$\angle 4$ = ... $\angle 6$ = ...
(ii)	$\angle 3$ = ... $\angle 5$ = ...	$\angle 4$ = ... $\angle 6$ = ...

Find out in which figure the pairs of interior alternate angles are equal?

What can you say about the lines '*l*' and '*m*' ?

What can you say about the lines '*p*' and '*q*'?

Thus, if a pair of lines are intersected by a transversal and the alternate interior angles are equal then the lines are parallel.

Table 3 : Fill the table with the measures of interior angles on the same side of the transversal

Figure	Pairs of interior angles on the same side of the transversal.			
	1st pair		2nd pair	
(i)	$\angle 3$= .... $\angle 6$=	$\angle 3+\angle 6$ = ...	$\angle 4$= .... $\angle 5$=	$\angle 4+\angle 5$= ....
(ii)	$\angle 3$= .... $\angle 6$=	$\angle 3+\angle 6$ = ...	$\angle 4$= .... $\angle 5$=	$\angle 4+\angle 5$= ....

In which figure the pairs of interior angles on the same side of the transversal are supplementary (i.e sum is 180^0)?

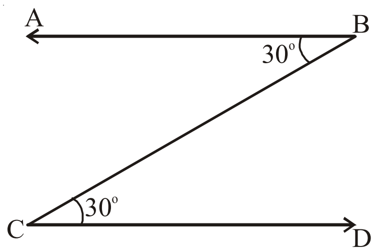
What can you say about the lines '*l*' and '*m*' ?

What can you say about the lines '*p*' and '*q*' ?

Thus, if a pair of lines are intersected by a transversal and the interior angles on the same side of the transversal are supplementary then the lines are parallel.

Example 2 : In the figure given below, two angles are marked as 30^0 each.

Is AB || CD? How?



Solution : The given angles form a pair of interior alternate angles with transversal $\overline{BC}$.

As the angles are equal, $\overline{AB} \parallel \overline{CD}$.

Exercise - 7

1. Fill up the blanks-

   (i) The line which intersects two or more lines at distinct points is called __________

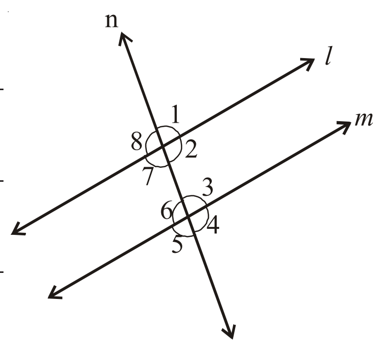
   (ii) If the pair of alternate interior angles are equal then the lines are ____________

   (iii) The sum of interior angles on the same side of the transversal are supplementary then the lines are ___________

   (iv) If two lines intersect each other then the number of common points they have ____________.

2. In the adjacent figure, the lines '*l*' and '*m*' are parallel and '*n*' is a transversal.

   Fill in the blanks for all the situations given below-



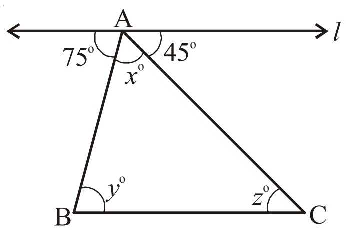
   (i) If $\angle 1 = 80^0$ then $\angle 2 =$ ______________

   (ii) If $\angle 3 = 45^0$ then $\angle 7 =$ ______________

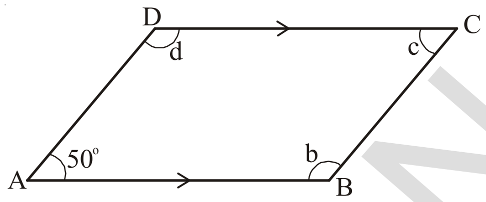
   (iii) If $\angle 2 = 90^0$ then $\angle 8 =$ ______________

   (iv) If $\angle 4 = 100^0$ then $\angle 8 =$ ______________

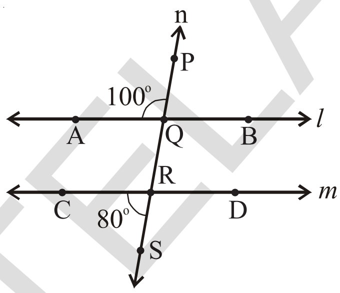
3. Find the measures of x,y and z in the figure, where l ll BC



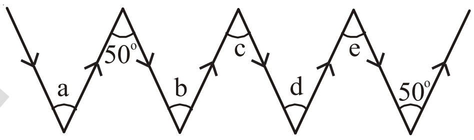
4. ABCD is a quadrilateral in which AB ll DC and AD ll BC. Find $\angle$b, $\angle$c and $\angle$d.



5. In a given figure, 'l' and 'm' are intersected by a transversal 'n'. Is l || m?



6. Find ∠a, ∠b, ∠c, ∠d and ∠e in the figure? Give reasons.



Note: Two arrow marks pointing in the same direction represent parallel lines.

Looking Back

1.(i) If the sum of two angles is equal to 90°, then the angles are called complementary angles.

(ii) Each angle in a pair of complementary angles is acute.

2.(i) If the sum of two angles is equal to 180°, then the angles are called supplementary angles.

(ii) Each angle in a pair of supplementary angles may be either acute or right or obtuse.

(iii) Two right angles always supplement to each other.

3. The angles formed on both sides of a common arm and a common vertex are adjacent angles.

4. A pair of complementary angles or a pair of supplementary angles need not be adjacent angles.

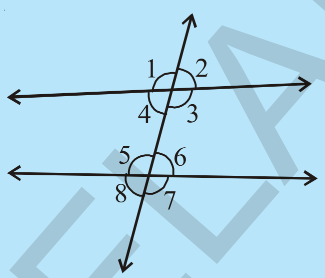
5. A pair of angles that are adjacent and supplementary form a linear pair.

6.(i) When two lines intersect each other at a point (vertex), the angles formed opposite to each other are called vertically opposite angles.

(ii) A pair of vertically opposite angles are always equal in measure

7.(i) A line which intersects two or more lines at distinct points is called a transversal to the lines.

(ii) A transversal makes eight angles with two lines as shown in the adjacent figure.



S.No.	Types of angles	No.of Pairs	Angles
1.	Interior angles	—-	$\angle 3, \angle 4, \angle 5, \angle 6$
2.	Exterior angles	—-	$\angle 1, \angle 2, \angle 7, \angle 8$
3.	Vertically opposite angles	4 pairs	$(\angle 1, \angle 3)$; $(\angle 4, \angle 2)$; $(\angle 5, \angle 7)$; $(\angle 8, \angle 6)$
4.	Corresponding angles	4 pairs	$(\angle 1, \angle 5)$; $(\angle 2, \angle 6)$; $(\angle 4, \angle 8)$; $(\angle 5, \angle 7)$
5.	Alternate interior angles	2 pairs	$(\angle 3, \angle 5)$; $(\angle 4, \angle 6)$
6.	Alternate exterior angles	2 pairs	$(\angle 1, \angle 7)$; $(\angle 2, \angle 8)$
7.	Interior angles on the same side of transversal	2 pairs	$(\angle 3, \angle 6)$; $(\angle 4, \angle 5)$

8. When a transversal intersects a pair of parallel lines, the angles in

(i) Each pair of corresponding angles are equal.

(ii) Each pair of alternate interior angles are equal.

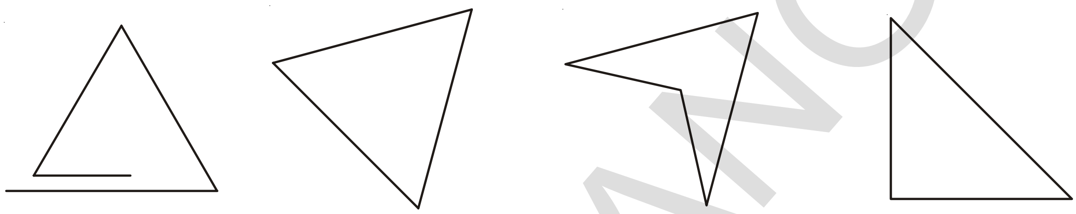
(iii) Each pair of alternate exterior angles are equal.

(iv) Each pair of interior angles on the same side of the transversal are supplementary.

TRIANGLE AND ITS PROPERTIES 5

5.0 Introduction

You have been introduced to triangles in your previous class. Look at the figures given below. Which of these are triangles?

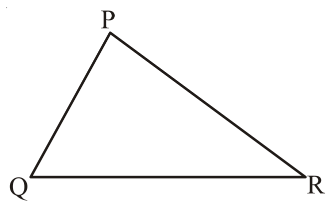


(i) (ii) (iii) (iv)

Discuss with your friends why you consider only some of these as triangles.

A triangle is a closed figure made up of three line segments.

In ΔPQR, the:



(i) Three sides are $\overline{PQ}, \overline{QR}, \overline{RP}$

(ii) Three angles are $\angle PQR$, $\angle QRP$, $\angle RPQ$

(iii) Three vertices are P, Q, R

The side opposite to vertex P is $\overline{QR}$ Can you name the sides which are opposite to vertices Q and R?

Likewise, the side opposite to $\angle QPR$ is $\overline{QR}$. Can you name the side which is opposite to $\angle PQR$

Try This

Uma felt that a triangle can be formed with three collinear points. Do you agree? Why?

Draw diagrams to justify your answer.

[If three or more points lie on the same line, then they are called collinear points]

Note: LM = Length of Line segment of LM ; $\overline{LM}$ = Line segment LM

$\overrightarrow{LM}$ = Ray LM ; $\overleftrightarrow{LM}$ = Line LM

5.1 Classification of triangles

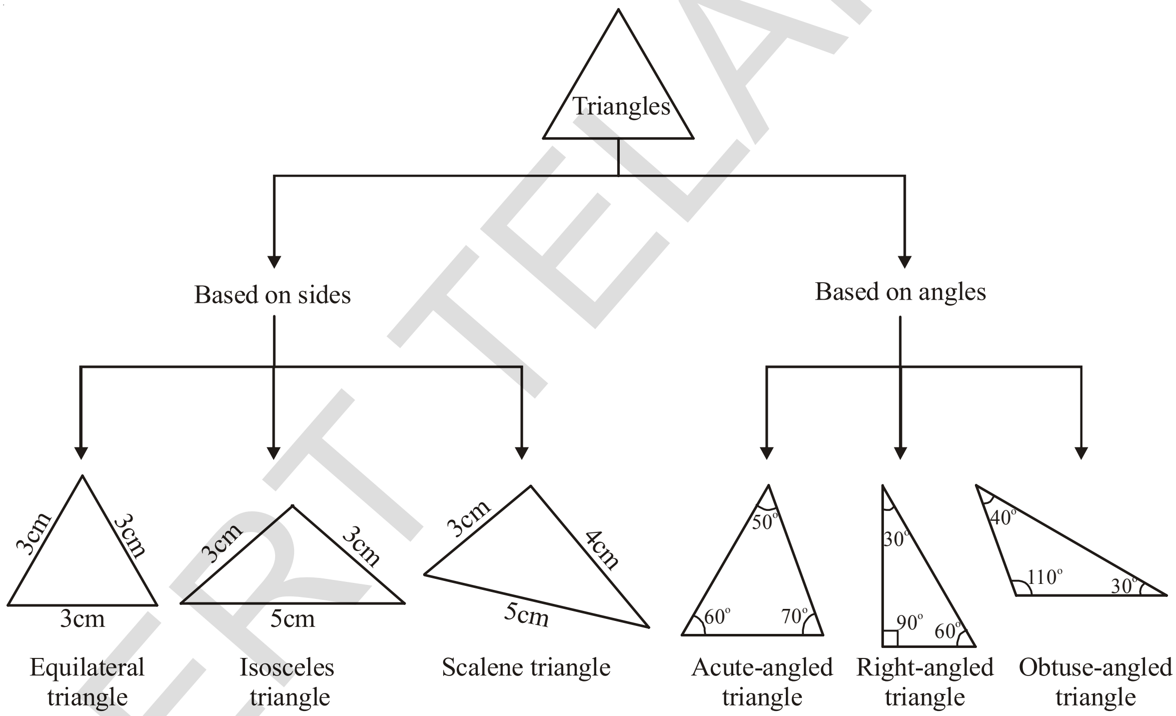
Triangles can be classified according to properties of their sides and angles.

Based on the sides, triangles are of three types:

- A triangle having all three sides of equal length is called an Equilateral Triangle.
- A triangle having two sides of equal length is called an Isosceles Triangle.
- If all the three sides of a triangle are of different length, the triangle is called a Scalene Triangle.

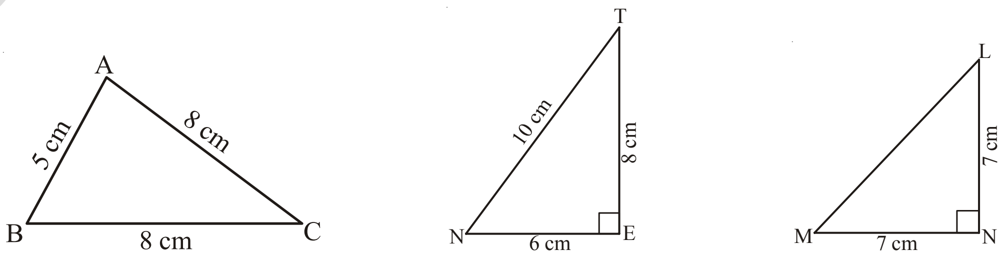
Based on the angles, triangles are again of three types:

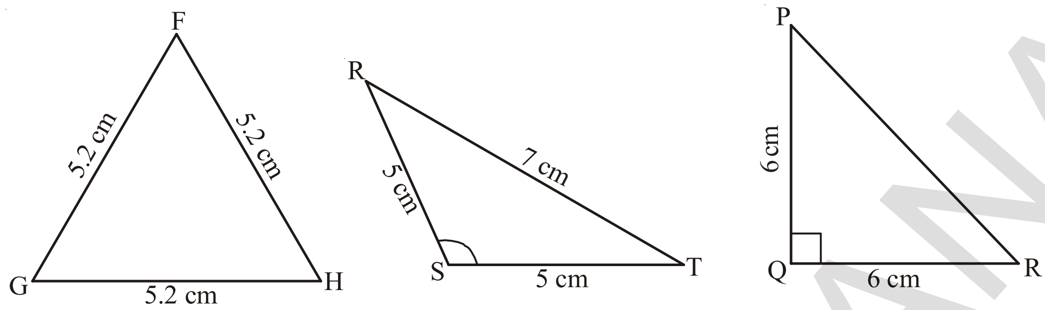
- A triangle whose all angles are acute is called an acute-angled triangle.
- A triangle whose one angle is obtuse is called an obtuse-angled triangle.
- A triangle whose one angle is a right angle is called a right-angled triangle.



Do This

1. Classify the following triangles according to their (i) sides and (ii) angles.





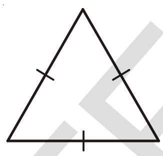
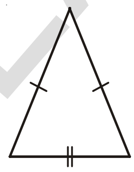
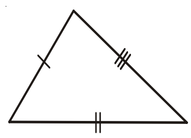
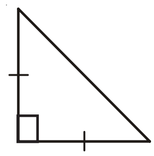
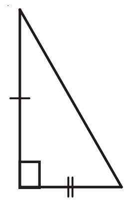
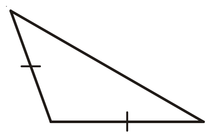
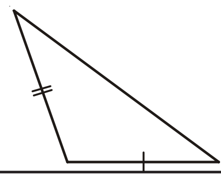
(2) Write the six elements (i.e. the 3 sides and 3 angles) of ΔABC.

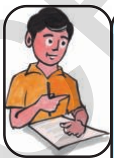
(3) Write the side opposite to vertex Q in ΔPQR.

(4) Write the angle opposite to side $\overline{LM}$ in ΔLMN.

(5) Write the vertex opposite to side $\overline{RT}$ in ΔRST.

If we consider triangles in terms of both sides and angles we can have the following types of triangles:

Type of Triangle	Equilaterial	Isosceles	Scalene
Acute-angled			
Right-angled			
Obtuse-angled			



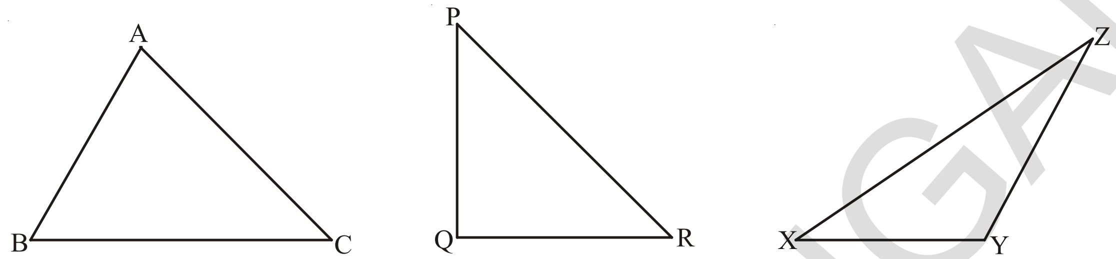
Try This

1. Make paper-cut models of the various types of triangles discussed above. Compare your models with those of your friends.
2. Rashmi claims that no triangle can have more than one right angle. Do you agree with her. Why?
3. Kamal claims that no triangle can have more than two acute angles. Do you agree with him. Why?

5.2 Relationship between the sides of a triangle

5.2.1 Sum of the lengths of two sides of a triangle

Draw any three triangles say ΔABC, ΔPQR and ΔXYZ as given below:



Use your ruler to find the lengths of their sides and tabulate your results as follows:

Name of Δ	Sides of Δ	Sum of two sides	Is this true?	Yes/No
ΔABC	AB =	AB+BC =	AB + BC > CA	
	BC =	BC+CA =	BC + CA > AB	
	CA=	CA+AB =	CA + AB > BC	
ΔPQR	PQ =	PQ+QR =	PQ + QR > RP	
	QR =	QR+RP =	QR + RP > PQ	
	RP =	RP+PQ =	RP + PQ > QR	
ΔXYZ	XY=	XY+YZ =	XY+YZ > ZX	
	YZ=	YZ+ZX =	YZ + ZX > XY	
	ZX	ZX+XY =	ZX + XY > YZ	

We can see that in all the above examples, the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

For eg. In ΔABC, $AB + BC > CA$

$BC + CA > AB$

$CA + AB > BC$

5.2.2 Difference between the lengths of two sides of a triangle

Take the same triangles as in the above example and tabulate your results as follows:

Name of Δ	Length of sides	Difference between two sides	Is this true?	Yes/No
ΔABC	AB =	BC–CA =	$BC - AB < AC$	
	BC =	CA–AB =	$CA - AB < BC$	
	CA=	AB–BC =	$AB - BC < CA$	
ΔPQR	PQ =	QR–RP =	$QR - RP < PQ$	
	QR =	RP–PQ =	$RP - PQ < QR$	
	RP =	PQ–QR =	$PQ - QR < RP$	
ΔXYZ	XY=	YZ–ZX =	$YZ - ZX < XY$	
	YZ =	ZX–XY =	$ZX - XY < YZ$	
	ZX =	XY–YZ =	$XY - YZ < ZX$	

From these observations we can conclude that the difference between the lengths of any two sides of a triangle is less than the length of the third side.

For eg. In ΔABC, $AB - BC < CA$; $BC - AB < CA$

$BC - CA < AB$; $CA - BC < AB$

$CA - AB < BC$; $AB - CA < BC$

Try This

The lengths of two sides of a triangle are 6 cm and 9 cm. Write all the possible lengths of the third side.

Example 1: Can a triangle have sides with lengths 6 cm, 5 cm and 8 cm?

Solution : Let the sides of the triangle be AB = 6 cm

BC = 5 cm

CA = 8 cm

Sum of any two sides i.e, $AB + BC = 6 + 5 = 11 > 8$

$BC + CA = 5 + 8 = 13 > 6$

$CA + AB = 8 + 6 = 14 > 5$

Since, the sum of the lengths of any two sides is greater than the lenght of the third side. The triangle is possible.

Exerciese - 1

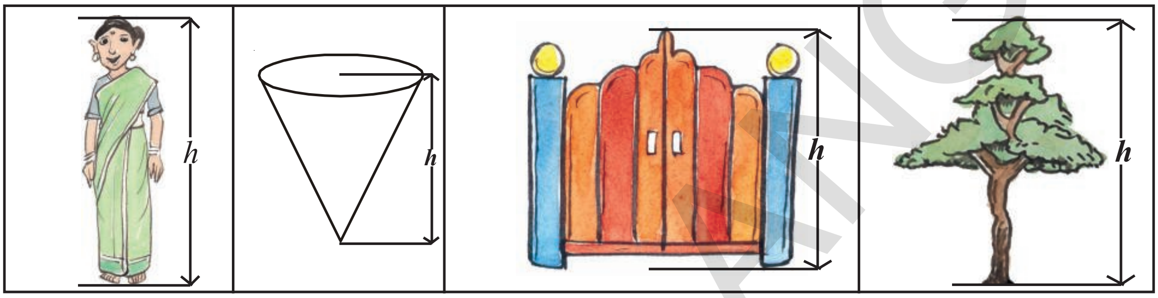
1. Is it possible to have a triangle with the following sides?

   (i) 3 cm, 4 cm and 5 cm. (ii) 6 cm, 6 cm and 6 cm.

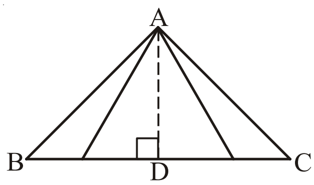
   (iii) 4 cm, 4 cm and 8 cm. (iv) 3 cm, 5 cm and 7 cm.

5.3 Altitudes of a triangle

In your daily life you might have come across the word 'height' in different situations. How will you measure the height of different figures given below:



You will measure it from the top point of the object to its base as shown in the figures. Let us use this creteria to measure the height for a triangle.



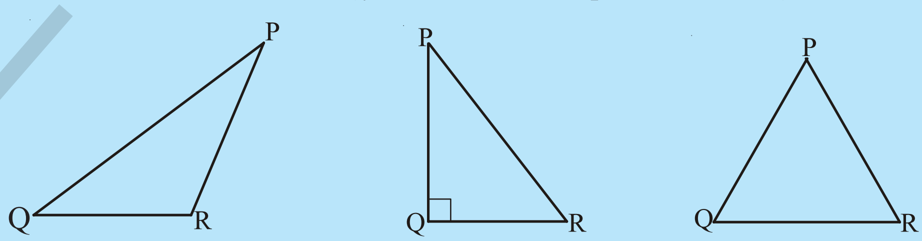
In a given Δ ABC, the height is the distance from vertex A to the base $\overline{BC}$. However, you can think of many line segments from A to $\overline{BC}$. Which among them will represent the height?

The height is given by lenght of the line segment that starts from A and is perpendicular to $\overline{BC}$.

Thus, the line segment $\overline{AD}$ is the altitude of the triangle and its length is height. An altitude can be drawn from each vertex.

Try This

(i) Draw altitudes from P to $\overline{QR}$ for the following triangles. Also, draw altitudes from the other two vertices.(you can use a set squares if needed)

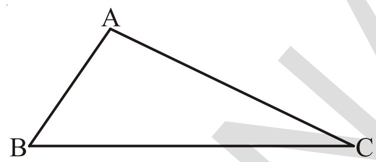


Obtuse-angled Right-angled Acute-angled

(ii) Will an altitude always lie in the interior of a triangle?

(iii) Can you think of a triangle in which the two altitudes of a triangle are two of its sides?

5.4 Medians of a triangle



Make a paper cut out of ΔABC.

Now fold the triangle in such a way that the vertex B falls on vertex C. The line along which the triangle has been folded will intersect side $\overline{BC}$ as shown in Figure 1. The point of intersection is the mid-point of side $\overline{BC}$ which we call D. Draw a line joining vertex A to this mid-point D (as can be seen in Figure 2).

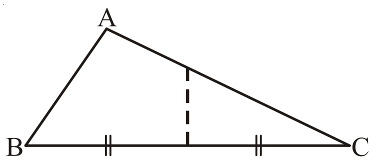


Figure 1

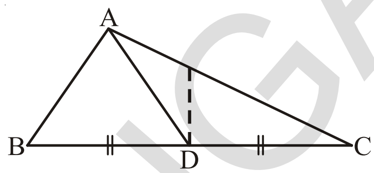
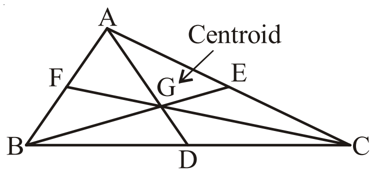


Figure 2

Similarly, fold the triangle in such a way that the vertex A falls on vertex C. The line along which the triangle has been folded will intersect side $\overline{AC}$. The point of intersection is the mid-point of side $\overline{AC}$. Draw a line joining vertex B to this mid-point, which we call E.

Lastly, fold the triangle in such a way that the vertex A falls on vertex B. The line along which the triangle has been folded will intersect side $\overline{AB}$. The point of intersection is the mid-point of side $\overline{AB}$. Draw a line joining vertex C to this mid-point, which we call F.



Line segments $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ join the vertices of the triangle to the mid-points of the opposite sides. They are called the medians of the triangle.

You will observe that the three medians intersect each other at a point in the interior of the triangle. This point of concurrent is called the Centroid.

Thus, line segments which join the vertex of the triangle to the mid-point of the opposite side are called medians of the triangle. Their point of concurrent is called the Centroid.

Try This

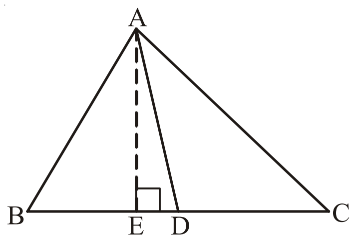
Take paper cut outs of right-angled triangles and obtuse-angled triangles and find their centroid.

Exercise - 2

1. In ΔABC, D is the midpoint of $\overline{BC}$.

   (i) $\overline{AD}$ is the ______________________

   (ii) $\overline{AE}$ is the ______________________



2. Name the triangle in which the two altitudes of the triangle are two of its sides.
3. Does a median always lie in the interior of the triangle?
4. Does an altitude always lie in the interior of a triangle?
5. (i) Write the side opposite to vertex Y in ΔXYZ.
   (ii) Write the angle opposite to side $\overline{PQ}$ in ΔPQR.
   (iii) Write the vertex opposite to side $\overline{AC}$ in ΔABC.

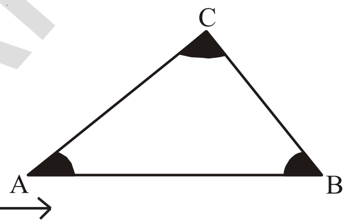
5.5 Properties of triangles

5.5.1 Angle-sum property of a triangle

Let us learn about this property through the following four activities

Activity 1

1. On a white sheet of paper, draw a triangle ABC. Using colour pencils mark its angles as shown.
2. Using a scissors, cut out the three angular regions.
3. Draw a line XY and mark a point 'O' on it.



4. Paste the three angular cut outs adjacent to each other to form one angle at 'O' as shown in the figure below.

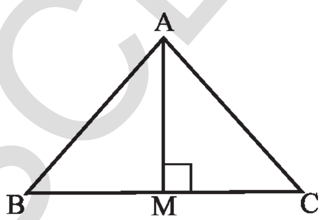
You will find that three angles now constitute a straight angle. Thus, the sum of the measures of angles of a triangle is equal to 180º.

Activity 2

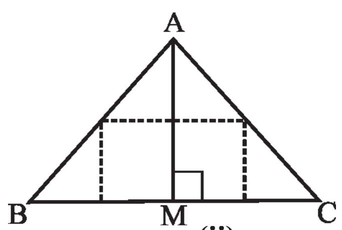
Take a piece of paper and cut out a triangle, say ABC.

Make the altitude $\overline{AM}$ by folding ΔABC.

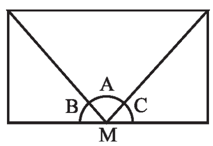
Now, fold the three corners such that all the vertices A,B and C touch at M as shown in the following figures.



(i)



(ii)

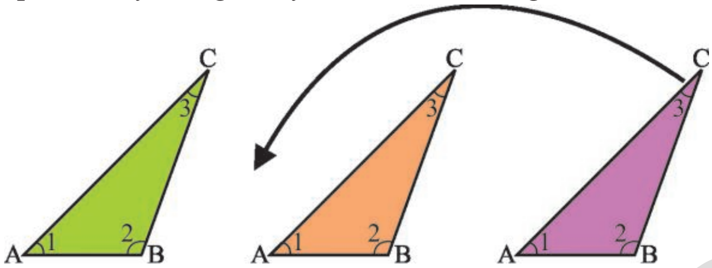


(iii)

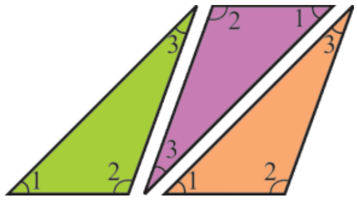
You will see that all the three angles A, B and C form a straight line and thus $\angle A + \angle B + \angle C = 180^\circ$.

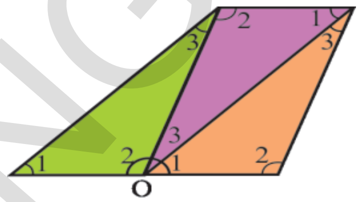
Activity 3

Take three copies of any triangle, say ABC. Mark its angles as 1,2 and 3 as shown below:



Arrange the triangular cut-outs as shown in the above and in the following figures.





What do you observe about $\angle 1 + \angle 2 + \angle 3$ at the point 'O'?

You will observe that three angles form a straight line and so measure 180°.

Activity 4

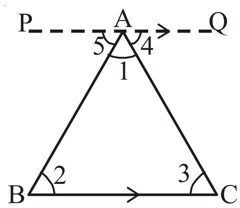
Draw any three triangles, say ΔABC, ΔPQR and ΔXYZ in your note book. Use your protractor and measure each of the angles of these triangles.

Name of the Triangle	Measure of angles	Sum of the measures of the three angles
ΔABC	$\angle A =$....., $\angle B =$....., $\angle C =$.....,	$\angle A + \angle B + \angle C =$
ΔPQR	$\angle P =$....., $\angle Q =$....., $\angle R =$.....,	$\angle P + \angle Q + \angle R =$
ΔXYZ	$\angle X =$....., $\angle Y =$....., $\angle Z =$.....,	$\angle X + \angle Y + \angle Z =$

Allowing marginal errors in measurements, you will find that the sum of the three angles of a triangle is 180°.

You are now ready to give a formal justification of your assertion that the sum of the angles of a triangle is equal to 180° through logical argumentation.

Proof of angle-sum property of a triangle



Statement : The sum of the angles of a triangle is 180°

Given : A triangle ABC

To prove : $\angle A + \angle B + \angle C = 180^\circ$

Construction : Through A draw a line segment $\overleftrightarrow{PQ}$ parallel to BC.

Proof :

Mark the angles as indicated in the figure:

$\angle 2$	$=$	$\angle 5$	(alternate interior angles)
$\angle 3$	$=$	$\angle 4$	(alternate interior angles)
$\angle 2 + \angle 3$	$=$	$\angle 5 + \angle 4$	(adding (1) and (2))
$\angle 1 + \angle 2 + \angle 3$	$=$	$\angle 1 + \angle 5 + \angle 4$	(adding $\angle 1$ to both sides)
But $\angle 1 + \angle 5 + \angle 4$	$=$	180°	(angles forming a straight line)

Therefore, $\angle 1 + \angle 2 + \angle 3 = 180^\circ$

$\therefore \angle A + \angle B + \angle C = 180^\circ$.

Thus, the sum of the angles of a triangle is 180°.

Example 1: In ΔABC, $\angle A = 30^\circ$, $\angle B = 45^\circ$, find $\angle C$.

Solution : In ΔABC, $\angle A + \angle B + \angle C = 180^\circ$ (angle-sum property of a triangle)

$30^\circ + 45^\circ + \angle C = 180^\circ$ (substituting given values in question)

$75^\circ + \angle C = 180^\circ$

$\angle C = 180^\circ - 75^\circ$

Therefore, $\angle C = 105^\circ$

Example 2 : In ΔABC, if $\angle A = 3\,\angle B$ and $\angle C = 2\,\angle B$. Find all the three angles of ΔABC.

Solution : $\angle A + \angle B + \angle C = 180^\circ$ [angle-sum property of a triangle]

$3\,\angle B + \angle B + 2\,\angle B = 180^\circ$ [$\angle A = 3\,\angle B$, $\angle C = 2\,\angle B$]

$6\,\angle B = 180^\circ$

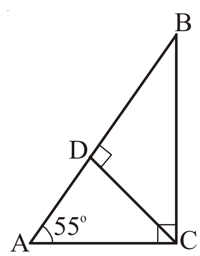
Therefore, $\angle B = 30^\circ$

Thus, $\angle A = 3\,\angle B = 3 \times 30^\circ = 90^\circ$

$\angle C = 2\,\angle B = 2 \times 30^\circ = 60^\circ$

Example 3 : ΔABC is right angled at C and $CD \perp AB$, $\angle A = 55^\circ$

Find (i) $\angle ACD$ (ii) $\angle BCD$ (iii) $\angle ABC$



Solution : In ΔACD,

$\angle CAD + \angle ADC + \angle ACD = 180^\circ$ (angle-sum property of a triangle)

$55^\circ + 90^\circ + \angle ACD = 180^\circ$ (substituting values given in question)

$$145^\circ + \angle ACD = 180^\circ$$

$$\angle ACD = 180^\circ - 145^\circ = 35^\circ$$

Therefore, $\angle ACD = 35^\circ$

(ii) In ΔABC,

$$\angle ACB = 90^\circ$$

Therefore, $\angle ACD + \angle BCD = 90^\circ$ (from the figure $\angle ACB = \angle ACD + \angle BCD$)

$35^\circ + \angle BCD = 90^\circ$ (from (i), $\angle ACD = 35^\circ$)

$$\angle BCD = 90^\circ - 35^\circ = 55^\circ$$

(iii) In ΔABC,

$\angle ABC + \angle BCA + \angle CAB = 180^\circ$ (angle-sum property of a triangle)

$\angle ABC + 90^\circ + 55^\circ = 180^\circ$ (given)

$$\angle ABC + 145^\circ = 180^\circ$$

$$\angle ABC = 180^\circ - 145^\circ$$

Therefore, $\angle ABC = 35^\circ$

Example 4 : The angles of a triangle are in the ratio 2 : 3 : 4. Find the angles.

Solution : The given ratio between the angles of the triangle = 2 : 3 : 4

Sum of the terms of the ratio = 2 + 3 + 4 = 9

Sum of the angles of a triangle = 180°

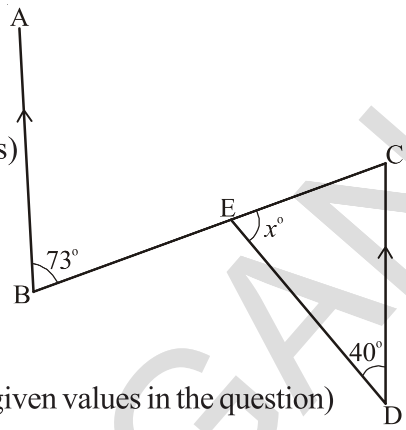
Therefore, 1st angle $= \frac{2}{9} \times 180^\circ = 40^\circ$

2nd angle $= \frac{3}{9} \times 180^\circ = 60^\circ$

3rd angle $= \frac{4}{9} \times 180^\circ = 80^\circ$

Thus, the angles of the triangle are 40°, 60° and 80°.

Example 5 : Find the value of angle ***'x'*** in the figure.



Solution : $\angle ECD = \angle ABC = 73^o$

(Since AB || CD these two are alternate angles)

In ΔECD,

$\angle CED + \angle EDC + \angle DCE = 180^o$

(angle-sum property of a triangle)

$x^o + 40^o + 73^o = 180^o$ (substituting given values in the question)

$x^o + 113^o = 180^o$

$x^o = 180^o - 113^o$

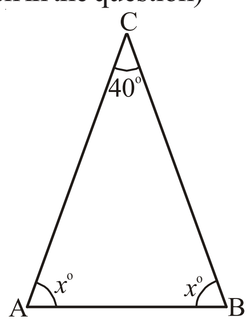
$x^o = 67^o$

Example 6 : One angle of ΔABC is 40^o and the other two angles are equal. Find the measure (value) of each equal angle.

Solution : Let $\angle C = 40^o$ and $\angle A = \angle B = x^o$

$\angle A + \angle B + \angle C = 180^o$ (angle-sum property of a triangle)

$x^o + x^o + 40^o = 180^o$ (substituting values given in the question)



$2x^o + 40^o = 180^o$

$2x = 180^o - 40^o$

$2x = 140^o$

$x^o = 70^o$

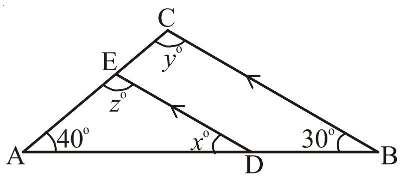
Thus, each angle is 70^o

Example 7 : In the figure, D and E are the points on sides AB and AC of ΔABC such that DE || BC. If $\angle B = 30^o$ and $\angle A = 40^o$, find (i) x (ii) y (iii) z

Solution : (i) $\angle ADE = \angle ABC$ (corresponding angles as DE || BC)

Therefore, $x^o = 30^o$

(ii) In ΔABC,



$\angle A + \angle B + \angle C = 180^o$ (angle sum property of a triangle)

$40^o + 30^o + y^o = 180^o$ (substituting values given in the question)

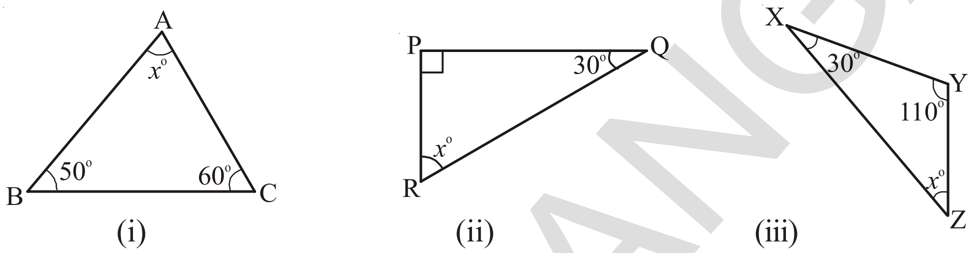
$70^o + y^o = 180^o$

Therefore, $y^o = 180^o - 70^o = 110^o$

(iii) $y^{o} = z^{o} = 110^{o}$ (corresponding angles since DE || BC)

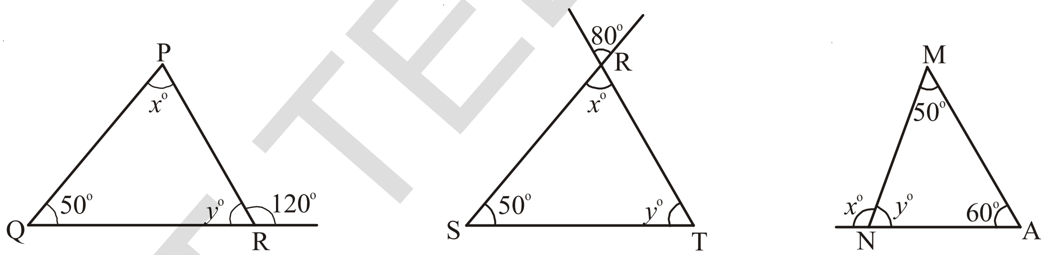
Exercise - 3

1. Find the value of the unknown 'x' in the following triangles.

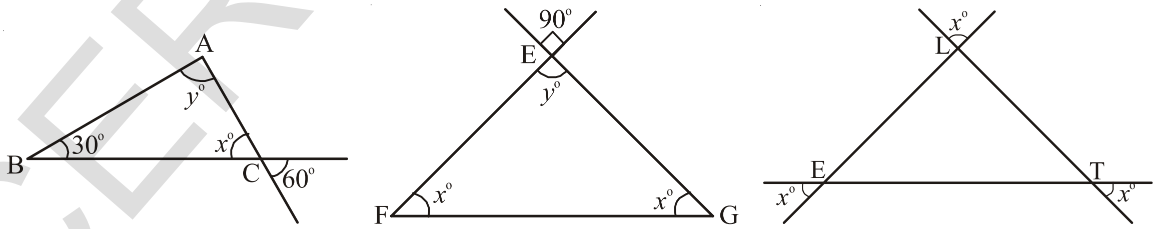


(i) (ii) (iii)

2. Find the values of the unknowns 'x' and 'y' in the following diagrams.



(i) (ii) (iii)



(iv) (v) (vi)

3. Find the measure of the third angle of triangles whose two angles are given below:

   (i) 38^o, 102^o (ii) 116^o, 30^o (iii) 40^o, 80^o

4. In a right-angled triangle, one acute angle is 30^o. Find the other acute angle.

5. State true or false for each of the following statements.

(i) A triangle can have two right angles.

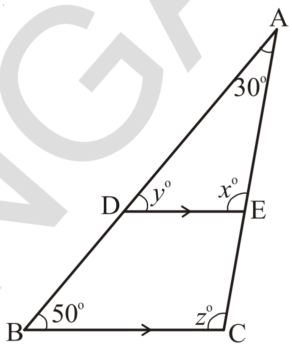
(ii) A triangle can have two acute angles.

(iii) A triangle can have two obtuse angles.

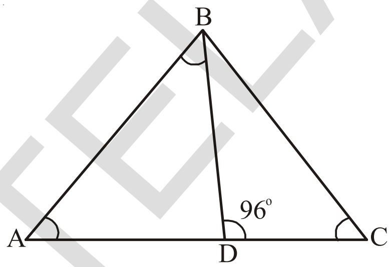
(iv) Each angle of a triangle can be less than 60°.

6. The angles of a triangle are in the ratio 1 : 2 : 3. Find the angles.

7. In the figure, $\overline{DE} \parallel \overline{BC}$, $\angle A = 30^\circ$ and $\angle B = 50^\circ$. Find the values of x, y and z.



8. In the figure, $\angle ABD = 3\ \angle DAB$ and $\angle BDC = 96^\circ$. Find $\angle ABD$.

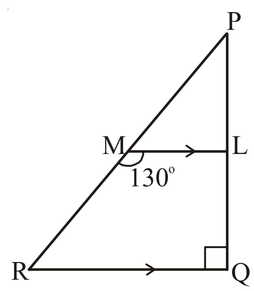


9. In ΔPQR $\angle P = 2\ \angle Q$ and $2\ \angle R = 3\ \angle Q$, calculate the angles of ΔPQR.

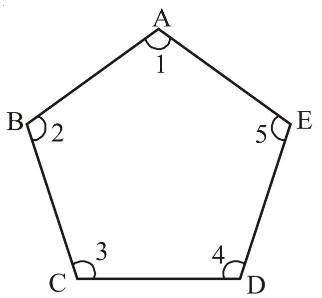
10. If the angles of a triangle are in the ratio 1 : 4 : 5, find the angles.

11. The acute angles of a right triangle are in the ratio 2 : 3. Find the angles of the triangle.

12. In the figure, ΔPQR is right angled at Q, $\overline{ML} \parallel \overline{RQ}$ and $\angle LMR = 130^\circ$. Find $\angle LPM$, $\angle PML$ and $\angle PRQ$.



13. In Figure ABCDE, find $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5$.



5.5.2 Exterior angle of a triangle

Draw ΔABC and produce one of its sides say $\overline{BC}$ as shown in the Figure 1. Observe the $\angle ACD$ formed at point C. This angle lies in the exterior of ΔABC. We call it the exterior angle of ΔABC formed at vertex C.

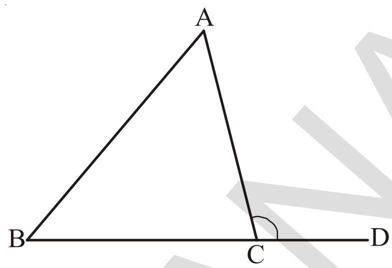


Figure 1

Clearly $\angle BCA$ is an adjacent angle to $\angle ACD$. The remaining two angles of the triangle namely $\angle BAC$ or $\angle A$ and $\angle CBA$ or $\angle B$ are called the two interior opposite angles of $\angle ACD$. Now cut out (or make trace copies of) $\angle A$ and $\angle B$ and place them adjacent to each other as shown in the Figure 2.

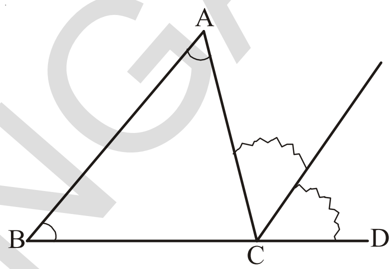


Figure 2

Do these two pieces together entirely cover $\angle ACD$?

Can you say that $\angle ACD = \angle A + \angle B$?

From the above activity, we can say that an exterior angle of a triangle is equal to the sum of two interior opposite angles.

Do This

Draw ΔABC and form an exterior $\angle ACD$. Now take a protractor and measure $\angle ACD$, $\angle A$ and $\angle B$.

Find the sum $\angle A + \angle B$ and compare it with the measure $\angle ACD$.

Do you observe that $\angle ACD$ is equal (or nearly equal) to $\angle A + \angle B$?

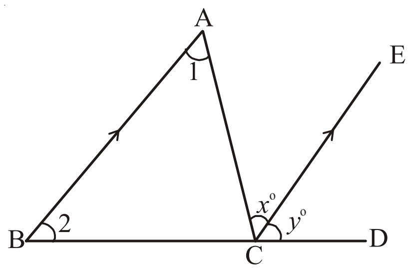
A logical step- by- step argument can further confirm that the exterior angle of a triangle is equal to the sum of the interior opposite angles.

Statement : An exterior angle of triangle is equal to the sum of its interior opposite angles.

Given : ΔABC with $\angle ACD$ as exterior angle

To prove : $\angle ACD = \angle A + \angle B$

Construction : Through C draw $\overline{CE}$ parallel to $\overline{BA}$



Justification

$\angle 1 = \angle x$ ($\overline{BA} \parallel \overline{CE}$ and $\overline{AC}$ is transversal therefore, alternate angles are equal)

$\angle 2 = \angle y$ ($\overline{BA} \parallel \overline{CE}$ and $\overline{BD}$ is transversal therefore, corresponding angles are equal)

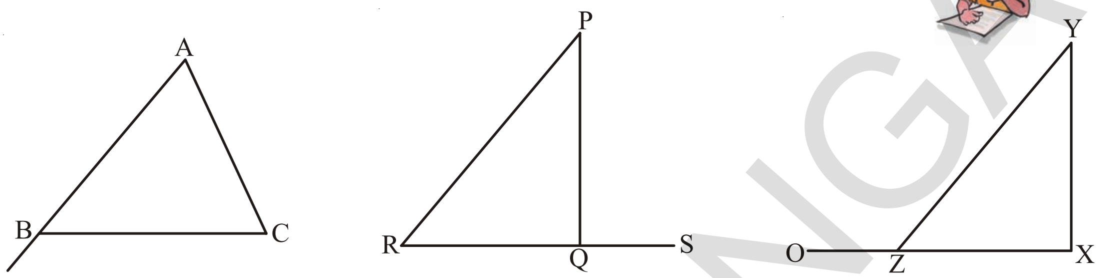
$\angle 1 + \angle 2 = \angle x + \angle y$

Therefore , $\angle ACD = \angle 1 + \angle 2$ (from the figure $\angle x + \angle y = \angle ACD$)

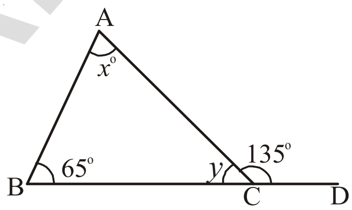
Thus, the exterior angle of a triangle is equal to the sum of the interior opposite angles. This property is called the exterior-angle property of a triangle.

Do This

Copy each of the following triangles. In each case verify that an exterior angle of a triangle is equal to the sum of the two interior opposite angles.



Example 8 : In the figure, find the values of x and y.



Solution : $\angle ACD = \angle ABC + \angle BAC$

(exterior angle property)

$$135^{o} = 65^{o} + x^{o}$$

$$135^{o} - 65^{o} = x^{o}$$

Therefore, $x^{o} = 70^{o}$

$\angle ABC + \angle BAC + \angle BCA = 180^{o}$ (angle-sum property of a triangle)

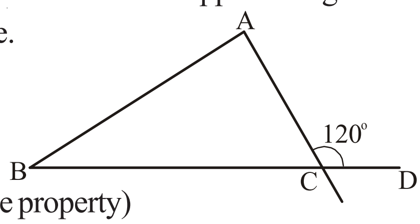
$$65^{o} + 70^{o} + y^{o} = 180^{o}$$

$$135^{o} + y^{o} = 180^{o}$$

$$y^{o} = 180^{o} - 135^{o}$$

Therefore, $y^{o} = 45^{o}$

Example 9 : One of the exterior angles of a triangle is 120^{o} and the interior opposite angles are in the ratio 1 : 5. Find the angles of the triangle.



Solution : $\angle ACD = 120^{o}$ (from the question)

$\angle ACD = \angle A + \angle B$ (exterior angle property)

$\angle A + \angle B = 120^{o}$

$\angle B : \angle A = 1 : 5$

$$\angle B = \frac{1}{6} \times 120^\circ = 20^\circ$$

$$\angle A = \frac{5}{6} \times 120^\circ = 100^\circ$$

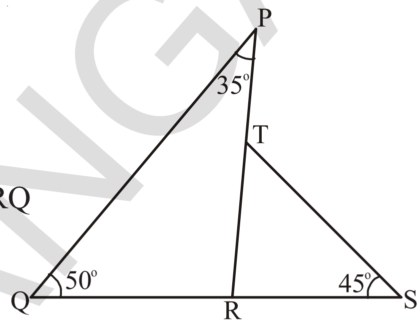
$\angle A + \angle B + \angle C = 180^\circ$ (angle-sum property of a triangle)

$100^\circ + 20^\circ + \angle C = 180^\circ$

Therefore, $\angle C = 180^\circ - 120^\circ = 60^\circ$

Example 10 : In the adjacent figure, find

(i) $\angle PRS$ (ii) $\angle PTS$ (iii) $\angle STR$ (iv) $\angle PRQ$



Solution : (i) In ΔPQR, $\angle PRS$ is the exterior angle and $\angle RQP$ and $\angle QPR$ are the interior opposite angles.

$\therefore \angle PRS = \angle RQP + \angle QPR$ (exterior angle property)

$\angle PRS = 50^\circ + 35^\circ = 85^\circ$

(ii) In ΔRST, $\angle PTS$ is the exterior angle and $\angle SRT$ and $\angle RST$ are the interior opposite angles.

Therefore, $\angle PTS = \angle SRT + \angle RST$

$\angle PTS = 85^\circ + 45^\circ$ ($\angle SRT = \angle PRS = 85^\circ$)

$\angle PTS = 130^\circ$

(iii) In Δ RST we have

$\angle STR + \angle RST + \angle SRT = 180^\circ$ (angle-sum property of a triangle)

$\angle STR + 45^\circ + 85^\circ = 180^\circ$

$\angle STR + 130^\circ = 180^\circ$

Therefore, $\angle STR = 180^\circ - 130^\circ = 50^\circ$

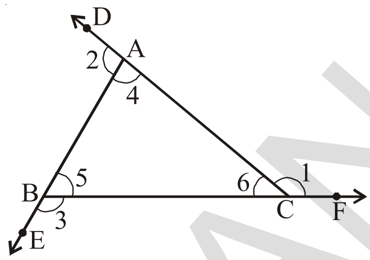
(iv) $\angle PRQ + \angle PRS = 180^\circ$ (liner pair property)

$\angle PRQ + 85^\circ = 180^\circ$

$\angle PRQ = 180^\circ - 85^\circ$

$\angle PRQ = 95^\circ$

Example 11 : Show that the sum of the exterior angles of ΔABC is 360^o.



Solution : $\angle 2 + \angle 4 = 180^o$ (linear pair)

$\angle 3 + \angle 5 = 180^o$ (linear pair)

$\angle 6 + \angle 1 = 180^o$ (linear pair)

Adding the angles on both sides, we get-

$\angle 2 + \angle 4 + \angle 3 + \angle 5 + \angle 6 + \angle 1 = 180^o + 180^o + 180^o$

$(\angle 4 + \angle 5 + \angle 6) + (\angle 1 + \angle 2 + \angle 3) = 540^o$

We know that, $\angle 4 + \angle 5 + \angle 6 = 180^o$ (angle-sum property of a triangle)

Therefore, $180^o + \angle 1 + \angle 2 + \angle 3 = 540^o$

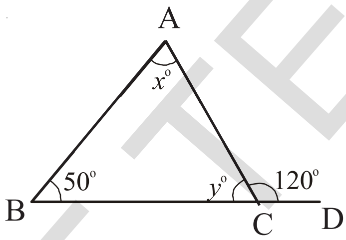
$\angle 1 + \angle 2 + \angle 3 = 540^o - 180^o$

$\angle 1 + \angle 2 + \angle 3 = 360^o$

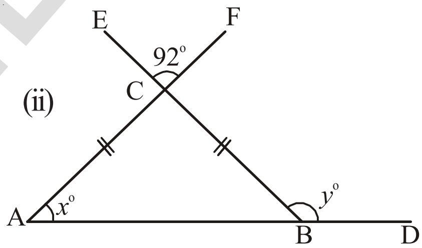
Thus, the sum of the exterior angles of a triangle is 360^o.

Example 12 : Find the angles x and y in the following figures.

(i)



(ii)



Solution : (i) $\angle BAC + \angle ABC = \angle ACD$ (exterior angle property)

$x^o + 50^o = 120^o$

$x^o = 120^o - 50^o = 70^o$

$\angle ACB + \angle ACD = 180^o$ (linear pair)

$y^o + 120^o = 180^o$

$y^o = 180^o - 120^o = 60^o$

(ii) $\angle ACB = \angle ECF = 92^o$ (vertically opposite angles)

$\angle CAB = \angle CBA$ (opposite angles of equal sides)

In ΔABC, $\angle BAC + \angle CBA + \angle ACB = 180^o$ (angle-sum property)

$x^o + x^o + 92^o = 180^o$

$2x = 180^o - 92^o = 88^o$

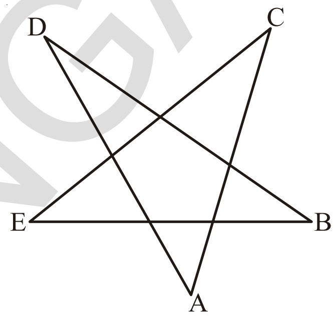
Therefore, $\quad x^o = \dfrac{88}{2} = 44^o$

Also $\quad \angle ABC + y^o = 180^o \quad$ (linear pair)

$y^o = 180^o - x^o$

Therefore., $\quad y^o = 180^o - 44^o = 136^o$

Example 13 : Find the value of $\angle A + \angle B + \angle C + \angle D + \angle E$ of the following figure.



Solution : Name the angles as shown in the figure.

In ΔGHC $\angle 3 + \angle 6 + \angle 7 = 180^o$ ....(1)

(angle-sum property of triangle)

In ΔEHB, $\angle 6 = \angle 5 + \angle 2$ ....(2)

In ΔAGD, $\angle 7 = \angle 1 + \angle 4$ ....(3)

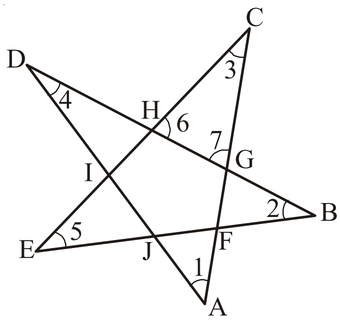
(exterior angle property of a triangle)

Substituting (2) and (3) in (1)

$\Rightarrow \quad \angle 3 + \angle 5 + \angle 1 + \angle 2 + \angle 4 = 180^o$

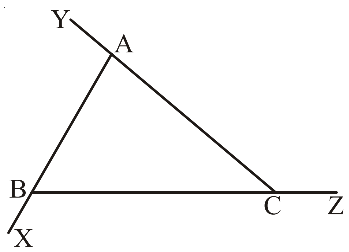
$\Rightarrow \quad \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 = 180^o$

Therefore, $\angle A + \angle B + \angle C + \angle D + \angle E = 180^o$

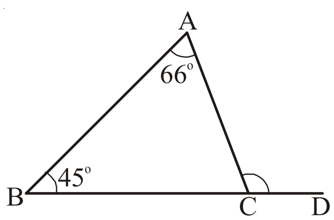


Exercise - 4

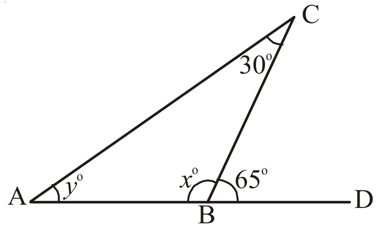
1. In ΔABC, name all the interior and exterior angles of the triangle.



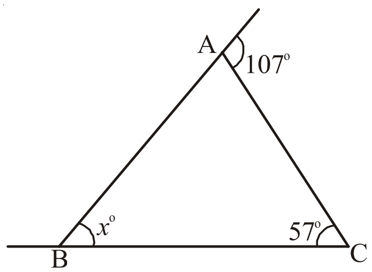
2. For ΔABC, find the measure of $\angle ACD$.

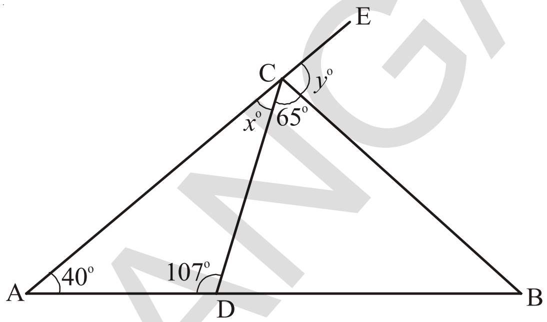


3. Find the measure of angles x and y.

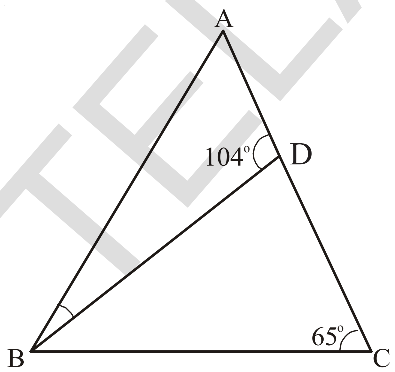


4. In the following figures, find the values of x and y.

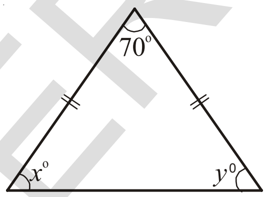




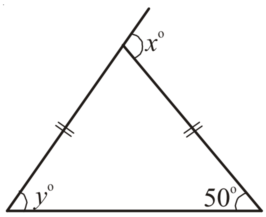
5. In the figure $\angle BAD = 3\angle DBA$, find $\angle CDB$, $\angle DBC$ and $\angle ABC$.



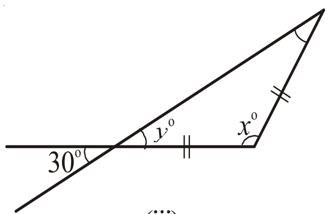
6. Find the values of x and y in the following figures.



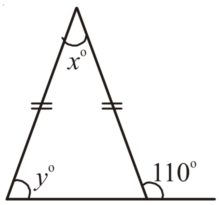
(i)



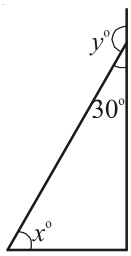
(ii)



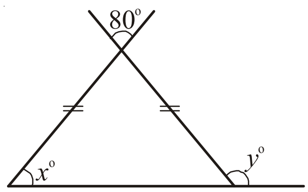
(iii)



(iv)



(v)



(vi)

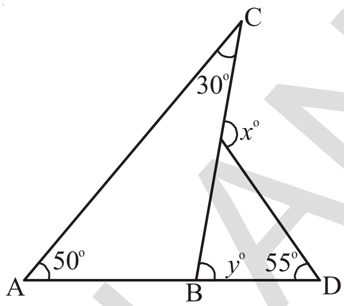
7. One of the exterior angles of a triangle is 125° and the interior opposite angles are in the ratio 2 : 3. Find the angles of the triangle.

8. The exterior $\angle PRS$ of ΔPQR is 105°. If $Q = 70^\circ$, find $\angle P$. Is $\angle PRS > \angle P$?

9. If an exterior angle of a triangle is 130° and one of the interior opposite angle is 60°. Find the other interior opposite angle.

10. One of the exterior angle of a triangle is 105° and the interior opposite angles are in the ratio 2 : 5. Find the angles of the triangle.

11. In the figure find the values of x and y.



Looking Back

1 (i) A triangle is a simple closed figure made up of three line segments.

(ii) Based on the sides, triangles are of three types

- A triangle having all three sides of same length is called an Equilateral Triangle.
- A triangle having at least two sides of equal length is called an Isosceles Triangle.
- If all the three sides of a triangle are of different length, the triangle is called a Scalene Triangle.

(iii) Based on the angles, triangles are of three types

- A triangle whose all angles are acute is called an acute-angled triangle.
- A triangle whose one angle is obtuse is called an obtuse-angled triangle.
- A triangle whose one angle is a right angle is called a right-angled triangle.

2. The six elements of a triangle are its three angles and the three sides.

3. Properties of the lengths of the sides of a triangle:

(i) The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

(ii) The difference between the lengths of any two sides of a triangle is smaller than the length of the third side.

4. The line segment joining a vertex of a triangle to the mid-point of its opposite side is called a median of the triangle. A triangle has 3 medians.

5. The perpendicular line segment from a vertex of a triangle to its opposite side is called the altitude of the triangle.

6. The total measure of the three angles of a triangle is 180^{o}. This is called the angle sum property of a triangle.

7. The measure of any exterior angle of a triangle is equal to the sum of its interior opposite angles. This is called the exterior angle property of the triangle.

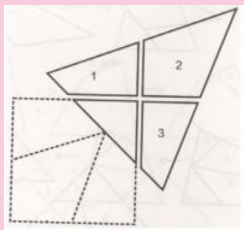
8. Representation of line, line segment and ray

LM = Lenght of Line segment of LM ; $\overline{LM}$ = Line segment LM

$\overrightarrow{LM}$ = Ray LM ; $\overleftrightarrow{LM}$ = Line LM

Fun with Card board shapes

Take square card board sheet. Mark the mid points of sides and draw lines as shown in the figure. Cut the square into four parts and rearrange them to get a triangle



RATIO - APPLICATIONS 6

6.0 Introduction

In your previous class, you have learnt how to use ratio and proportion to compare quantities. In this class, we will first review our understanding of the same and then learn about ratios expressed in the form of percentages.

6.1 Ratio

- Madhuri's weight is 50 kg and her daughter's weight is 10 kg. We say that Madhuri's weight is 5 times her daughter's weight. We can also say that the daughter's weight is one-fifth of her mother's weight. Thus, the ratio of Madhuri's weight and her daughter's weight is 50 : 10 or 5 : 1. Inversely, the ratio of the daughter's weight and the mother's weight is 1 : 5.

- In a class there are 60 boys and 40 girls. The number of boys is $\frac{3}{2}$ times the number of girls.

  we can also say that the number of girls is two-thirds of the boys. Thus, the ratio of the number of boys and the number of girls is 60 : 40 or 3 : 2. Inversely, the ratio of number of girls and number of boys is 2 : 3.

Anand has a wire of length 100 cm and Rashmi has a wire of length 5 m. Anand said to Rashmi, "the wire with me is 20 times longer than yours." You know that this is not true as 5 m is much longer than 100 cm. The length of Rashmi's wire has been expressed in meters and that of Anand has been expressed in centimeters. Both have to be expressed in the same units before they are compared.

We know that 1 m = 100 cm. So the length of the wire with Rashmi is 5 m = 5 × 100 = 500 cm. Thus, the ratio of Rashmi and Anand's wire is 500 : 100 or 5 : 1. We can also say that the length of Rashmi's wire is 5 times that of Anand.

In all the above examples quantities have been comapred in the form of ratios. Thus, a ratio is an ordered comparison of quantities of the same units. We use the symbol ':' to represent a ratio. The ratio of two quantities a and b is a: b and we read this as "a is to b". The two quantities 'a' and 'b' are called terms of the ratio. The first quantity 'a' is called first term or antecedent and the second quantity 'b' is called second term or consequent.

Try This

Think of some real life situations in which you have to compare quantities in the form of a ratio.

Exercise 1

1. What is the ratio of ₹ 100 and ₹10 ? Express your answer in the simplest form.

2. Sudha has ₹ 5. Money with Radha is 3 times the money with Sudha. How much money does Radha have?

   (i) What is the ratio of Radha's money and Sudha's money?

   (ii) What is the ratio of Sudha's money and Radha's money?

3. Divide 96 chocolates between Raju and Ravi in the ratio 5 : 7

4. The length of a line segment AB is 38 cm. A point X on it divides it in the ratio 9 : 10. Find the lengths of the line segments AX and XB.

5. A sum of ₹ 1,60,000 is divided in the ratio of 3 : 5. What is the smaller share?

6. To make green paint, a painter mixes yellow paint and blue paint in the ratio of 3 : 2. If he used twelve liters of yellow paint, how much blue paint did he use?

7. A rectangle measures 40 cm at its length and 20 cm at its width. Find the ratio of the length to the width.

8. The speed of a Garden-Snail is 50 meters per hour and that of the Cheetah is 120 kilometers per hour. Find the ratio of the speeds.

9. Find (i) The ratio of boys and girls in your class.

   (ii) The ratio of number of doors and number of windows of your classroom.

   (iii) The ratio of number of text books and number of note books with you

Classroom Project

1. Take a tape and with the help of your friend measure the length and breadth of your classroom Find the ratio of length and breadth.

2. Take a ₹ 10 note. Find its length and breadth. Roundoff the answers to the nearest whole number, with the help of your teacher, find the ratio of the length and breadth.

   Repeat this activity with ₹ 20 and ₹ 50 notes and record the lengths in your note book.

6.2 Proportion

Srilekha's mother prepares tea by using 2 spoons of tea powder for 1 cup of tea. One day 3 guests visited their home. How many spoons of tea powder must she use to prepare 3 cups of tea? Yes, you are right. She uses 6 spoons of tea powder to prepare 3 cups of tea. Here, Srilekha's mother used the 'law of proportion' to solve the problem.

Let us see one more example:

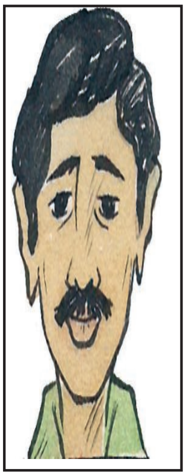
Ravi took a photo. He got the picture developed in a photo lab in a size 4 cm × 6 cm .

4 cm

6 cm

4 cm

12 cm



He wanted to get the photo enlarged so he went to the photo lab again. The lab-man gave him this photo. In turn Ravi said, "there seems to be something wrong with this picture".

Do you think, is Ravi right?

Can you say what is wrong with this picture?

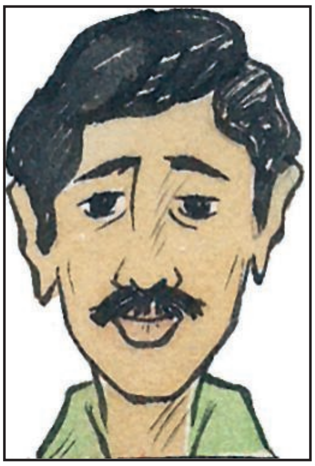
Ravi decided to measure the length and breadth of the photo. He knew that the ratio of length and breadth of the original photo should be equal to the ratio of length and breadth of the enlarged photo.

Ratio of length and breadth of the original photo = 4 : 6 = 2 : 3

Ratio of length and breadth of the enlarged photo = 4 : 12 = 1 : 3

Are the two ratios equal? Ravi also realised that the ratio of length and breadth of the enlarged photo was not equal to that of the original photo. He understood that the second picture was not proportionate to the first.

8 cm



12 cm

He asked the lab-man to develop another enlarged photo.

This time the photo was good. He again measured the length and breadth and calculated the ratio.

Ratio of length and breadth = 8 : 12 = 2 : 3

Now, Ravi understood that the original photo and the new enlarged photo looked fine to him because the ratios of their length and breadth were equal i.e., they were in proportion.

Thus, two ratios are said to be in proportion when they are equal. The symbol we use for propotion is '**: :**' (is as). If two ratios **a : b** and **c : d** are equal, we write **a : b = c : d** or **a : b : : c : d**. We read this as '**a is to b is proportionate to c is to d**'. This can also be read as '**a is to b is as c is to d**'.

The four quantities a, b, c and d are called first, second, third and fourth terms respectively. The first and fourth terms are known as extreme terms or extremes. The second and third terms are known as middle terms or means.

In a proportion, $a : b = c : d$

$$\text{i.e. } \frac{a}{b} = \frac{c}{d}$$

Therefore, $ad = bc$

Thus, The product of the means is equal to the product of the extremes.

i.e.,
$$\underbrace{a : \overbrace{b = c}^{\text{Means}} : d}_{\text{Extremes}}$$

Here 'd' is called the fourth proportional and $d = \dfrac{b.c}{a}$

Let us consider some examples

Example 1 : Find $\square$ to complete the proportion.

(i) $2 : 5 = 6 : \square$

Solution: The product of the means is equal to the product of the extremes,

i.e. $\underbrace{2 : \overbrace{5 = 6} : \square}$

Therefore, $2 \times \square = 5 \times 6$

$$\square = \frac{30}{2} = 15$$

(ii) $16 : 20 = \square : 35$

The product of the means is equal to the product of the extremes,

i.e. $\underbrace{16 : \overbrace{20 = \square} : 35}$

Therefore, $20 \times \square = 16 \times 35$

$$\square = \frac{560}{20} = 28$$

$\therefore$ $6 : 20 = \boxed{28} : 35$

Exercise 2

1. Find the missing numbers in the following proportions in the table given below .

S.No.	Proportion	Product of extremes	Product of means
(i)	1 : 2 : : 4 : 8		
(ii)	5 : 6 : : 75 : 90		
(iii)	3 : 4 : : 24 : 32		
(iv)	2 : 5 : : $\square$: 15	30	
(v)	3 : 6 : : 12 : $\square$		72

2. Write true or false.

(i) 15 : 30 : : 30 : 40

(ii) 22 : 11 : : 12 : 6

(iii) 90 : 30 : : 36 : 12

(iv) 32 : 64 : : 6 : 12

(v) 25 : 1 : : 40 : 160

3 Madhu buys 5 kg of potatoes at the market. If the cost of 2 kg is ₹. 36, how much will Madhu pay?

4. A man whose weight is 90 kgs weighs 15 kg on the moon, what will be the weight of a man whose weight is 60 kg?

5. A disaster relief team consists of engineers and doctors in the ratio of 2 : 5.

(i) If there are 18 engineers, find the number of doctors.

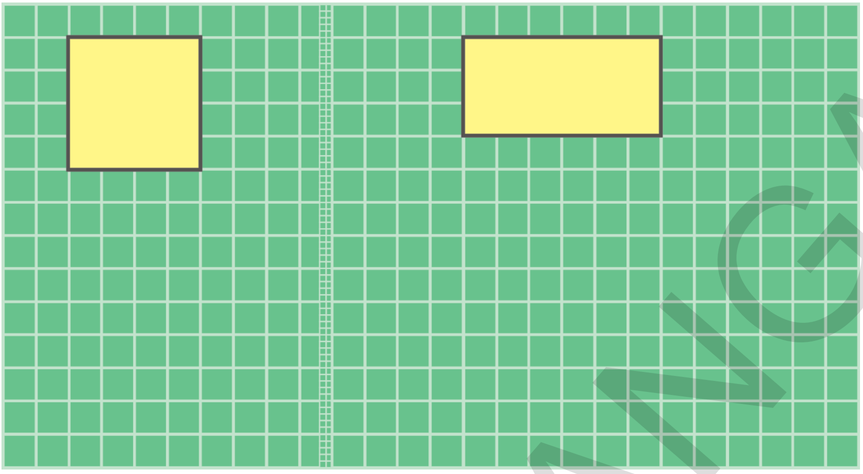
(ii) If there are 65 doctors, find the number of engineers.

6. The ratio of two angles is 3 : 1. Find the

(i) larger angle if the smaller is 180^{o} (ii) smaller angle if the larger is 63^{o}.

Do This

Enlarge the square and rectangle such that the enlarged square and rectangle remain proportional to the original square and rectangle.



6.3 Rate

Sometimes ratios appear as rates. Some examples are given below :

i) My father drives the vehicle with a speed of 60 km per hour.

ii) I bought apples at the rate of ₹ 120 per kg.

iii) My heart rate is 72 beats per minute.

iv) The cost of eggs is ₹ 60 per dozen.

v) The birth rate of India is 21 (approximately). (Birth rate is the number of live births per thousand people in a given time - Refer: *http://www.indexmundi.com/g/g.aspx?c=in&v=25*)

In the first example the distance travelled by the vehicle is compared with the time taken. In the second example cost of apples is compared to the quantity of apples. In the third example the number of heart beats is compared to the time taken. In the fourth example, the cost of eggs is compared to the quantity of eggs. In the fifth example, the number of live births is compared to 1000 poeple.

The word per can be replaced with the symbol '/' and the above examples can be written as 60km/hour, ₹ 120/kg, 72 beats/ minute, ₹ 60/dozen and 21 births per1000 people.

6.4 Unitary Method

The method in which we first find the value of one unit and then the value of the required number of units is known as unitary method.

Example 2 : A shopkeeper sells 5 tumblers for ₹ 30. What would be the cost of 10 such tumblers?

Solution Cost of 5 tumblers = ₹ 30

Therefore, Cost of 1 tumbler = $\frac{30}{5}$ = ₹ 6

Thus, cost of 10 tumblers = 6×10 = ₹60.

Example 3 : What is the cost of 9 bananas, if the cost of a dozen bananas is ₹ 20?

Solution 1 dozen = 12 units.

Cost of 12 bananas = ₹ 20

Therefore, cost of 1 banana = ₹ $\frac{20}{12}$

Thus, cost of 9 bananas = $\frac{20}{12} \times 9$ = ₹ 15

Do This

1. 40 benches are required to seat 160 students. How many benches will be required to seat 240 students at the same rate?
2. When a Robin bird flies, it flaps wings 23 times in ten seconds. How many times will it flap its wings in two minutes?"
3. The average human heart beats at 72 times per minute. How many times does it beat in 15 seconds? How many in an hour? How many in a day?

6.5 Direct Proportion

There are various situations in day-to-day life, when a change in one quantity leads to a change in the other quantity.

For example**:**

- If the number of things purchased increases, the cost incurred also increases. Alternately, if the number of things purchased decreases, the cost incurred also decreases.
- If the money deposited with a bank increases, the interest earned on that sum also increases. Alternately, if the money in the bank decreases, the interest also decreases.
- At a constant speed, if the distance travelled increases, the time taken for it also increases. Alternately, if the distance travelled decreases, time also decreases.

In the above examples, when one quantity increases the other also increases and vice-versa. Let us understand such situations with the help of an example.

A tap takes 1 hour to fill 300 litres of a tank. How many litres will be filled up in 2 hours?

The tank will filled up by 600 litres. How many in 4 hours, how many in 8 hours? How do you make this calculation?

Look at the table given below **:**

Time taken to fill tank (hours)	1	2	4	8
Capacity filled (lts)	300	600	1200	2400

You will find that in each case above, if the time taken increases the quantity of water filled also increases such that the ratio of the time taken and the ratio of the quantity filled is same. Thus, when the time taken doubles, the quantity filled will also doubled; when the time taken is 4 times, the quantity filled is also four times the original. And when the time taken is 8 times, the quantity filled is also 8 times. The ratio of the time taken is 1 : 2 and the ratio of quantity filled is also 1 : 2. Thus, we can say that time taken to fill the tank and quantity filled are in direct proportion.

Example 4 : A shopkeeper sells 6 eggs for ₹ 30. What would be the cost of 10 eggs?

Solution : Let the cost of 10 eggs be ₹ x.

We know that as the number of eggs increases, the cost will also increases such that the ratio of the number of eggs and the ratio of their costs will remain the same. In other words, the ratio of the number of eggs and the ratio of the cost of eggs is in proportion.

Thus, $6 : 10 = 30 : x$

Since the product of the means is equal to the product of the extremes **:**

$6 \times x = 10 \times 30$

$6x = 10 \times 30$

$$x = \frac{10 \times 30}{6} = 50$$

$x = ₹\ 50$

Thus, the cost of 10 eggs is equal to ₹50.

This problem can be solved by using unitary method too i.e. finding the cost of one egg and then multiplying the unit cost with the number of eggs required.

Cost of 6 eggs is ₹ 30

Therefore, cost of 1 egg $= \frac{30}{6} =$ ₹ 5

Cost of 10 eggs $= 5 \times 10 =$ ₹ 50

Example 5: 20 kgs of rice is needed for a family of 4 members. How many kgs of rice will be required if the number of members in the house increases to 10?

Method 1 : Girija said as the number of members increases, the amount of rice required will also increase such that the ratio of number of members and the ratio of the amount of rice is the same. Thus, the number of members and amount of rice are in direct proportion.

Solution : Let x be the amount of rice required for 10 members

Then $x : 20 = 10 : 4$

Since the product of the means is equal to the product of the extremes:

$$4x = 20 \times 10$$

$$x = \frac{20 \times 10}{4} = 50$$

$$x = 50 \text{ kgs}$$

$\therefore$ Amount of Rice required for 10 members = 50 Kgs.

Method 2 : Sarala used the unitary method to solve this problem **:**

Amount of Rice required for 4 members = 20 kgs.

Thus, amount of Rice required for one member = $\frac{20}{4}$ = 5 kgs.

$\therefore$ Amount of Rice required for 10 members = 10 × 5 = 50 kgs.

Example 6 : A jeep travels 90 km in 3 hours at a constant speed. In how many hours will the jeep covers 150 kms?

We know that as the distance travelled increases the time taken will also increases such that the ratio of the number of km and the ratio of the times taken is the same. Thus, the number of kms and the time taken is directly proportional.

Solution : Let x be the number of hours for the jeep to cover 150 kms.

Thus, $x : 3 = 150 : 90$

Since the product of the means is equal to the product of the extremes

$$90\,x = 150 \times 3$$

$$x = \frac{150 \times 3}{90} = 5$$

$$x = 5$$

$\therefore$ Time taken to cover 150 Km = 5 hours.

Example 7 : The scale of a map is given as 1:30000. Two cities are 4 cm apart on the map. Find the actual distance between them.

Solution : Let the actual distance be x cm. Since the distance on the map is directly proportional to the actual distance,

$1:30000 = 4 : x$

Since the product of the means is equal to the product of the extremes

$x = 4 \times 30,000$

$= 1,20,000$ cm

$= 1.2$ kms (1 km = 1,00,000 cm)

Thus, two cities, which are 4 cm apart on the map, are actually 1.2 kms away from each other.

Try This

1. Place a 1litre empty bottle under a tap from which water is falling drop by drop due to leakage. How much time did it take to fill the bottle? Calculate how much water would be wasted in a year?

2. Take a clock and fix its minutes hand at 12.

Note the angles made by minutes hand in the given intervals of time :

Time Passed	(T_1)	(T_2)	(T_3)	(T_4)
(in minutes)	15	30	45	60
Angle turned	(A_1)	(A_2)	(A_3)	(A_4)
(in degree)	90	....	....	....

Is the angle turned through by the minute hand directly proportional to the time that has passed? Yes!

From the above table, you can also see

$T_1 : T_2 = A_1 : A_2$, because

$T_1 : T_2 = 15 : 30 = 1 : 2$

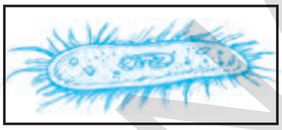
$A_1 : A_2 = 90 : 180 = 1 : 2$

Check if $T_2 : T_3 = A_2 : A_3$ and $T_3 : T_4 = A_3 : A_4$

You can repeat this activity by choosing your own time interval.

Exercise - 3

1. A length of a bacteria enlarged 50,000 times attains a length of 5 cm. What is the actual length of the bacteria? If the length is enlarged 20,000 times only, what would be its enlarged length?



2. Observe the following tables and find if x is directly proportional .

(i)

x	20	17	14	11	8	5	2
y	40	34	28	22	16	10	4

(ii)

x	6	10	14	18	22	26	30
y	4	8	12	16	20	24	28

(iii)

x	5	8	12	15	18	20	25
y	15	24	36	60	72	100	125

3. Sushma has a road map with a scale of 1 cm representing 18 km. She drives on a road for 72 km. What would be her distance covered in the map?

4. On a Grid paper, draw five squares of different sizes. Write the following information in a tabular form.

	Square 1	Square 2	Square 3	Square 4	Square 5
Length of a side (L)					
Perimeter (P)					
Area (A)					

Find whether the length of a side is in direct proportion to:

(i) the perimeter of the square.

(ii) the area of the square.

Ratios also appear in the form of percentages. We will learn about percentages and the various ways in which we use them in day-to-day life.

6.6 Percentages

- Soumya got 65% marks in Mathematics and Ranjeet got 59% marks.
- A cloth seller in whole-sale market makes a profit of 25% on silk sarees in the retail-market makes a profit of 10%.

- Anita borrowed a loan of ₹ 10000 from the bank for one year. She has to pay a 10% interest at the end of the year.
- During festival season a T.V. seller was offering a discount of 10% and another was offening a discount of 15%.

The word ‘**percent**’ means ‘**per every hundred**’ or ‘for a hundred’. The symbol ‘%’ is used to represent percentage. Thus, 1% (one percent) means 1 out of a 100; 27% (27 percent) means 27 out of 100 and 93% (ninty three percent) means 93 out of a 100.

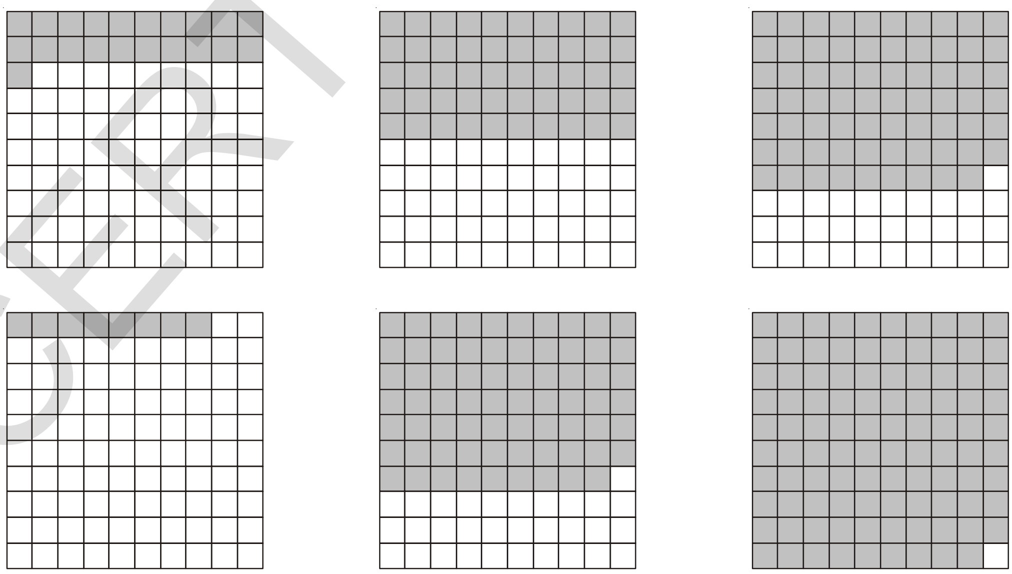
1% can also be written as $\frac{1}{100}$ or 0.01

27% can also be written as $\frac{27}{100}$ or 0.27

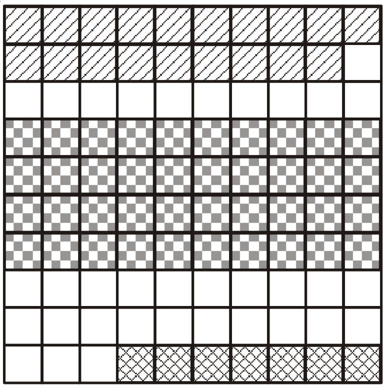
93% can also be written as $\frac{93}{100}$ or 0.93

Do This

1. Given below are various grids of 100 squares.
   Each has a different number of squares coloured.
   In each case, write the coloured and white part in the form of a (1) Percentage, (2) Fraction and (3) Decimal.



2. Look at the grid paper given below. It is shaded in various designs.



Find the percentage of each design

What percent represents ?

What percent represents ?

What percent represents ?

What percent represents ?

3. The strength particular of a school are given below. Express the strength of each class as a fraction, percentage of total strength of the school.

Class	No. of children	As a fraction	As a percentage
VI	17		
VII	15		
VIII	20		
IX	30		
X	18		
Total	100		

In all the above examples the total number is 100. How do we find percentages when the total is not hundred?

Example. 8 : In a class there are 35 girls and 15 boys. What is the percentages of boys and what is the percentage of girls?

Sudhir solved it like this;

Table - 1

Solution:

Student	Number	Fraction	Converting denominator into hundred	As a percentage
Girls	35	$\frac{35}{50}$	$\frac{35}{50} \times \frac{100}{100} = \frac{70}{100}$	70%
Boys	15	$\frac{15}{50}$	$\frac{15}{50} \times \frac{100}{100} = \frac{30}{100}$	30%
Total	50			

Table - 2

Anwar found the percentage of girls and boys like this.

Total students $35 + 15 = 50$

Out of 50 students there are 35 girls

Thus, out of 100 students there will be $\frac{35}{50} \times 100 = 70$ girls

Table - 3

Reena solved it like this

$\frac{35}{50} \times \frac{2}{2} = \frac{70}{100} = 70\%$

We see that there are three methods that can be used to find percentage when the total does not add up to 100. In the first table, we multiply the fraction by $\frac{100}{100}$. This does not change the value of the fraction. Subsequently, only 100 remains in the denominator. Reena has multiplied by it $\frac{2}{2}$ to get 100 in the denominator. Anwar has used the unitary method. You can choose any of the methods or you can also find your own method.

Does Anwar's method work for all ratios? Does the method be used by Reena also work for all ratios?

Anwar says Reena's method can be used only if you can find a natural number which on multiplication with the denominator gives 100. Since denominator was 50, she could multiply it by 2 to get 100. If the denominator was 60, she would not have been able to use this method. Do you agree?

Example 9 : Shirt "A" has $\frac{3}{5}$ cotton where as shirt "B" has $\frac{3}{4}$ cotton.

(i) What is the percentage of cotton in each shirt?

(ii) Which shirt has more percentage of cotton?

Solution : The percentage of cotton in shirt "A" = $\frac{3}{5} \times 100 = 60\%$

The percentage of cotton in shirt "B" = $\frac{3}{4} \times 100 = 75\%$

shirt "B" has more percentage of cotton.

Example 10 : Ganga went to a tailor with 1 mt. cloth. She asked him to make a blouse to her. The tailor used 0.75 mts of cloth to make the blouse and returned the remaining cloth to Ganga.

What percentage of the cloth (i) is used in making the blouse (ii) is given back to Ganga?

Solution : The tailor used 0.75 mts of cloth.

The percentage of cloth used $= 0.75 \times 100\%$

$$= \frac{75}{100} \times 100\,\%$$

$$= 75\%$$

The tailor returned $1 - 0.75 = 0.25$ mts of cloth.

The percentage of cloth returned $= 0.25 \times 100\%$

$$= \frac{25}{100} \times 100\ \%$$

$$= 25\ \%$$

Example 11 : Last year the cost of a commodity was ₹ 40. This year, the cost of the comodity increased to ₹ 50. What is the percentage change in its price?

Solution : Percentage increase in price $= \frac{\text{amount changed}}{\text{original amount}} \times 100\ \%$

$$= \frac{50-40}{40} \times 100\,\%$$

$$= \frac{10}{40} \times 100\% = \frac{1000}{40}\% = 25\ \%$$

Example 12 : Shyam's monthly income is ₹ 10,000. He spends 60% of it on family expenses, 10% on medical expenses, 5% on donations and saves by 25%. Find the amount he spends on each item?

Solution : Amount spent on family expenses = 60% of total income

$= 60\%$ of ₹.10000

$= \frac{60}{100} \times 10000 =$ ₹.6000

Similarly, amount spent on medical expenses $= \frac{10}{100} \times 10000 =$ ₹.1000

Amount spent on donations $= \frac{5}{100} \times 10000 =$ ₹.500

Amount saved $= \frac{25}{100} \times 10000 =$ ₹. 2500

Exercise 4

1. In a school X, 48 students appeared for 10th class exam out of which 36 students passed. In another school Y, 30 students appeared and 24 students passed. If the District Educational Officer wants to give an award on the basis of pass percentage. To which school will he give the award?

2. Last year the cost of 1000 articles was ₹ 5000. This year it goes down to ₹ 4000. What is the percentage of decrease in price?

3. Sri Jyothi has a basket full of bananas, oranges and mangoes. If 50% are bananas, 15% are oranges, then what percent are mangoes?

4. 64% + 20% + ...?....... = 100%

5. On a rainy day, out of 150 students in a school 25 were absent. Find the percentage of students absent from the school? What percentage of students are present?

6. Out of 12000 voters in a constituency, 60% voted. Find the number of people voted in the constituency?

7. A local cricket team played 20 matches in one season. If it won 25% of them and lost rest. How many matches did it loose?

8. In every gram of gold, a goldsmith mixes 0.25 grams of silver and 0.05 grams of copper. What is the percentage of gold, silver and copper in every gram of gold?

9. 40% of a number is 800 then find the number?

Try This

1. Population of our country as per 2011 census is about 12×10^8 (120,00,00,000)

   If the population of our country increases by 3% every year what will be the population by 2012?

2. (i) Can you eat 75% of a dosa?

   (ii) Can the price of an item go up by 90%?

   (iii) Can the price of an item go up by 100%?

Project Work

Fill up the following table showing the amount of time you spend on various activities in a day and calculate the percentage of time on each activity.

Activity	No. of hours	% of a day
For brushing bathing and getting ready for school		
In school		
For reading and doing home work		
For playing / watching TV/helping parents		
For sleeping		

6.7 Some situations in which we use percentages

We use percentages to express profit and loss, discount and interest. Expressing these in percentages makes comparisons easy.

6.7.1 Profit and Loss

- A potter makes pots on the wheel, then bakes them in a kiln and decorates them with paint. He spends ₹ 3 on material, ₹ 2 on baking and ₹ 1 on painting the each pot. He sells each pot for ₹ 10. Does the potter make profit or loss?
- A toy maker makes a toy for ₹ 50 and sells it for ₹ 75. Does he make profit or loss?
- A trader buys shirts at ₹ 540 each. The shirts remain unsold till the end of the year. The trader sells them at ₹ 500 each at year end. Did the trader make a profit or a loss?

- Amar is a gold merchant. He bought 10 gms of gold worth ₹ 15000 in the last year. Now its rate has gone up to ₹ 20000. Will Amar make a profit or a loss on selling the gold?

For each of the above situations you can calculate the amount of profit or loss. However, many a times percentages are used in expressing the profit or loss made in a transaction.

Example 13 : Ramayya bought pens for ₹200 and he sold them for ₹ 240 whereas Somayya bought pens for ₹ 500 and he sold them for ₹ 575. Who made more profit?

Solution : Ramayya's Profit = ₹ 240 – ₹ 200 = ₹ 40

Somayya's Profit = ₹ 575 – ₹ 500 = ₹ 75

It appears like Somayya made more profit as he made a profit of ₹ 75 where as Ramayya made a profit of ₹ 40 only. Is this correct?

Ramayya made a profit of ₹ 40 when he invested an amount of ₹200 where as Somayya made a profit of ₹ 75 when he invested an amount of ₹ 500.

Thus, Ramayya's ratio of profit and cost $= \frac{40}{200}$ and

Somayya's ratio of profit and cost is $= \frac{75}{500}$

To compare profit, cost ratios we convert them in to percentages.

Thus,

Ramayya's profit percentage

$= \frac{40}{200} \times 100\%$

= 20 %

Somayya's profit percentage

$= \frac{70}{500} \times 100\%$

= 15%

Ramayya earn a profit of 20% or ₹ 20 on investment of ₹ 100 and Somayya earns a profit of 15% or ₹ 15 on investment of ₹100. Thus, Ramaya earns more profit percent than Somayya.

Example 14 : A shop keeper bought a TV for ₹ 9000 and he sold it for ₹ 10,000. Find the profit or loss? calculate percentage.

Solution : **Gopal solved the problem in the following way:**

Cost price (CP) of the TV = ₹ 9000

Selling price (SP) of the TV = ₹ 10,000

As SP is greater than CP, the shopkeeper makes a profit:

Profit (P) = ₹ 10000 – ₹ 9000 = ₹ 1000

Thus, when the CP is ₹ 9000, the shopkeeper makes a profit of ₹ 1000

The ratio of profit and cost is $\frac{1000}{9000}$

To find the profit percentage we multiply this ratio with 100%

i.e. $\frac{1000}{9000} \times 100\% = \frac{100}{9}\% = 11\frac{1}{9}\%$

Madhu solved this problem using proportion.

When the CP is ₹ 9000, the profit is ₹ 1000.

Now, when CP is ₹ 100, let the profit be ₹ x.

We know that the CP and profit are directly proportional thus, ratio of profit and the ratio of cost price (CP) will be same in both cases.

Therefore, $x : 1000 = 100 : 9000$

$$\frac{x}{1000} = \frac{100}{9000}$$

$$9000 \times x = 1000 \times 100$$

$$x = \frac{1000 \times 100}{9000} = 11\frac{1}{9}.$$

Thus, the profit % $= 11\frac{1}{9}$ %

Try This

The cost price of 12 mangoes is equal to the selling price of 15 mangoes. Find the loss percent?

Example 15 : Suppose a person buys an article for ₹ 650/- and gains 6% on selling it. Find the selling price?

Solution : Ravi solved it like this:

CP = ₹ 650

Gain % = 6%

So, if the CP is ₹ 100 then gain is ₹ 6 and SP is 100 + 6 = ₹ 106

Now, when the CP is ₹ 650 let the SP be ₹ x.

The CP and SP are directly proportional

Therefore, The ratio of CP = ratio of SP

$100 : 650 = 106 : x$

$$\frac{100}{650} = \frac{106}{x}$$

Therefore, $100x = 106 \times 650$

Therefore, $x = \frac{106 \times 650}{100} = 689$

Thus, the SP = ₹ 689

Arun solved it like this:

CP = ₹ 650

Profit % = 6%

Thus, profit = 6% of 650

$$\frac{6}{100} \times 650 = 39$$

We know that SP = CP + Profit

$= 650 + 39 = 689$

Thus, the SP = ₹ 689

Example 16 : Ramesh sold a D.V.D player for ₹2800 at a gain of 12%. For how much did he buy it?

Solution : Naik uses proportion:

Gain % = 12%

SP = ₹ 2800

So, If CP is ₹ 100, then SP is ₹ 112

When SP = ₹ 2800, let its CP be ₹ x.

CP and SP are directly proportional

Thus, ratio of CP = ratio of SP

$x : 100 = 2800 : 112$

$$\frac{x}{100} = \frac{2800}{112}$$

Therefore, $112 \times x = 100 \times 2800$

Therefore, $x = \frac{100 \times 2800}{112} =$ ₹ 2500

Thus, CP = ₹ 2500

Meena uses the unitary method:

S.P = 2800

Gain = 12%

If CP is 100, then profit is 12

SP= 100+12=112

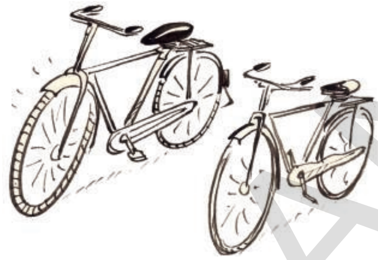
So, when SP in ₹ 112 then CP is ₹ 100

Therefore, when SP is 1 then CP is $\frac{100}{112}$

Thus, when SP is ₹2800 then CP is $\frac{100}{112} \times 2800$ = ₹ 2500

CP = ₹ 2500

Example 17 : A man sold two cycles for ₹ 3000 each, gaining 20% on one and losing 20% on the other. Find his gain or loss percentage on the whole transaction?



Solution: SP = ₹ 3000

Gain% on first cycle = 20%

Loss% on second cycle = 20%

Method (i) : Using the unitary method:

For first cycle

If CP is ₹ 100, then the profit is ₹ 20 and SP = 100 + 20 = ₹120

Thus, if SP is ₹ 120 then CP is ₹100

Now, if SP is 1 then CP is $= \frac{100}{120}$

Now, if SP is ₹ 3000 then CP $= \frac{100}{120} \times 3000 =$ ₹ 2500

For second cycle

If CP is ₹ 100 then the loss is 20 and the SP is 100 – 20 = ₹ 80

Thus, if SP is ₹ 80 then CP is = ₹ 100

Now, if SP is Rs. 1 then CP is $= \frac{100}{80}$

Now, if SP is ₹ 3000 then CP is $= \frac{100}{80} \times 3000 =$ ₹ 3750

Total CP = ₹ 2500 + ₹ 3750 = ₹ 6250

Total SP = ₹ 3000 + ₹ 3000 = ₹ 6000

Since SP is less than CP, loss = 6250 – 6000 = ₹ 250

$$\text{Loss } \% = \frac{loss}{CP} \times 100 = \frac{250}{6250} \times 100 = 4\%$$

Method (ii): Using proportion:

On the first cycle:

When CP increases SP will increase, thus CP and SP are in direct proportion.

CP	SP
100	120
x	3000

Thus, the ratio of CP = ratio of SP

$100 : x = 120 : 3000$

$$\frac{100}{x} = \frac{120}{3000}$$

$100 \times 3000 = 120\,x$

$$\frac{100 \times 3000}{120} = x$$

$x = 2500$

Thus, CP of first cycle = ₹ 2500

On the second cycle:

CP	SP
100	80
x	3000

$100 : x = 80 : 3000$

$$\frac{100}{x} = \frac{80}{3000}$$

$$x = \frac{100 \times 3000}{80} = ₹\ 3750$$

Therefore, total CP of two cycles = ₹ 2500 + ₹ 3750

= ₹ 6250

Total SP of cycles = ₹ 6000

Since SP is less than CP, he has a loss

Loss = ₹ 6250 – ₹ 6000 = ₹ 250

Therefore, loss percentage = $\frac{Loss}{C.P} \times 100 = \frac{250}{6250} \times 100 = 4\%$

Method (iii): SP of first cycle = ₹ 3000

Gain% = 20%

Let the CP be ₹ x

Then, the profit = $\frac{20}{100} \times x = \frac{20}{100} x$

We know that SP = CP + profit

Thus, $x+\frac{20}{100}x = 3000$

$\frac{100x+20x}{100} = 3000$

$\frac{120x}{100} = 3000$

$x = \frac{3000 \times 100}{120} = ₹\,2500$

Thus, CP of the first cycle = ₹ 2500

SP of second cycle = ₹ 3000

Loss % = 20%

Let the CP be ₹ x

Then, the loss $\frac{20}{100} \times x = \frac{20}{100}x$

We know that SP = CP – loss

Thus, $x - \frac{20}{100}x = 3000$

$\frac{80}{100}x = 3000$

80 x = 3000 × 100

$x = \frac{3000 \times 100}{80} = ₹\,3750$

Thus, CP of the second cycle = ₹ 3750

Therefore, total CP of two cycles = ₹ 2500 + ₹ 3750 = ₹ 6250

Total SP of cycles = ₹ 6000

Since SP is less than CP, he has a loss

Loss = ₹ 6250 – ₹ 6000 = ₹ 250

Therefore, loss = $\frac{Loss}{C.P} \times 100 = \frac{250}{6250} \times 100 = 4\%$

Example 18 : The cost of an article goes down every year by 20% of its previous value. Find its original cost if the cost of it after 2 years is ₹19,200?

Solution : Cost of an article at the end of 2nd year = ₹ 19,200

The cost decreases every year by 20%

Let cost at the beginning of 1st year be 100. At the begining of 2nd year it will be ₹ 80 (i.e. 100–20% of 100)

At the begning of the 3rd year = ₹ 64 (80 – 20% of 80)

Thus, an article that costs ₹ 100 will cost ₹ 64 at the begining of third year.

The cost of an article is ₹ 19200 after 2 years

Let the original cost be ₹ x.

Thus, ratio of the original cost = ratio of cost after 2 years

$x : 100 = 19200 : 64$

$$\frac{x}{100} = \frac{19200}{64}$$

$64\,x = 19200 \times 100$

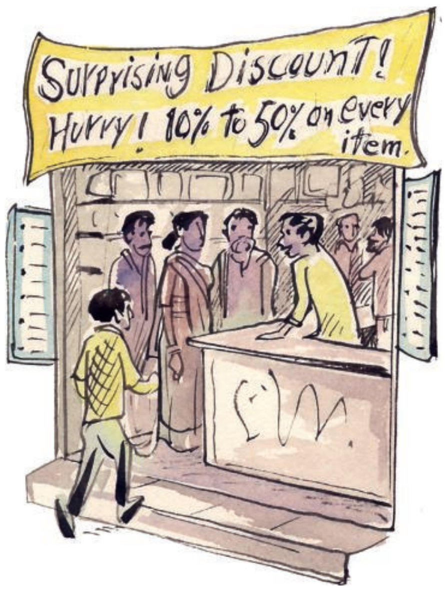
$$x = \frac{19200 \times 100}{64}$$

$= 30000$

Thus, the original cost of an article was ₹ 30000.

6.7.2 Discount

Situation 1 : Vijay opened a new cloth shop. To attract people, he advertised in the following way.



Situation 2 : On some special occassions such as dussera, deepawali, sankranthi business men offer discounts on marked price.

Situation 3 : Some times to clear their old stock or out dated stock, businessmen offer clearance sales in the form of discounts in the following way.

Example 19 : A shopkeeper marks his goods 25% above the cost price and allows a discount of 12% on them. What percent does he gain?

Solution: Let the cost price be ₹ 100.

Then marked price (MP) = ₹ 100 + ₹ 25 = ₹ 125.

Discount percent on marked price = 12%

$$\text{Discount} = \frac{12}{100} \times 125 = ₹\ 15$$

SP = MP – Discount

= 125 – 15 = 110

Gain = SP – CP

=110 – 100

= ₹ 10

$$\text{Gain\%} = \frac{10}{100} \times 100 = 10\%$$

Thus, the shopkeeper gains 10% after discount.

Exercise - 5

1. A shopkeeper bought a suit case for ₹ 480 and sold it for ₹ 540. Find his gain percent?
2. Ajay bought a TV for ₹ 15000 and sold it for ₹14100. Find the loss percent?
3. Ramu sold a plot of land for ₹ 2,40,000 gaining 20%. For how much did he purchase the plot?
4. On selling a mobile for ₹ 750, a shop keeper looses 10%. For what amount should he sell it to gain 5%?
5. A farmer sold 2 bullocks for ₹24000 each. On one bullock he gained 25% and on the other he lost 20%. Find his total profit or loss percent?
6. Sravya bought a watch for ₹ 480. She sold it to Ridhi at a gain of $6\frac{1}{4}$ %. Ridhi sold it to Divya at a gain of 10%. How much did Divya pay for it?
7. The marked price of a book is ₹ 225.The publisher allows a discount of ₹10% on it. Find the selling price of it?
8. A carpenter allows 15% discount on his goods. Find the marked price of a chair which is sold by him for ₹ 680?
9. A dealer allows a discount of ₹ 10% and still gains by 10%. What should be the marked price if the cost price is ₹ 900?

6.7.3 Simple Interest

Ramayya has ₹ 10,000. He requires ₹ 15,000 for agriculture. He approaches an agricultural bank manager. The conversation with the bank manager is as follows:

Ramayya: Sir, I need some money for agricultural purposes.

Bank manager : How much money do you require?

Ramayya : ₹ 5000

Bank manager : How long will you take to repay?

Ramayya : One year.

Bank manager : You have to pay an interest of 6% on the loan along with the lent amount after one year.

Ramayya : Yes sir, I will repay after one year the whole amount.

Bank manager : Do you know how much you have to pay after one year.

Ramayya : Yes, On ₹ 100 I have to pay ₹ 6.

So, on ₹ 1, I have to pay ₹ $\frac{6}{100}$ and on ₹ 5000, I have to pay ₹ $\frac{6}{100} \times 5000$

that is ₹ 300.Thus, I have to pay a total amount of ₹ 5300.

The money borrowed or lent out for a certain period is called the **Principle**. This money would be used by the borrower for some time before it is returned. For keeping this money for some time the borrower has to pay some extra money to the bank. This is known as **Interest**.

The amount that is to be repayed back is equal to the sum of the borrowed principle and the interest. That is, **Amount = Principle + Interest.**

Interest is generally expressed as percent of the principle for a period of one year. It is written as say 10% per year or per anmum or in short as 10% p.a.

10% p.a. means on every ₹100 borrowed, ₹ 10 is the interest you have to pay for one year. Let us take an example and see how this works.

Example 20 : Sunita takes a loan of ₹ 5000 at 12% rate of interest. Find the interest she has to pay at the end of one year.

Solution : Principle = ₹5000, Rate of interest = 12 % per year

If ₹ 100 is borrowed, sunita has to pay ₹ 20 interest for one year. Since the amount borrowed is ₹5000 the interest she has to pay for one year

$$= \frac{12}{100} \times 5000 = ₹\ 600$$

So, at the end of the year she has to pay an amount of ₹ 5000 + ₹ 600 = ₹ 5600

> In general, when P is principle, R% is rate of interest per annum and I is the interest, the amount to be received at the end of the year is:
>
> $$A = P + \frac{P \times R}{100}$$

If Ramayya, due to unavoidable circumstances, can not pay the total amount as requested by the manager in one year then the loan can be extended for one more year, The interest for next year will also be ₹ 300. Thus, Ramayya will pay ₹600 interest for 2 years.

For ₹100 borrowed for 3 years at 18%, the interest be paid at the end of 3 years will be 18 + 18 + 18 = 3 ×18 = ₹ 54

As the number of year increase the interest also increases. This interest being charged unformly for each year is called simple interest.

In general, for Principle = P, Rate of Interest = R and Time = T years.

Interest to be paid (I) = P×R%×T or $P \times \frac{R}{100} \times T = \frac{PRT}{100} = \frac{PTR}{100}$

Do This

1. Find the interest on a sum of ₹ 8250 for 3 years at the rate of 8% per annum.
2. ₹ 3000 is lent out at 9% rate of interest. Find the interest which will be recieved at the end of 2½ years.

Example 21 : In what time will ₹ 6880 amount to ₹ 7224, if simple interest is calculated at 10% per annum?

Solution : Amount = ₹ 7224

Principle = ₹ 6880

S.I = Amount - Principle = ₹ 7224 – ₹ 6880 = ₹ 344

R% = 10%

Now I = $P \times \frac{R}{100} \times T$

$$344 = 6880 \times \frac{10}{100} \times T$$

344 × 100 = 6880 × 10 × T

Therefore, T = $\frac{344 \times 100}{6880 \times 10} = \frac{1}{2}$ year = 6 months

Example 22 : What sum will yield an interest of ₹ 3927 in 2 years and 4 months at 8% per annum?

Solution : S.I = ₹ 3927,

R% = 8 %

T = 2 year + 4 months

$$\left(2 + \frac{4}{12}\right) \text{Yrs.}$$

$$\left(2 + \frac{1}{3}\right) \text{Yrs.} = \frac{7}{3} \text{Yrs.}$$

Substituting in $I = P \times \frac{R}{100} \times T$

$3927 = P \times \frac{8}{100} \times \frac{7}{3}$

$3927 \times 100 \times 3 = P \times 8 \times 7$

Therefore, $P = \frac{3927 \times 100 \times 3}{8 \times 7}$

Thus, P = ₹ 21037.50

Therefore, Principle = ₹ 21037.50

Example 23 : At what rate per annum will ₹ 6360 yield an interest of ₹1378 in $2\frac{1}{2}$ years?

Solution : Principle (P) = ₹ 6360

Time (T) $= 2\frac{1}{2}$ years $= \frac{5}{2}$ years

Simple interest(S.I) = ₹ 1378

Substituting in $I = P \times \frac{R}{100} \times T$

$1378 = 6360 \times \frac{R}{100} \times \frac{5}{2}$

$1378 \times 100 \times 2 = 6360 \times 5 \times R$

Therefore, $R = \frac{1378 \times 100 \times 2}{6360 \times 5} = \frac{26}{3} = 8\frac{2}{3}\%$

Example 24 : At what rate per annum will the principle triples in 16 years?

Solution : Let the principle be ₹ x

Amount after 16 years = ₹ $3x$

Amount – Principle = Interest

Therefore, $3x - x = 2x$

For $P = x$, $T = 16$, $I = 2x$

$$I = P \times \frac{R}{100} \times T$$

$$2x = x \times \frac{R}{100} \times 16$$

$$2x \times 100 = x \times 16 \times R$$

Therefore, $R = \frac{2x \times 100}{x \times 16} = \frac{25}{2} = 12\frac{1}{2}\ \%$

Exercise - 6

1. How long will it take for a sum of ₹ 12600 invested at 9% per annum become to ₹ 15624?

2. At what rate a sum doubles itself in 8 year 4 months?

3. A child friendly bank announces a savings scheme for school children. They will give kiddy banks to children. Children have to keep their savings in it and the bank collects all the money once in a year. To encourage children savings, they give 6% interest if the amount exceeds by ₹ 10000, and other wise 5%. Find the interest received by a school if they deposit is ₹ 9000 for one year.

4. A sum of money invested at 8% per annum for simple interest amounts to ₹ 12122 in 2 years. What will it amounts to in 2 year 8 months at 9% rate of interest?

5. In 4 years, ₹ 6500 amounts to ₹ 8840 at a certain rate of interest. In what time will ₹1600 amounts to ₹ 1816 at the same rate?

Lets earn Interest

Children! Let us play a game on simple interest.

5 members can play this game.

1. Take 3 bowls each labelled as P, R and T. Drop 5 pieces of paper in each bowl such that every paper is marked with a number.
   (Hint: All the numbers in bowl P must be multiples of 100 or 1000.
2. Pick out 3 pieces of papers, one from each of the bowls, one after another.
3. The number on the paper picked from bowl P relates to principle, number on the paper picked from bowl T relates to time, number on the paper picked from bowl R relates to rate of interest.

4. Now calculate interest and tell I, P, T and R to every one.

5. If you say the right answer enter the interest amount in your account other wise put a 0 in your account.

Note : Repeat two or three rounds as per your wish and note down the values in the table given below:

Interest amount				
Name	1^{st} round	2^{nd} round	3^{rd} round	Total

Looking Back

- Many times in day-to-day life we compare quantities using ratios. For e.g., my income is ₹ 10000 and my friend's is ₹ 20000. Thus, my income is half of my friend's income or we can say that my friend's income is twice my income. The ratio of my income and my friends income is 1 : 2. and the ratio of my friend's income and my income is 2 : 1.

- When two ratio's are equal they are said to be in a proportion. The idea of proportion helps us solve various problems in our daily life.

- If some increase (decrease) in one quantity leads increase (decrease) in other quantity, the quantities are said to be in direct proportion.

- Ratio's can be expressed in the form of percentages. The word 'percent' means per hundred or out of every hundred. The symbol for percentage is '%'. 13% means 13 out of 100.

  $$13\% = \frac{13}{100} = 0.13$$

- Percentages are used in various situations like profit and loss, discount and simple interest etc.,

Fun with Fascinating Ratios

The digits 1,2,3,...9 can be arranged to form two numbers whose ratio is 1:2, as $\frac{7329}{14658} = \frac{1}{2} = 1:2$. This is interesting itself.

But even more fascinating is the fact that the nine digits can also be arranged to form numbers whose ratio is 1:3, 1:4, 1:5, 1:6, 1:7, 1:8 and 1:9. Enjoy by finding them.

DATA HANDLING 7

7.0 Introduction

Ravi is reading the sports section of a newspaper. There are two tables on the sports page of the newspaper.

Top 5 Batsmen in World Cup 2011

Name of the Batsman	Runs scored
T Dilshan (Sri Lanka)	500
Sachin Tendulkar (India)	482
K. Sangakkara (Sri Lanka)	465
Jonadhan Trott (England)	422
U Tharanga (Sri Lanka)	395

Table - 1

Top 5 Bowlers in World Cup 2011

Name of the Bowler	Wickets Taken
Shahid Affridi (Pakistan)	21
Zahir Khan (India)	21
TG Southee (New Zealand)	18
Robin Peterson (South Africa)	15
M. Muralitharan (Sri Lanka)	15

Table - 2

What do the two tables tell us?

Table 1 tells us the names of batsmen who scored the most runs in the World Cup, 2011 as well as the number of runs they scored. This information can help in taking decisions or in drawing conclusions. For e.g. it can help the organisers of the World Cup in deciding whom to award the prize for the best batsman.

Table-2 tells us the names of bowlers who took the most wickets in the World Cup, 2011 as well as the number of wickets they took. This information can also help in taking decisions or in drawing conclusions. For e.g. it can help the organisers of the World Cup in deciding whom to award the prize for the best bowler.

Information which is in the form of numbers or words and helps in taking decisions or drawing conclusions is called data. The names of batsmen and the runs they scored as well as the names of bowlers and the number of wickets they took is data. Tables and graphs are the ways in which data is presented.

The numerical entries in the data are called 'Observations'.

Try This

Look at your school information board. Do you find any data tables there? Find out who uses this data.

7.1 Organising data

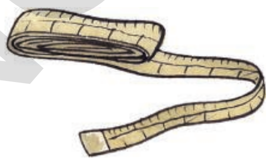
Details of seven students of class VIII in a school are collected under the Javahar Bala Arogya Raksha Scheme.

Krishna noted the heights of the following students in his notebook as

Amala-125cm, Lekhya-133cm, Thabasum-121cm, Sudha-140cm, Vanaja-117cm, Lenin-129cm and Rajesh-132cm.

Another student Kumar wrote the same data in the form of a table and arranged the heights in ascending order.

Name of the Student	Height (in cms)
Vanaja	117
Thabassum	121
Amala	125
Lenin	129
Rajesh	132
Lekhya	133
Sudha	140



Now, let us answer these questions.

(i) Who is the tallest amongst the students?

(ii) Who is the shortest amongst the students?

(iii) Whose height is between that of Amala and Rajesh?

Did you use the data written by Krishna? or by Kumar? to answer the question. You must have used Kumar's data as it is organised and thus easier to read and understand.

Do This

In a unit test Amar secured 20, 18, 23, 21, 24 and 22 marks in Telugu, Hindi, English, Mathematics, Science and Social Science respectively. Peter got 23, 21, 20, 19, 24 and 17 marks in the above subjects respectively. Interpret the data in an organized manner.

Classroom Project

Use the weighing machine to find the weights of all your classmates. Organise this data in the form a table. Make sure to arrange the weights in either ascending or descending order. Then answer the following questions:

a. Who is the lightest student in your class?

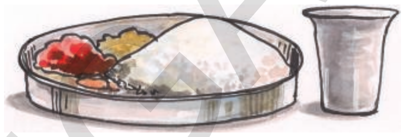
b. How many students weigh more than 25 kg?

c. How many students weigh between 20 and 30 kg?

7.2 Representative Values

In a hostel

- Average consumption of rice per child per day is 150 g.
- Average age of children is 13 years.
- Average height of children is 135 cm.



On studying this data, can we say that every child consumes exactly 150 gms of rice per day? Can we say that the age of each child in the class is 13 years? Can we say that the height of each child in class is 135 cm? Obviously not, we know that some children may take more than 150 gms of rice some may take less and some may take exactly 150 gms. A similar situation will hold for children's weight and height.

At the same time, 150 gms gives us an idea of the amount of rice consumed by each child in the hostel. It is a representative value of the amount of rice consumed by each child. Similarly, 13 years gives us an idea of the age of each child in the hostel. It is a representative value of the age of each child. The same holds for the height. All the above examples are of a particular representative value called arithmetic mean. In the section ahead, we shall learn about 'arithmetic mean' and also two other types of representative values called 'median' and 'mode'.

7.3.1 Arithmetic Mean

The physical education teacher in a school instructed his students to practice regularly Rajender had his practice sessions for a week as follows.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Minutes	20	35	40	30	25	45	15

Can we compute the time spent by Rajender for practice in terms of time spent per day? Let us observe.

What is the total time Rajender spent during the week on practice?

Total time = 20 + 35 + 40 + 30 + 25 + 45 + 15 = 210 minutes

Now to find the time spent on practice, per day, we divide the total time spent by the number of days.

i.e. $\frac{20+35+40+30+25+45+15}{7} = \frac{210}{7} = 30$ minutes

This is the average time spent on practice per day or the average practice session per day.

Example 1 : Earnings (in rupees) of a vegetable vendor in a week are ₹ 200, ₹150, ₹180, ₹300, ₹160, ₹170 and ₹170. Find his average earning, per day.

Solution : Total earnings (in rupees) = 200+150+180+300+160+170+170

= ₹1330

Number of days = 7

Average earning or mean earning $= \dfrac{1330}{7} =$ ₹ 190

The average of a data is also called Arithmetic Mean or Mean.

Average or Mean or Arithmetic Mean (A.M) $= \dfrac{\textbf{Sum of all observations}}{\textbf{Number of observations}}$

Try This

1. The ages (in years) of players are in a team of 16, 16, 16, 14, 17, 18. Then find the following:

   (i) Age of the youngest and the oldest player.

   (ii) Mean age of the players.

   What is the average number of glasses of water that you drink per day? in a week. How did you find the average?

7.3.2 Where does the mean lie?

The marks obtained by Anil, Amar, Anthony and Inder in Telugu, Hindi and English are given below.

	Telugu	Hindi	English
Anil	15	8	10
Amar	10	10	12
Antony	11	6	11
Inder	12	12	13

Now let us calculate the average marks obtained by the students in each subject.

Telugu	Hindi	English
$AM = \frac{15+10+11+12}{4}$	$AM = \frac{8+10+6+12}{4}$	$AM =$………
$= \frac{48}{4}$	$= \frac{36}{4}$	= ………
= 12	= ……..	= ………
Highest marks = 15	Highest marks = …….	Highest marks = …….
Least marks = 10	Least marks = …….	Least marks = …….
Mean = 12	Mean = …….	Mean = …….

Does the mean lie between the minimum and maximum value in each case?

You will find this to be true.

The arithmetic mean always lies between the highest and lowest observations of the data.

7.3.3 A property of mean

Example 2 : In a family, ages (in years) of members; Krishna, Radhika, Niharika and Nikhil are 44, 39, 17 and 12. (i) Find the arithmetic mean of their ages. (ii) What were their ages 5 years before? Find their mean age. (iii) Can you see a relationship between the change in mean and the number of years.

Solution : Present ages of family members are = 44, 39, 17, 12 years

Number of family members = 4

Therefore, Arithmetic Mean of their ages $= \frac{44+39+17+12}{4} = \frac{112}{4} = 28$ years

Ages of family members, 5 years ago = 44 – 5, 39 – 5, 17 – 5, 12 – 5

= 39, 34, 12, 7

$\therefore$ mean of their ages 5 years ago $= \frac{39+34+12+7}{4} = \frac{92}{4} = 23$ years

Thus, on reducing the age of each family member by 5 years, we find that the mean age of the family also decreases by 5 years from the present mean age.

Now calculate the mean age of the family, 3 years from now. What do you think will be the mean age of the family 10 years from now?

You will find that when all the values of data set are increased or decreased by a certain number, the mean also increases or decreases by the same number.

Try This

1. A data of 10 observations has a minimum value 15 and maximum value 25. What is the mean of the data?

   (i) 12 (ii) 15 (iii) 21 (iv) 27

2. Observations of a data are 23, 45, 33, 21, 48, 30, 34, 36 and 35. Without actual calculation choose the mean of the data.

   (i) 20 (ii) 35 (iii) 48 (iv) 50

Exercise - 1

1. Maximum day time temperatures of Hyderabad in a week (from 26th February to 4th March, 2011) are recorded as 26 °C, 27 °C, 30 °C, 30 °C, 32 °C, 33 °C and 32 °C.

   (i) What is the maximum temperature of the week?

   (ii) What is the average temperatures of the week?

2. Rice consumed in a school under the mid-day meal program for 5 consecutive days is 15.750 kg, 14.850 kg, 16.500 kg, 14.700 kg, and 17.700 kg. Find the average rice consumption for the 5 days.

3. In a village three different crops are cultivated in four successive years. The profit (in rupees) on the crops, per acre is shown in the table below-

year / Crop	2005	2006	2007	2008
Ground nuts	7000	8000	7500	7500
Jawar	6000	1000	8000	1000
Millets	9000	5000	3000	4000

   (i) Calculate the mean profit for each crop over the 4 years.

   (ii) Based on your answers, which crop should be cultivated in the next year?

4. The number of passengers who travelled in APSRTC bus from Adilabad to Nirmal in 4 trips in a day are 39, 30, 45 and 54. What is the occupancy ratio (average number of passengers travelling per trip) of the bus for the day?

5. The following table shows the marks scored by Anju, Neelesh and Lekhya in four unit tests of English.

Name of the Student	Unit Test I	Unit Test II	Unit Test III	Unit Test IV
Anju	Absent	19	23	21
Neelesh	0	20	22	24
Lekhya	20	24	24	24

(i) Find the average marks obtained by Lekhya.

(ii) Find the average marks secured by Anju. Will you divide the total marks by 3 or 4? Why?

(iii) Neelesh has given all four tests. Find the average marks secured by him. Will you divide the total marks by 3 or 4? Why?

(iv) Who performed best in the English?

6. Three friends went to a hotel and had breakfast to their taste, paying ₹ 16, ₹ 17 and ₹ 21 respectively (i) Find their mean expenditure.(ii) If they have spent 3 times the amount that they have already spent, what would their mean expenditure be? (ii) If the hotel manager offers 50% discount, what would their mean expenditure be? (iii) Do you notice any relationship between the change in expenditure and the change in mean expenditure.

7. Find the mean of the first ten natural numbers.

8. Find the mean of the first five prime numbers.

9. In a set of four integers, the average of the two smallest integers is 102, the average of the three smallest integers is 103, the average of all four is 104. Which is the greatest of these integers?

10. Write at least two questions to find the mean, giving suitable data.

Project Work

Find out the number of family members in the houses on your street. Calculate the average family size of your street.

7.4 Mode

The second type of representative value that we will learn about is mode.
Let us read the example given below-

Example 3 : A shop keeper wants to find out which cooking oil he should stock in more number. For this, he maintains a record of cooking oil sale for the week in the form of the table given below.

Day	Packets of oil sold
Mon	GGGSSSSPP
Tue	GGGSSSSSPP
Wed	GGSSSSSP
Thu	GGGSSSP
Fri	GGGSSPP
Sat	GSSSSSSSS
Sun	GGGSSSP

G = Ground nut oil packet, S = Sunflower oil packet, and P= Palmolien oil packet.

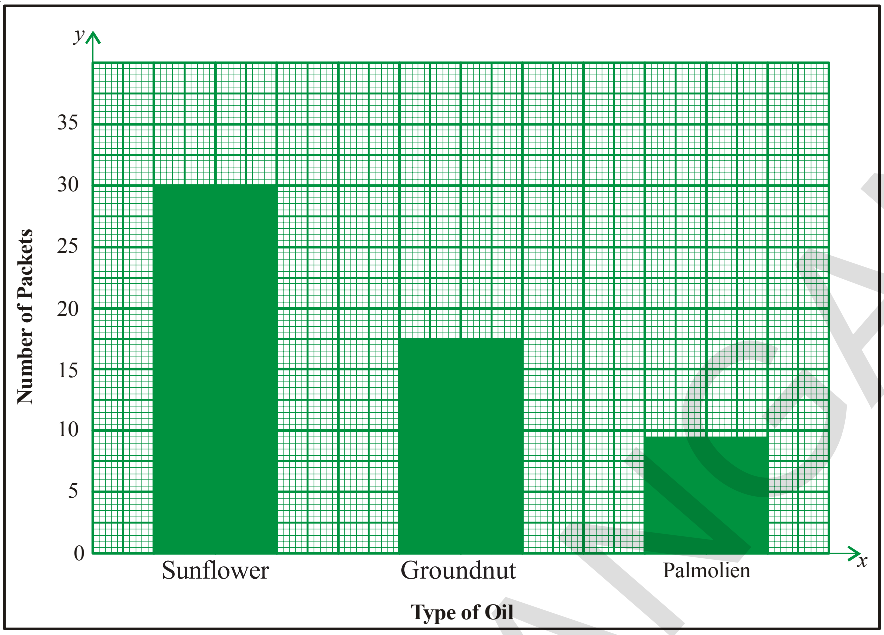
In such a situation will calculating the mean number of oil packets sold help the shopkeeper to take a decision?

Solution : The shopkeeper first calculates the averrage number of packets that he can order.

$$\text{Average number of packets} = \frac{18+30+9}{3} = \frac{57}{3} = 19.$$

Should the shopkeeper stock 19 packets for each type of oil? The shopkeeper looked at his sales figures again. He finds sunflower oil to be the most frequently demanded oil and palmolien oil to be the least demanded oil. If he was to order 19 packets of each he would fall short of sunflower oil and palmolien oil would be in surplus. The shopkeeper decides to stock more packets of sunflower oil and lesser number of packets of palmolien oil. Thus, the number of packets of sunflower oil i.e. 30 is the representative value for the shopkeeper's data as it tells him the most frequently purchased oil.

The most frequently occurring value for a set of observations is called the mode. The longest bar in a bar graph represents the mode, as can be seen in the bar graph given in the next page.



Example 4 : Find the mode of the given set of numbers- 2,3,5,3,4,7,3,2,1,7,3

Solution : Arranging the numbers with same value together, we get 1,2,2,3,3,3,3,4,5,7,7

3 occurs more frequently than the other observations. Thus, Mode = 3

Example 5 : Find the mode of the data 3, 5, 9, 6, 5, 9, 2, 9, 3, 5.

Solution : Arranging the numbers with the same value together we get 2, 3, 3, 5, 5, 5, 6, 9, 9, 9.

Here both 5 and 9 occurs more and equal number of times i.e., 3 times.

Thus, the given data contains two modes, i.e., 5 and 9

This kind of data is called **Bimodal Data**.

Note : If each observation in a data set is repeated an equal number of times then the data set has no mode.

Try This

1. Find the modes of the following data.

(i) 5, 6, 3, 5, 4, 9, 5, 6, 4, 9, 5

(ii) 25, 14, 18, 15, 17, 16, 19, 13, 12, 24

(iii) 10, 15, 20, 15, 20, 10, 15, 20, 10

Example 6 : Following are the marks obtained by 50 students in a unit test, which is administered for 10 marks. Find the mode of the data.

Marks obtained	No of students
00	2
1	1
2	2
3	1
4	-
5	4
6	10
7	15
8	9
9	5
10	1
Total	50

Solution: In the data marks are observations. From the data table it is clear that 7 marks are obtained by many students.

Mode of the data is 7

Note: The observation 7 that repeats fifteen times is the mode and number of times i.e.15 should not be confused as the mode.

Example 7 : In which of the following situations, is the mode an appropriate representative value?

(a) A shopkeeper selling shirts, needs to decide which size of shirts to order more.

(b) For purchasing rice for a party of 20 people.

(c) For finding the height of the door in your house.

Solution : (a) Let us look at the first situation. Supposing the shopkeeper is selling 4 sizes of shirts and his sale for the month of February is-

Shirt Size	Number
M	15
L	18
XL	40
XXL	22
Total	92

The average number of shirts sold by the shopkeeper is $\dfrac{12+18+40+22}{4} = 23$ shirts.

In such a situation does it make sense for the shopkeeper to order 23 shirts of each size? The shopkeeper looks at his data again. He finds that the most frequently purchased size is XL, If he orders 23 shirts of each size, he will fall short of size XL shirts. He thus finds it more sensible to order more shirts of this size and lesser of the rest.

Thus, the shopkeeper uses mode or the most frequently occurring value to take his decision.

(b) Neither we know how many take maximum and how much nor how many take minimum and how much. If we purchase 20 times of maximum, it would be waste, or if we purchase 20 times of munimum, it is not sufficient. So mode cann't be suggested here.

(c) If there are 5 members in the house, and whose heights are 134cm, 125cm, 100cm, 125cm and 144cm, as mode of the data is 125cm, we may suggest the height of the door must be 125cm. But it is difficult for the person of height 144cm. Even if we take mean of their heights, it is difficult for tall persons. So neither the mode nor the mean can be used here.

Try This

1. One situation where mean would be an appropriate representative value.
2. One situation where mode would be an appropriate representative value.

Exercise - 2

1. Long jumps by 7 students of a team are 98cm, 125cm, 140cm, 155cm, 174cm, 140cm and 155cm. Find the mode of the data.

2. Ages of players in a cricket team are 25, 26, 25, 27, 28, 30, 31, 27, 33, 27, 29.
(i) Find the mean and mode of the data.(ii) Find the minimum number of players to be added to the above team so that mode of the data changes and what must be their ages.

3. Find the mode of the following data. 12, 24, 36, 46, 25, 38, 72, 36, 25, 38, 12, 24, 46, 25, 12, 24, 46, 25, 72, 12, 24, 36,25, 38 and 36.

4. Decide whether mean or mode is a better representative value in the following situations.

   (i) A shop keeper, who sells tooth paste tubes of different sizes, wants to decide which size is to be ordered more.

(ii) An invigilator wants to bring sufficient number of additional papers to the examination hall.

(iii) Preparation of the number of laddus for a marriage.

(iv) For finding the favorite cricketer in a class.

7.5 Median

We have looked at situations where mean and mode are representative values of the data. Now let us look at another situation. The following are the salaries (in rupees) earned by the manager and the workers in a production unit.

Manager	-	₹ 40,000
Worker 1	-	₹ 3,300
Worker 2	-	₹ 5,000
Worker 3	-	₹ 4,000
Worker 4	-	₹ 4,200
Worker 5	-	₹ 3,500
Worker 6	-	₹ 4,500
Worker 7	-	₹ 4,200
Worker 8	-	₹ 4,300
Worker 9	-	₹ 3,500
Worker 10	-	₹ 3,500

Will the mean salary or the mode of salaries be a representative value for this data?

Let us calculate the mean salary in the production unit.

$$\text{Mean salary} = \frac{\text{Total salary}}{\text{Number of employees}}$$

$$= \frac{3300+5000+4000+4200+3500+4500+4200+4300+3500+3500+40000}{11}$$

$= ₹.7272.72$

Is this salary a representative of the salaries of either the manager or the workers? No it is not. It is much lesser than the manager's salary and more than the salary of all the workers.

Now let us consider the mode. 3500 is the most frequently occurring value in the data. However, it occurs only thrice thus, cannot be a representative of the data.

Now, let us use another way of calculating the representative value.

Let us arrange the numbers in ascending order-

3300, 3500, 3500, 3500, 4000, 4200, 4200, 4300, 4500, 5000, 40000

The middle value of this data is 4200 as it divides employees into 2 equal groups – 5 are earning more than 4200 and 5 are earning less. This value is called **Median** and as you can see it provides a representative picture for all.

In the above example, the number of observations is 11 i.e. an odd number, thus the median divides the data into 2 equal groups.

Now what if the number of observations were even?

Let us the take the example of the production unit again. What if a new worker earning ₹ 4000 joined the production unit?

Arranging the number in ascending order we get-

3300, 3500, 3500, 3500, 4000, 4000, 4200, 4200, 4300, 4500, 5000, 40000

Here both 4000 and 4200 lie in the middle of the data. Here the median will be calculated by finding the average of these two values. Thus, the median salary $= \frac{4000+4200}{2} =$ ₹.4100.

Example 8 : The monthly incomes of 7 graduates is ₹ 8000, ₹ 9000, ₹ 8200, ₹ 7900, ₹ 8500, ₹ 8600 and ₹ 60000. Find the median income.

Solution : Arranging the incomes in ascending order we get : 7900, 8000, 8200, 8500, 8600, 9000, 60000

Number of observations = 7

Middle term, i.e., 4^{th} term in the data = 8500

Thus, the median income = ₹ 8500

Example 9 : Find the median of 49, 48, 15, 20, 28, 17, 14 and 110.

Solution : Ascending order of observations = 14, 15, 17, 20, 28, 48, 49, 110

Number of observations = 8

Middle terms, i.e. the 4^{th} and 5^{th} values are 20 and 28.

$$\text{Median} = \text{average of } 4^{th} \text{ and } 5^{th} \text{ values} = \frac{20+28}{2} = 24$$

Thus, median of the given data is 24

Exercise - 3

1. Say true or false and why?

   (i) The difference between the largest and smallest observations in a data set is called the mean.

   (ii) In a bar graph, the bar which has greater length may contains mode.

   (iii) Value of every observation in the data set is taken into account when median is calculated.

   (iv) The median of a set of numbers is always one of the numbers

2. The monthly income (in rupees) of 7 households in a village are 1200, 1500, 1400, 1000, 1000, 1600, 10000. (i) Find the median income of the house holds. (ii) If one more household with monthly income of ₹1500 is added, what will the median income be?

3. Observations of a data are16, 72, 0, 55, 65, 55, 10, and 41. Chaitanya calculated the mode and median without taking the zero into consideration. Did Chaitanya do the right thing?

4. How many distinct sets of three positive integers have a mean of 6, a median of 7, and no mode?

5. Four integers are added to a group of integers 3, 4, 5, 5 and 8 and the mean, median, and mode of the data increases by 1 each. What is the greatest integer in the new group of integers?

Play the Game

Take a dice numbered 1, 2, 3, 4, 5 and 6 on its faces. Make a group of three students. Ask each student to roll the dice and record the number, turn by turn. Repeat the process for 10 rounds. Now each student will have 10 numbers each. Find the mean, median and mode of data of each student.

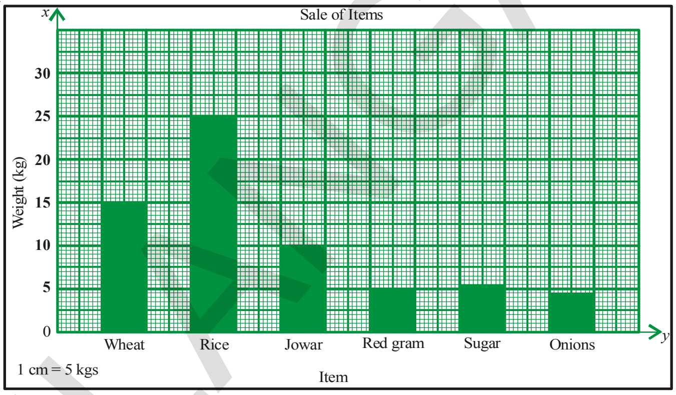
7.6 Presentation of data

We have already learnt how to present data in bar graphs and pictographs in class 6. Pictographs represent data using pictures of objects. However, presenting data by a pictograph is often time consuming and difficult. Bar graphs help in presenting data with much more ease.

7.6.1 Bar Graph

In this section we will learn a little more about bar graphs. We know that bar graphs are made up of bars of uniform width which can be drawn horizontally or vertically with equal spacing between them. The length of each bar tells us the frequency of the particular item. We also know that the length of the bar graph is as per scale.

Example 10 : The bar graph shows the one day sales of various items in a shop.



(i) What are taken on x-axis and y axis?

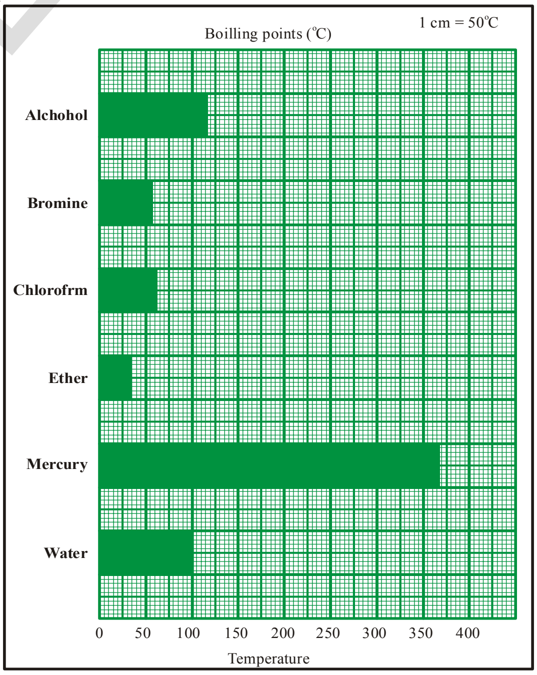
(ii) What is the scale selected on the y-axis?

(iii) Which of these provisions has most sale? How much?

(iv) Is the sale of onions more than red gram?

(v) What is the ratio between the sale of jowar and the sale of red gram?

Example 11 : Observe another bar graph.



(i) What information does the graph give us?

(ii) What are taken on x-axis and y-axis?

(iii) Which of these liquids has highest boiling point?

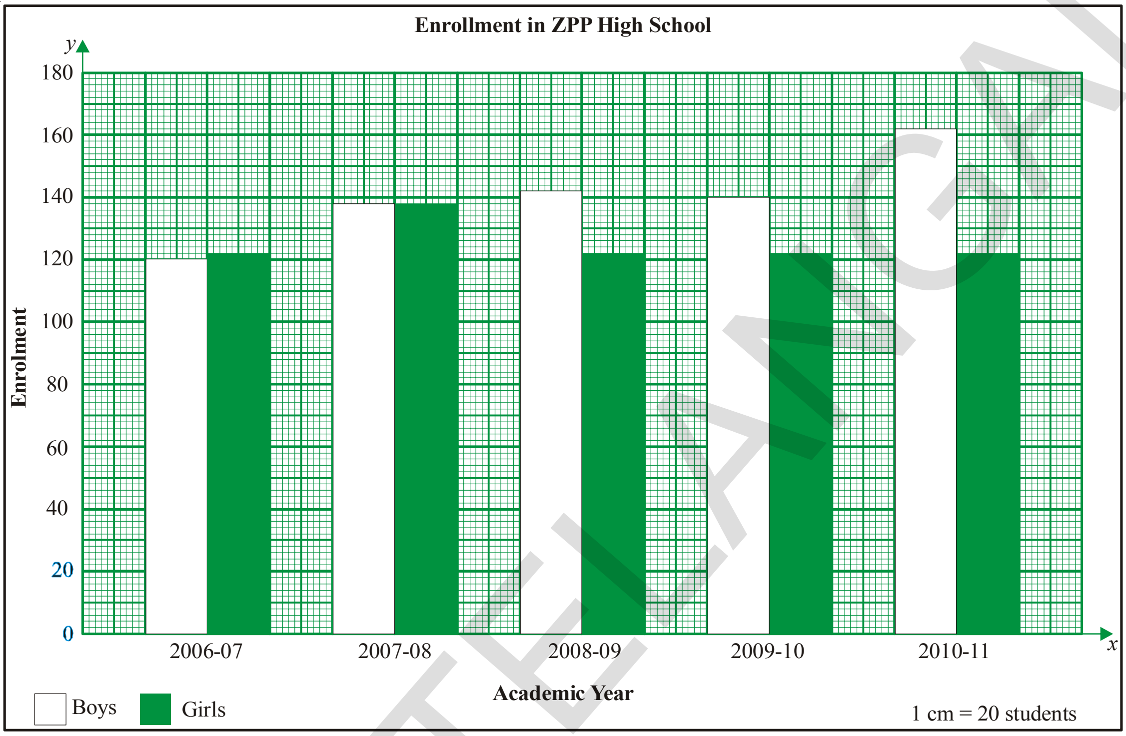
(iv) Which of these liquids has the lowest boiling point?

(v) What is the approximate ratio between the boiling point of mercury and the boiling point of ether?

7.6.2 Double Bar Graph

Now let us learn about another type of bar graph

Example 12 : Study the following graph representing the total enrolment of boys and girls in ZPP High School and answer the following questions.



Did you notice that there are two bars for each year? What does the first bar tell you? What does the second bar tell you? This kind of bar graph is called **Double bar graph**. It presents two observations side by side.

(i) In which year is the enrolment of girls more than the boys?

(ii) In which year is the enrolment of boys and girls the same?

(iii) In which year is the enrolment of girls minimum?

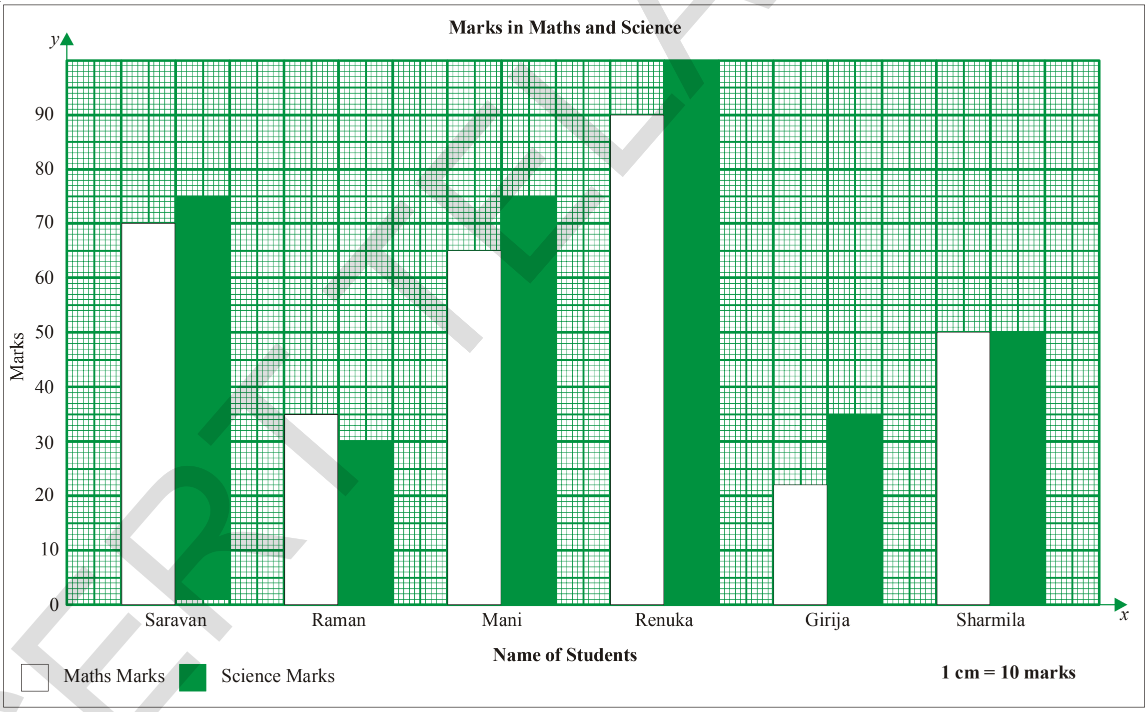
(iv) What is the total enrolment in the year 2007-08?

Example 13 : The following are the marks in Maths and Science of five students in class VII. Present this data in the form of a double bar graph.

Name of Student	Maths	Science
Saravan	70	75
Raman	35	30
Mani	65	75
Renuka	90	100
Girija	22	35
Sharmila	50	50

Solution : Steps in drawing a double bar graph.

1. Draw x-axis (horizontal line) and y-axis (vertical line) on the graph paper and mark their intersection as O.
2. Take names of students on x-axis.
3. Take Maths and Science marks on y-axis,
4. Take an appropriate scale on y-axis so that maximum marks of both the subjects fit on the graph sheet. Here the maximum value to be plotted on Y- axis is 100, so the scale 1 cm = 10 marks, is appropriate.
5. Find the length of each bar by dividing the value by 10 (Scale is 1 cm = 10 marks).
6. Draw bars representing 'Maths marks ' and 'Science marks' side by side of every student.

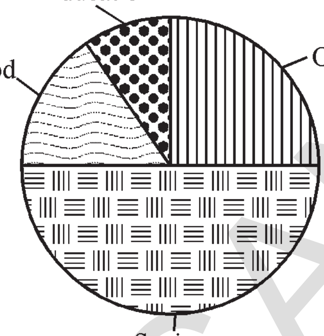


7.6.3 Pie Charts

Another way in which data can be presented is through pie charts.

The monthly budget of a family is given in the table on the left. This data has been presented in a pie chart on the right. The higher the share of expenditure of perticular item of the total income, the more the area occupied by the item in the pie chart.

Budget head	Amount (₹)
Food	1500
Education	750
Others	2250
Savings	4500
Total income	9000



Looking at the pie chart answer the following questions.

(i) What is the shape of the pie chart?

(ii) What is the name of each shape used to present different items in the pie chart?

(iii) Say true or false (a) The largest part of the income is saved.

(b) Least amount of money is spent on education.

7.6.4 Drawing a pie chart

Now, let us learn about how data is presented on a pie chart.

The pie chart represents each item as a portion of the circle, as how much part of the total incomeis is shared by the particular item.

We know that the total angle at the centre of a circle is 360°. We can assume that it represents the total of all observations i.e. ₹ 9000.

Each item of expenditure is a part of the total income thus, the angle of the sector or the area of the sector will depend on the ratio between the item of expenditure and total income.

Thus, the angle of each sector = $\frac{\text{Amount of Expenditure}}{\text{Total Income}} \times 360$

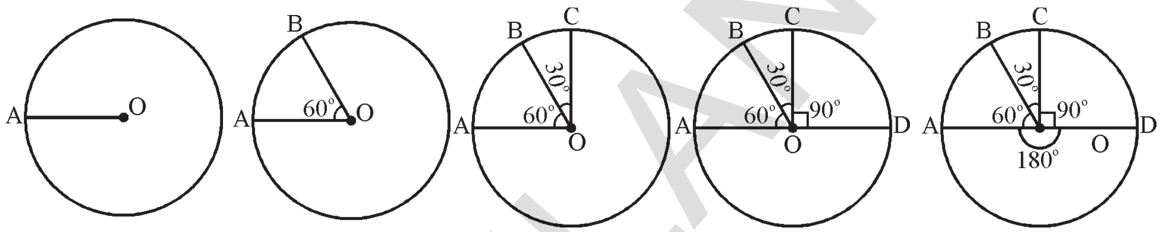
We make a table to find the angle of the sectors. The table is shown below.

Budget head	Amount of expenditure	Ratio between expenditure and total income	Angle of sector or area of the sector
Food	1500	$\frac{1500}{9000} = \frac{1}{6}$	$\frac{1}{6} \times 360^{o} = 60^{o}$
Education	750	$\frac{750}{9000} = \frac{1}{12}$	$\frac{1}{12} \times 360^{o} = 30^{o}$
Others	2250	$\frac{2250}{9000} = \frac{1}{4}$	$\frac{1}{4} \times 360^{o} = 90^{o}$
Savings	4500	$\frac{4500}{9000} = \frac{1}{2}$	$\frac{1}{2} \times 360^{o} = 180^{o}$

Note: Check whether the sum of all the angles of the sectors equal to 360 ° ?

Steps of construction

1. Draw a circle with any convenient radius and mark its centre 'O'.
2. Mark a point A, somewhere on the circumference and join OA.
3. Construct angle of the sector for food = 60º. Draw $\angle AOB = 60º$.
4. Construct angle of the sector for education = 30º. Draw $\angle BOC = 30º$.
5. Construct angle of the sector for other = 90º. Draw $\angle COD = 90º$.
6. Now $\angle DOA = 180º$ represents the angle sector for savings.



1 *2* *3* *4* *5*

Exercise - 4

1. Draw a bar graph for the following data.

   Population of India in successive census years-

Year	1941	1951	1961	1971	1981	1991	2001
Population (in millions) (approx)	320	360	440	550	680	850	1000

Source : Data from census of India 1991 and 2001.

2. Draw a pie chart for the following data.

Item of expenditure	Food	Health	Clothing	Education	Savings
Amount spent in rupees	3750	1875	1875	1200	7500

3. Draw a double bar graph for the following data.

Birth and Death rates of different states in 1999.

State	Birth Rate (Per 1000)	Death Rate (Per 1000)
Andhra Pradesh	22	8
Karnataka	22	8
Tamil Nadu	19	8
Kerala	18	6
Maharashtra	21	8
Orissa	24	11

Source : The table is taken from vittal statistics SRS 1999.

4. Draw a pie chart for the following data.

Time spent by a child during a day-

Time spent for	Sleep	School	Play	Others
Time spent	8 hrs	6 hrs	2 hrs	8 hrs

5. The adjoining pie chart gives the expenditure on various items during a month for a family. (The numbers written around the pie chart tell us the angles made by each sector at the centre.)

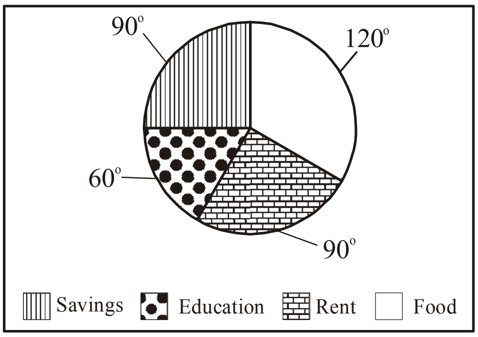
Answer the following-

(i) On which item is the expenditure minimum?

(ii) On which item is the expenditure maximum?

(iii) If the monthly income of the family is ₹ 9000, what is the expenditure on rent?

(iv) If the expenditure on food is ₹ 3000, what is the expenditure on education of children?



Project Work

1. Gather information of the number of different kinds of houses in your locality (ward / colony / village). Then find mode.
2. Collect the item-wise expenditure of your family in a month and represent it as a pie chart.
3. Collect different data presented in the form of bar graphs and pie charts in magazines, newspapers etc. and present them on your school bulletin board.

Looking back

- Mean, mode and median are representative values for a data set.
- Arithmetic mean or mean is equal to sum of all the observations of a data set divided by the number of observations. It lies between the lowest and highest values of the data.
- An observation of data that occurs most frequently is called the mode of the data. A data set may have one or more modes and sometimes none.
- Median is simply the middle observation, when all observations are arranged in ascending or descending order. (In case of even number of observations meadian is the average of middle two observations.)
- A pie chart is a circular chart /graph divided into sectors, and is used to present data.
- The central angle of each sector (and consequently its area) in a pie chart, is proportional to the quantity that it represents.

Dr.C.R.Rao (India)
1920 AD

A well known Statistician, famous for his "Theory of Estimation" (1945).

He worked on Cramer-Rao Inequality and Fisher-Rao theorm.

CONGRUENCY OF TRIANGLES

8

8.0 Introduction

If we take a pile of one rupee coins and place them one on top of the other, they would match perfectly. Do you know why this happens? This is because all the coins have the same size and shape. In the same way papers of a blank note book have the same size and shape.

Look around you and find some examples of objects that share this kind of similarity i.e. they are identical in shape and size. Think of at least 5 such examples.

When we talk about objects of the same size and shape we say that the objects are congruent. A practical test of congruence is to place one object over the other and see if they superimpose exactly.

Activity:

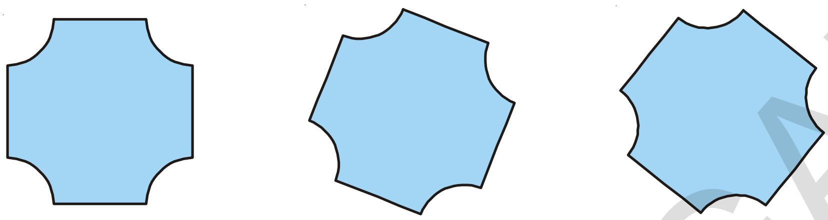
Are all ten rupee notes congruent? How will you check?

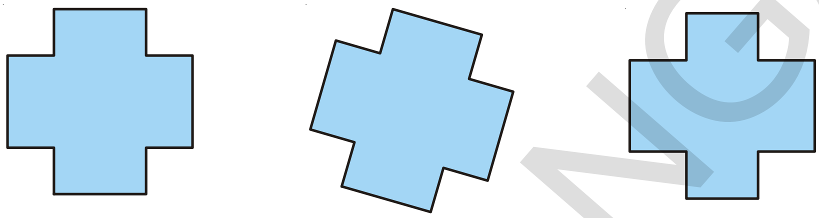
Similarly, check if the five rupee note you find are congruent.

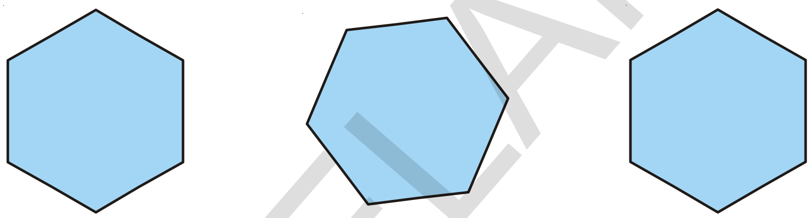
We see many examples of congruent objects all around us. Now, think of some shapes that are congruent.

Do This

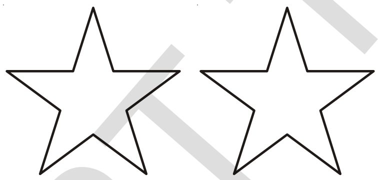
1. Here are some shapes. See whether all the shapes given in a row are congruent to each other or not. You can trace the figures and check.

(i) 

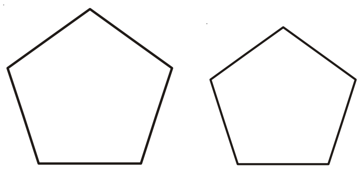
(ii) 

(iii) 

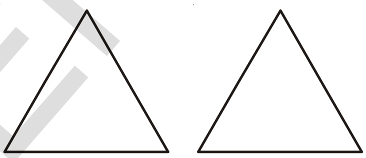
2. Which of the following pairs of figures are congruent?



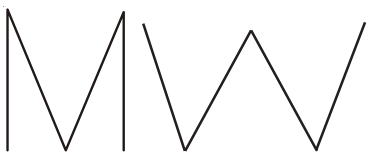
(i)



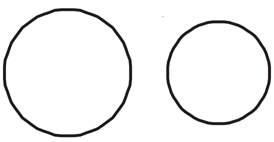
(ii)



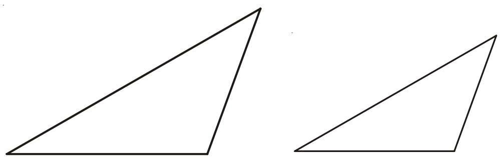
(iii)



(iv)



(v)



(vi)

8.1 Congruency of line segments

Observe the two pairs of line segments given below.

Figure 1

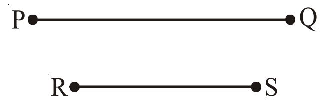


Figure 2

Copy the line segment AB on a tracing paper. Place it on CD. You will find that AB covers CD. Hence the line segments are congruent. We write $\overline{AB} \cong \overline{CD}$.

Repeat this activity for the pair of line segments in Figure 2. What do you find? Are they congruent?

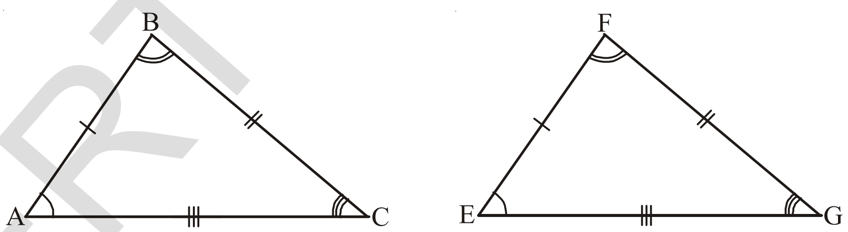
You will notice that the pair of line segments in Figure 1 match with each other because they have same length and this is not the case in Figure 2.

The line segment has only one dimension i.e., length. So if two line segments have the same length, they are congruent. Conversely, if two line segments are congruent, they have the same length.

When we write AB = CD, what we actually mean is $\overline{AB} \cong \overline{CD}$.

8.2 Congruency of triangles

We learnt that two line segments are congruent when their lengths are equal. We extend this idea to triangles. Two triangles are congruent if they are copies of one another and when superimposed, they cover each other exactly.



ΔABC and ΔEFG cover each other exactly i.e. they are of the same size and shape. They are congruent triangles. We express congruency of the two triangles as $\Delta ABC \cong \Delta EFG$.

If two triangles are congruent then all the corresponding six elements of the two triangles i.e. the three angles and three sides are congruent. We also say that if the corresponding parts of two triangles are congruent, then the triangles are congruent. This means that, when you place ΔABC on ΔEFG, their corresponding corners coincides with each other. A lies on E, B lies on F and C lies on G. Also $\angle$A coincides with $\angle$E, $\angle$B coincides with $\angle$F and $\angle$C coincides with $\angle$G and lastly AB coincides with EF, BC coincides with FG and AC coincides with EG.

Thus, for two triangles that are congruent, their corresponding parts i.e. vertices, angles and sides match one another or are equal.

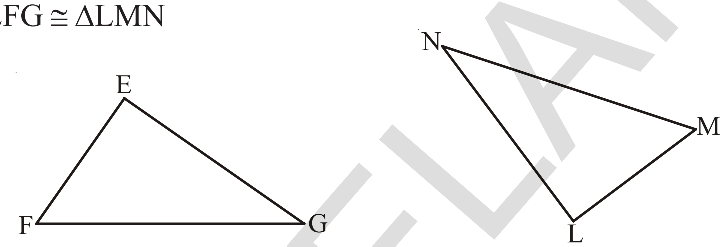
In ΔABC and ΔEFG

$A \rightarrow E$	$B \rightarrow F$	$C \rightarrow G$	(corresponding vertices)
$\angle A \cong \angle E$	$\angle B \cong \angle F$	$\angle C \cong \angle G$	(corresponding angles)
$\overline{AB} \cong \overline{EF}$	$\overline{BC} \cong \overline{FG}$	$\overline{AC} \cong \overline{EG}$	(corresponding sides)

So, when we say that $\Delta ABC \cong \Delta EFG$. The order of the alphabet in the names of congruent triangles displays the corresponding relationships.

Do This

1. $\Delta EFG \cong \Delta LMN$

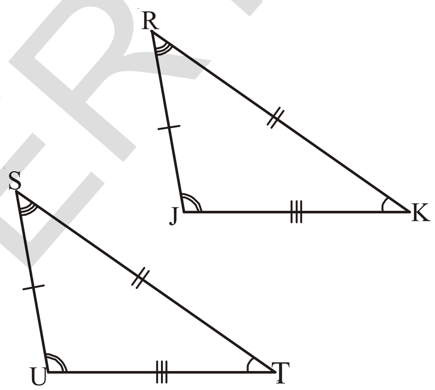


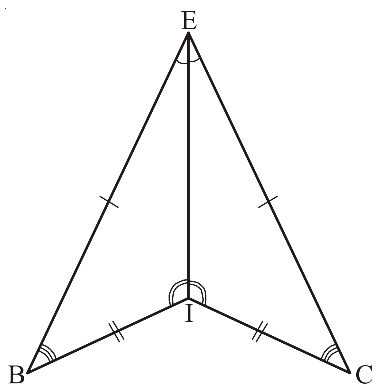
Write the corresponding vertices, angles and sides of the two triangles.

2. If $\Delta ABC \cong \Delta DEF$ write the parts of ΔABC that correspond to-

(i) DE (ii) $\angle E$ (iii) DF (iv) EF (v) $\angle F$

3. Name the congruent triangles in each of the following pairs. Write the statement using '$\cong$'.





4 . Name the congruent angles and sides for each pair of congruent triangles.

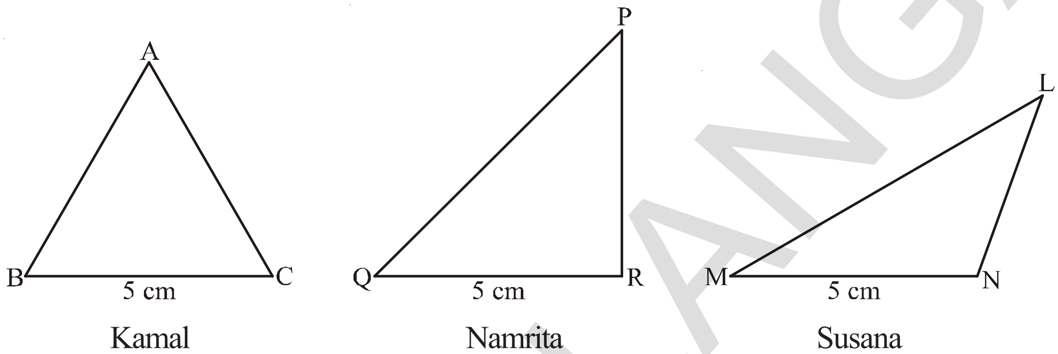
1. $\Delta TUV \cong \Delta XYZ$ 2. $\Delta CDG \cong \Delta RSW$

8.3 Criterion for congruency of triangles

Is it necessary for congruency to check whether all the corresponding parts of two triangles are congruent? How can we check if the given triangles are congruent using a minimum number of measurements. Let us explore and find out.

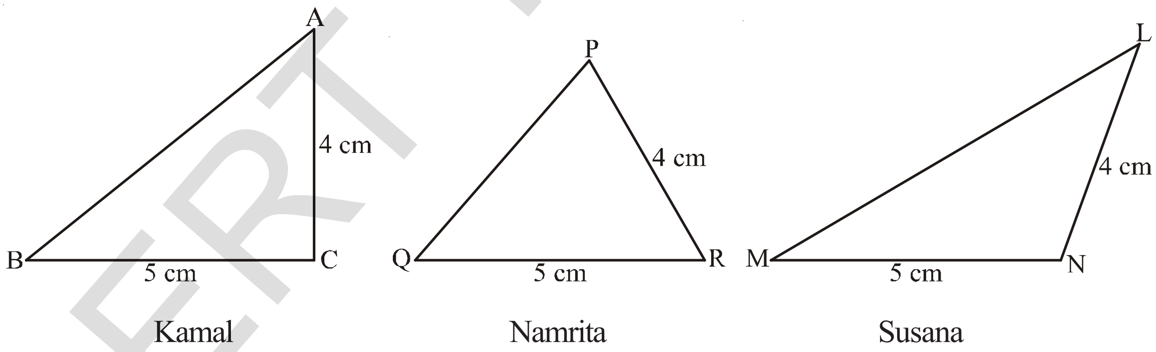
8.3.1 Side-Side-Side congruency (SSS)

Will all of you draw the same triangle if you only knew that the measure of one side of the triangle is 5 cm? Kamal, Namrita and Susana have drawn them like this.

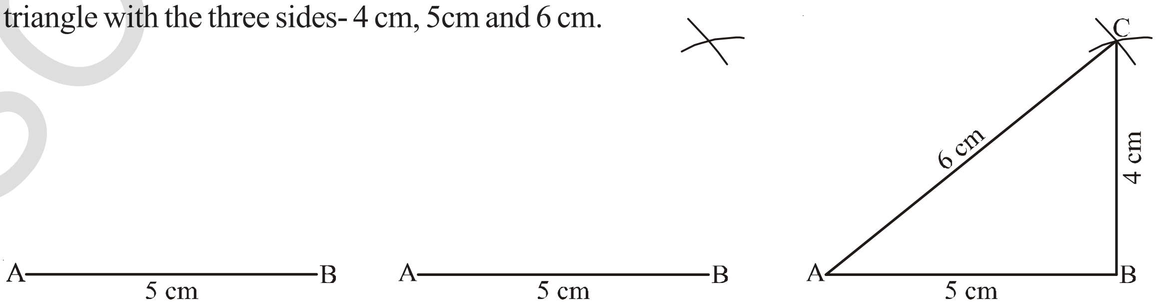


As you can see all the triangles are different. Kamal drew an equilateral triangle, Namrita drew a right-angled triangle and Susana drew an obtuse-angled triangle.

Now can all of you draw the same triangle, if you knew the measures of only two sides of a triangle say, 4 cm and 5 cm. Again Kamal, Namrita and Susana drew different triangles.



What if you know all the sides of the triangle? Kamal, Namrita and Susana all drew the same triangle with the three sides- 4 cm, 5cm and 6 cm.



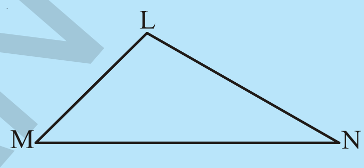
Thus, if we want to make a copy of ABC or a triangle congruent to ABC, we need the lengths of the three sides. This is referred to as the Side-Side-Side(SSS) criterion for congruency of triangles.

If two triangles are congruent because the lengths of their corresponding sides are equal, then will their angles also be equal?

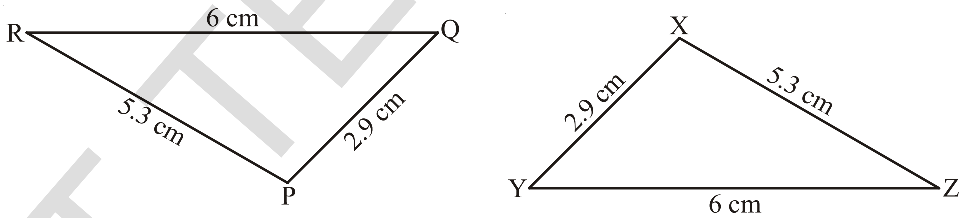
Side-Side-Side (SSS) criterion for congruence of triangles: If three sides of a triangle are equal to the corresponding three sides of another triangle, then the triangles are congruent.

Try This

Measure the lengths of ΔLMN. Now, construct a triangle with these measurements on a sheet of paper. Place this triangle over ΔLMN. Are the triangles congruent? What criterion of congruency applies over here?



Example 1 : Is $\Delta PQR \cong \Delta XYZ$? Also, write the corresponding angles of the two triangles.



Solution : According to the given figure of ΔPQR and ΔXYZ, we have

$PQ = XY = 2.9$ cm

$QR = YZ = 6$ cm

$RP = ZX = 5.3$ cm

Therefore, by Side-Side-Side congruence criterion, $\Delta PQR \cong \Delta XYZ$

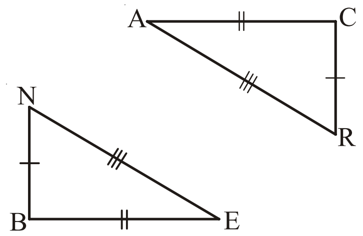
Clearly, the point P corresponds to point X, point Q corresponds to point Y and the point R corresponds to point Z.

So, $\angle P, \angle X$; $\angle Q, \angle Y$; $\angle R, \angle Z$ are pairs of corresponding angles.

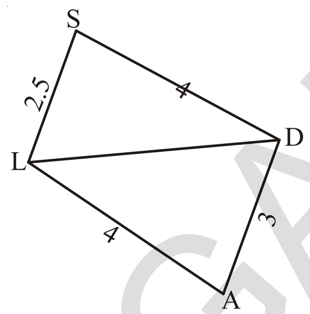
Exercise - 1

1. Decide whether the SSS congruence is true with the following figures. Give reasons

(i)

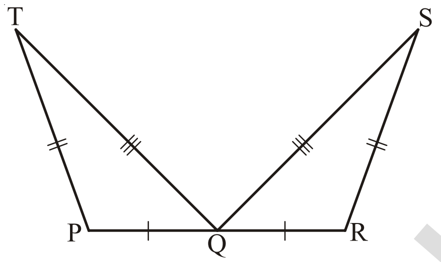


(ii)

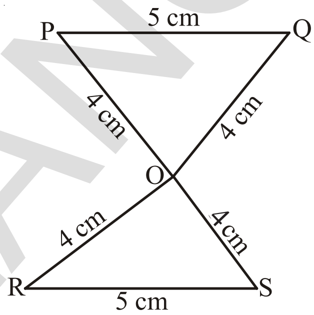


2. For the following congruent triangles, find the pairs of corresponding angles.

(i)



(ii)



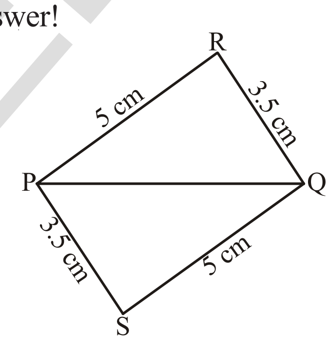
3. In adjacent figure, choose the correct answer!

(i) $\Delta PQR \cong \Delta PQS$

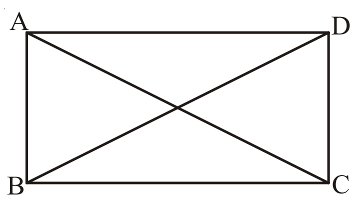
(ii) $\Delta PQR \cong \Delta QPS$

(iii) $\Delta PQR \cong \Delta SQP$

(iv) $\Delta PQR \cong \Delta SPQ$

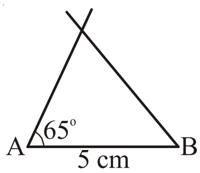


4. In the figure given below, $AB = DC$ and $AC = DB$. Is $\Delta ABC \cong \Delta DCB$.

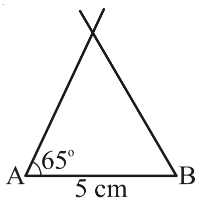


8.3.2 Side-Angle-Side Congruence

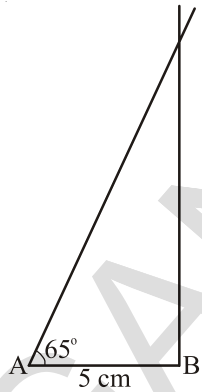
We have seen that it is not possible to draw congruent triangles, if we are given only the measurements of one side. Now, what if you were given one angle and one side? Kamal, Namrita and Susana were told to draw triangles with one side equal to 5 cm and one angle equal to 65°. They drew the following dissimilar triangles.



Kamal



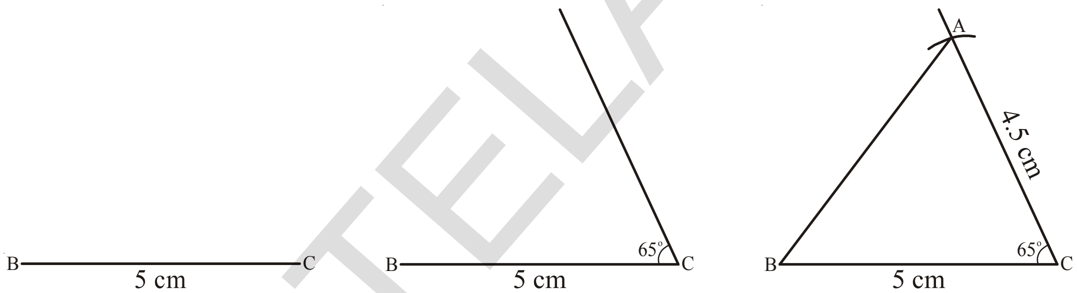
Namrita



Susana

Now, what if the three of them knew the two sides of the triangle and the angle included between these sides. The three children decided to draw triangles with sides 5 cm and 4.5 cm and the included angle of $\angle 65$.

Kamal drew ΔABC. He drew BC as the base= 5 cm. He then made $\angle C = 65^\circ$ using a protractor and then marked point A at a length of 4.5 cm on the angular line. He then joined points A and B.



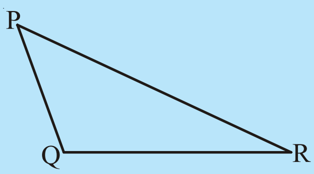
Can you draw the 65° angle at point B with side AB = 4.5 cm. Will the triangle that is formed be congruent to Kamal's triangle? Can you take the base to be 4.5 cm, side = 5 cm and included angle = 65°? Will the triangle that is formed be congruent to Kamal's triangle? You will find that the triangles formed in all these situations are congruent triangles.

Therefore, if we want to make a copy of ΔABC or a triangle congruent to ΔABC, we need the lengths of the two sides and the measure of the angle between the two sides. This is referred to as the Side-Angle-Side(SAS) criterion for congruence of triangles.

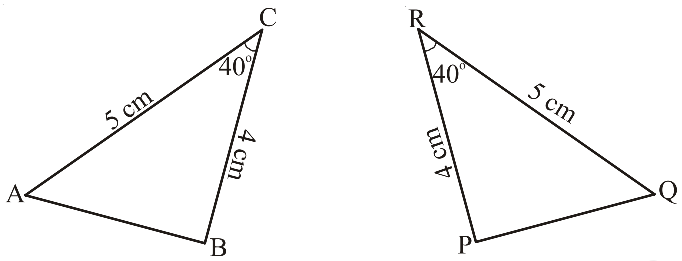
Side-Angle-Side(SAS) criterion for congruence of triangles: If two sides and the angle included between the two sides of a triangle are congruent to the corresponding two sides and the included angle of another triangle, then the triangles are congruent.

Try This

In ΔPQR measure the lengths PQ and QR as well as $\angle Q$. Now, construct a triangle with these three measurements on a sheet of paper. Place this triangle over ΔPQR. Are the triangles congruent? What criterion of congruency applies over here?



Example 2 : See the measurements of the triangles given below. Are the triangles congruent? Which are the corresponding vertices and angles?



Solution : In ΔABC and ΔPQR $AC = QR$ and $BC = PR$ and included angle $\angle C \cong \angle R$

So, $\Delta ABC \cong \Delta QPR$.

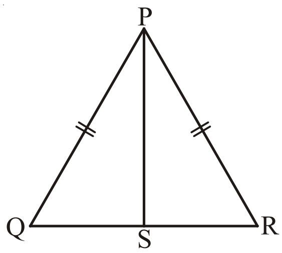
The correspondence is as follows:

$A \leftrightarrow Q$, $B \leftrightarrow P$ and $C \leftrightarrow R$

Therefore, $\angle A \cong \angle Q$, $\angle B \cong \angle P$ and $\angle C \cong \angle R$

Example 3 : In ΔPQR, $PQ = PR$ and PS is angle bisector of $\angle P$.

Are ΔPQS and ΔPRS congruent? If yes, give reason.



Solution : In ΔPQS and ΔPRS

$PQ = PR$ (given)

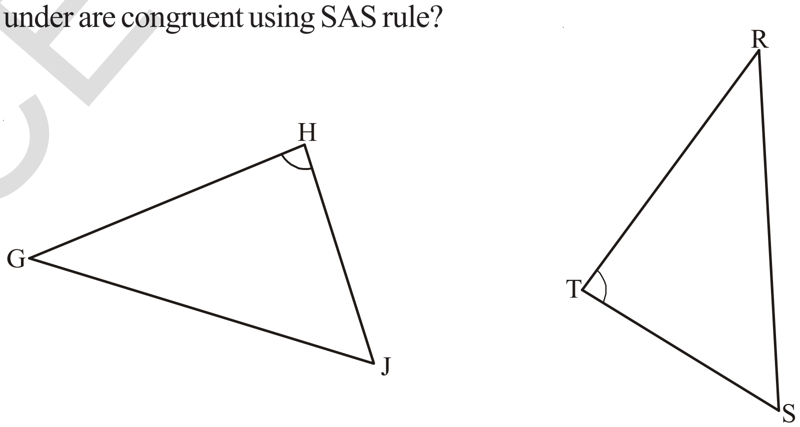
$PS = PS$ (common side in both the triangles)

and included angle $\angle QPS \cong \angle RPS$ (PS is the angle bisector)

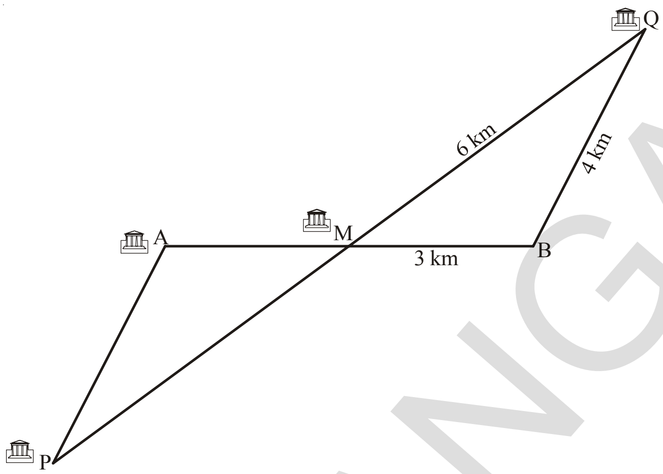
Therefore, $\Delta PQS \cong \Delta PRS$ (by SAS rule)

Exercise - 2

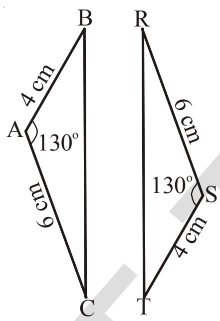
1. What additional information do you need to conclude that the two triangles given here under are congruent using SAS rule?

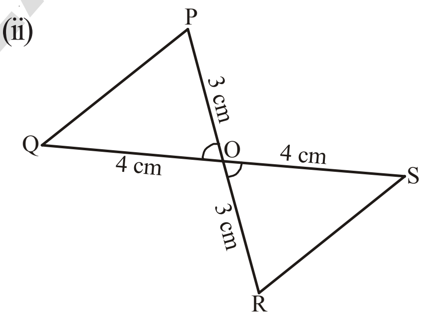


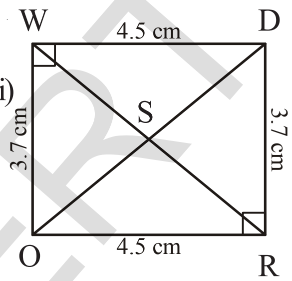
2. The map given below shows five different villages. Village M lies exactly halfway between the two pairs of villages A and B as well as and P and Q. What is the distance between village A and village P. (Hint: check if $\Delta PAM \cong \Delta QBM$)

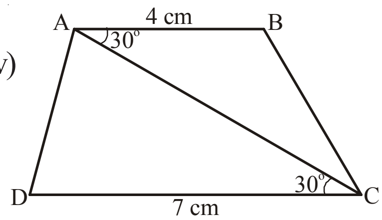


3. Look at the pairs of triangles given below. Are they congruent? If congruent write the corresponding parts.

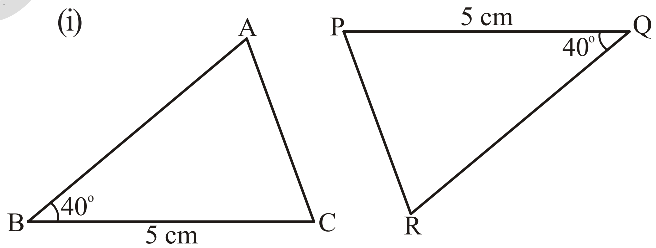
(i)


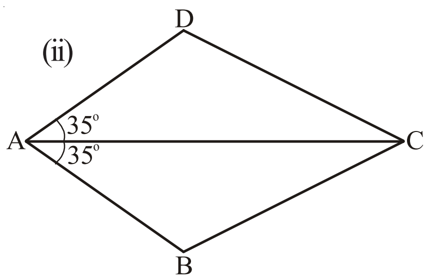
(ii)


(iii)


(iv)


4. Which corresponding sides do we need to know to prove that the triangles are congruent using the SAS criterion?

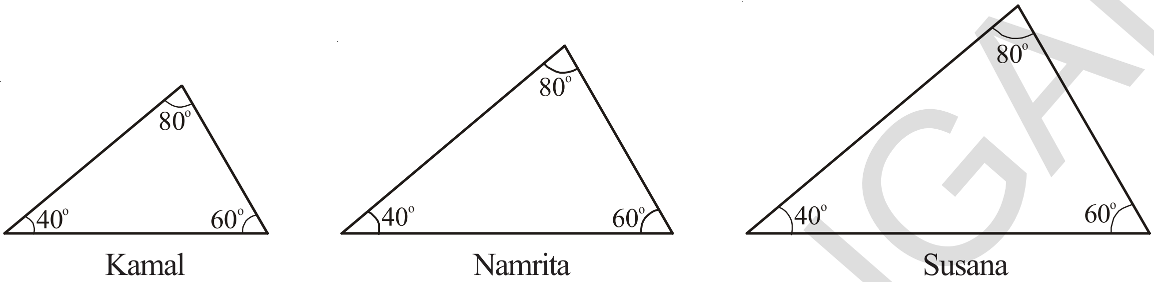
(i)


(ii)


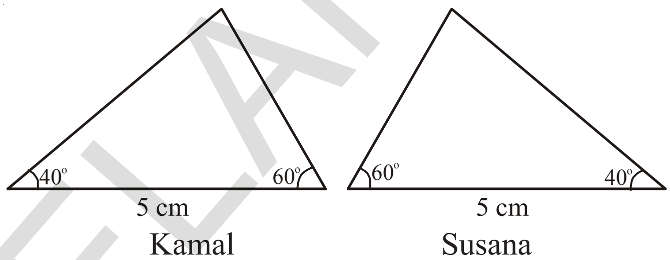
8.3.3 Angle-Side-Angle congruency (ASA)

Can the children construct a triangle if they know only one angle of the triangle? What if they know two angles? Will children be able to draw congruent triangles if they know all the angles of the triangle?

Kamal, Namrita and Susana drew the following triangles of angles 40°, 60° and 80°.



Therefore, though the angles of all the triangles are congruent, the lengths of their sides is not and they are not congruent.



Thus, we need to know the length of the sides, to draw congruent triangles. What if we have two angles and one side? Kamal and Namrita drew the following triangles with angles 60° and 40° and side 5 cm. When both the children constructed their triangles they made the given side, the included side.

We can conclude that if we want to make a copy of a triangle or a triangle congruent to another triangle, then we need to know two angles and the length of the side included between the two angles. This is referred to as the Angle-Side-Angle criterion of congruence.

Angle-Side-Angle criterion of congruence: If two angles and the included side of a triangle are congruent to the two corresponding angles and included side of another triangle then the triangles are congruent.

Try This

Teacher has asked the children to construct a triangle with angles 60°, 40° and with a side 5 cm. Sushma calculated the third angle of the triangle as 80° using angle - sum property of triangle. Then Kamal, Sushma and Namratha constructed triangles differently using the following measurements.

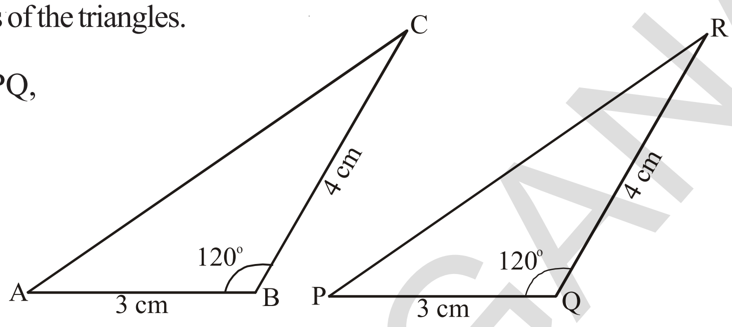
Kamal: 60°, 40° and 5cm side (as teacher said)

Sushma: 80°, 40°, and 5 cm side

Namratha: 60°, 80° and 5 cm side.

They cut these triangles and place them one upon the other. Are all of them congruent? You also try this.

Example 4 : Two triangles ΔCAB and ΔRPQ are given below. Check whether the two are congruent? If they are congruent, what can you say about the measures of the remaining elements of the triangles.



Solution : In ΔCAB and ΔRPQ,

$BC = QR = 4$ cm

$\angle B = \angle Q = 120°$

$AB = PQ = 3$ cm

Thus, two sides and included angle of ΔCAB are equal to the corresponding sides and included angle of ΔRPQ.

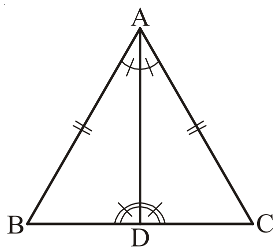
Therefore, by Side-Angle-side criterion of congruency $\Delta CAB \cong \Delta RPQ$

Thus, in the two triangles

$AC \cong PR$

$\angle C \cong \angle R$ and $\angle A \cong \angle P$

Example-5 : In the following picture, the equal angles in the two triangles are shown. Are the triangles congruent?



Solution : In ΔABD and ΔACD

$\angle BAD \cong \angle CAD$ (given in the question)

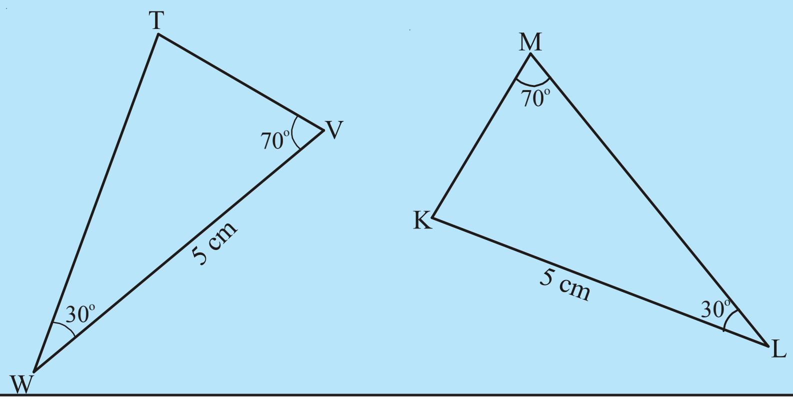
$\angle ADB \cong \angle ADC$ (given in the question)

$AD \cong AD$ (common side, seen in the figure)

Thus, by Angle-side-Angle congruence criterion $\Delta ABD \cong \Delta ACD$

Try This

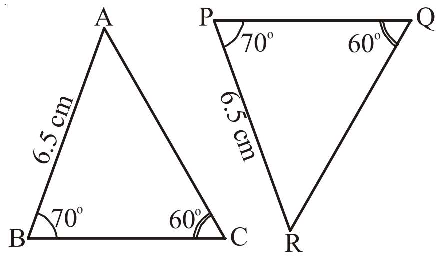
Is the following pair of triangles congruent? Give reason to support your answer.



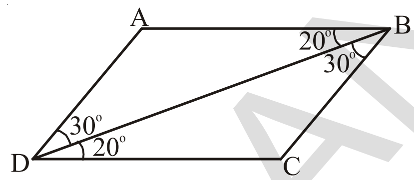
Exerciese - 3

1. In following pairs of triangles, find the pairs which are congruent? Also, write the criterion of congruence.

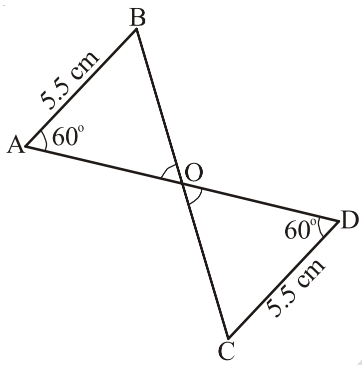
(i)



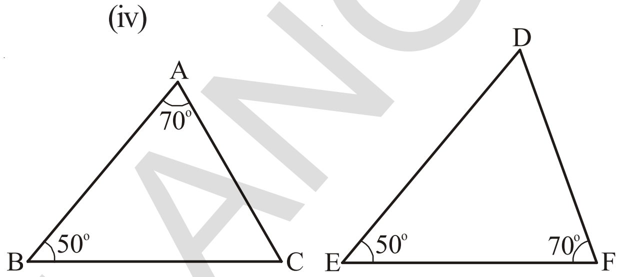
(ii)



(iii)



(iv)

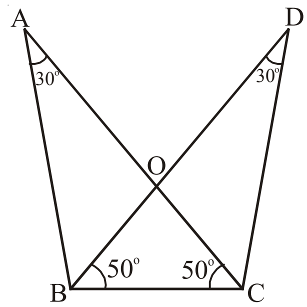


2. In the adjacent figure.

   (i) Are ΔABC and ΔDCB congruent?

   (ii) Are ΔAOB congruent to ΔDOC?

   Also identify the relation between corresponding elements and give reason for your answer.



8.3.4 Right-Angle Hypotenuse Side congruence

In right-angled triangles we already know that one of the angles is a right angle. So what else do we need to prove that the two triangles are congruent?

Let us take the example of ΔABC with $\angle B = 90$. Can we draw a triangle congruent to this triangle, if,

(i) only BC is known

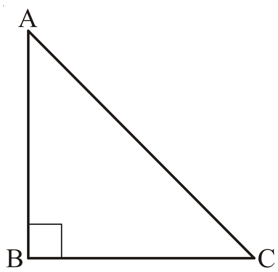
(ii) only $\angle C$ is known

(iii) $\angle A$ and $\angle C$ are known

(iv) AB and BC are known

(v) $\angle C$ and BC are known

(vi) BC and the hypotenuse AC are known



When you try to draw the rough sketches of these triangles, you will find it is possible only in cases (iv), (v) and (vi).

The last of the situations is new to us and it is called the Right-Angle Hypotenuse Congruence Criterion.

Right-Angle Hypotenuse Congruence Criterion: If the hypotenuse and one side of a right angled triangle are equal to the corresponding hypotenuse and side of the other right angled triangle, then the triangles are congruent.

Example 6 : Given below are measurements of some parts of two triangles. Examine whether the two triangles are congruent or not, using RHS congruence rule. In case of congruent triangles, write the result in symbolic form :

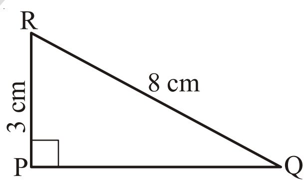
	ΔABC	ΔPQR
(i)	$\angle B = 90^\circ$, AC = 8 cm, AB = 3 cm	$\angle P = 90^\circ$, PR = 3 cm, QR = 8 cm
(ii)	$\angle A = 90^\circ$, AC = 5 cm, BC= 9 cm	$\angle Q = 90^\circ$, PR = 8 cm, PQ = 5 cm

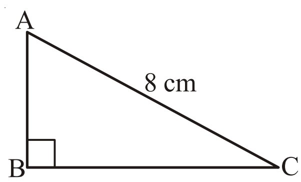
Solution :

(i) Here, $\angle B = \angle P = 90^\circ$

hypotenuse, AC = hypotenuse, RQ (= 8 cm) and

side AB = side RP (= 3 cm)





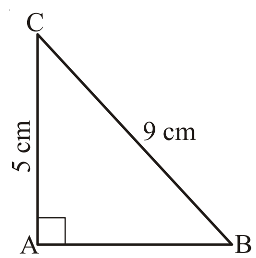
So, $\Delta ABC \cong \Delta RPQ$ (By RHS Congruence rule).

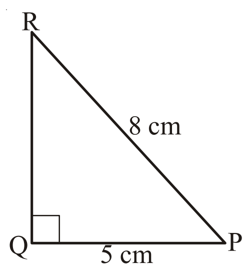
(ii) Here, $\angle A = \angle Q = 90^\circ$ and

side AC = side PQ (= 5 cm).

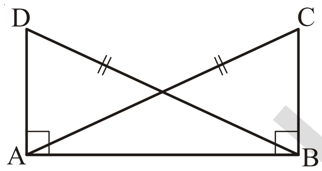
hypotenuse, BC $\neq$ hypotenuse, PR

So, the triangles are not congruent.





Example 7 : In the adjacent figure, $\overline{DA} \perp \overline{AB}$, $\overline{CB} \perp \overline{AB}$ and AC = BD.



State the three pairs of equal parts in ΔABC and ΔDAB.

Which of the following statements is meaningful?

(i) ΔABC ≅ ΔBAD (ii) ΔABC ≅ ΔABD

Solution : The three pairs of equal parts are :

$\angle ABC = \angle BAD$ (= 90°)

AC = BD (Given)

AB = BA (Common side)

ΔABC ≅ ΔBAD (By RHS congruence rule).

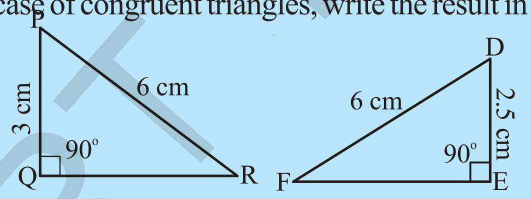
From the above,

statement (i) is true;

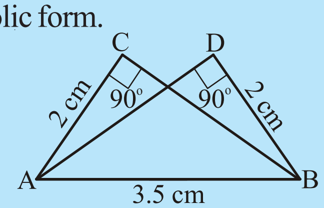
statement (ii) is not meaningful, in the sense that the correspondence among the vertices is not staisfied.

Try This

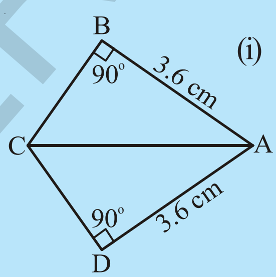
1. In the figures given below, measures of some parts of triangles are given. By applying RHS congruence rule, state which pairs of triangles are congruent. In case of congruent triangles, write the result in symbolic form.



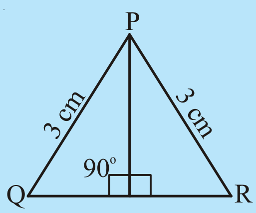
(i)



(ii)



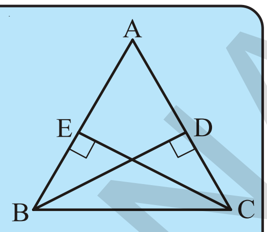
(iii)



(iv)

2. It is to be established by RHS congruence rule that ΔABC ≅ ΔRPQ. What additional information is needed, if it is given that $\angle B = \angle P = 90°$ and AB = RP?

3. In the adjacent figure, BD and CE are altitudes of ΔABC such that $BD = CE$.

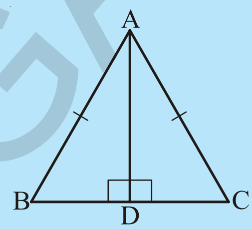


   (i) State the three pairs of equal parts in ΔCBD and ΔBCE.

   (ii) Is $\Delta CBD \cong \Delta BCE$? Why or why not?

   (iii) Is $\angle DBC = \angle EBC$? Why or why not?

4. ABC is an isosceles triangle with $AB = AC$ and AD is one of its altitudes (fig ...).



   (i) State the three pairs of equal parts in ΔADB and ΔADC.

   (ii) Is $\Delta ADB \cong \Delta ADC$? Why or why not?

   (iii) Is $\angle B \cong \angle C$? Why or why not?

   (iv) Is $BD \cong CD$? Why or why not?

Exercise - 4

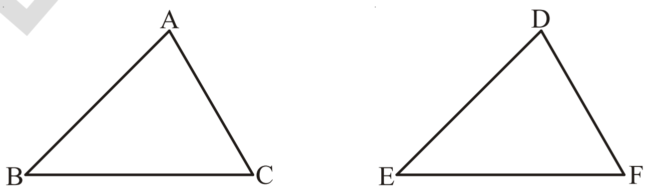
1. Which congruence criterion do you use in the following?

   (i) Given : $AC = DF$

   $AB = DE$

   $BC = EF$

   So, $\Delta ABC \cong \Delta DEF$

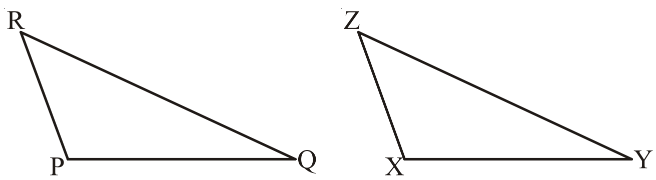


   (ii) Given : $ZX = RP$

   $RQ = ZY$

   $\angle PRQ \cong \angle XZY$

   So, $\Delta PQR \cong \Delta XYZ$

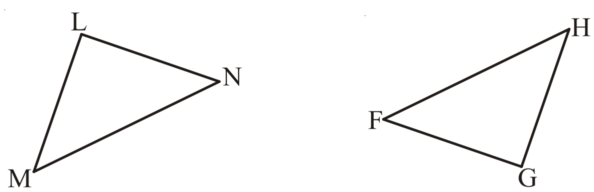


   (iii) Given : $\angle MLN \cong \angle FGH$

   $\angle NML \cong \angle GFH$

   $ML = FG$

   So, $\Delta LMN \cong \Delta GFH$

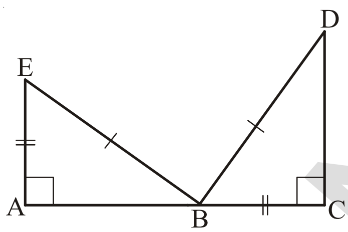


(iv) Given : EB = DB

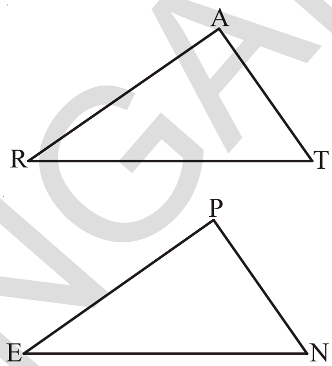
AE = BC

$\angle A = \angle C = 90^o$

So, $\Delta ABE \cong \Delta CDB$



2. You want to show that $\Delta ART \cong \Delta PEN$,



(i) If you have to use SSS criterion, then you need to show

(a) AR = (b) RT = (c) AT =

(ii) If it is given that $\angle T = \angle N$ and you are to use SAS criterion, you need to have

(a) RT = and (ii) PN =

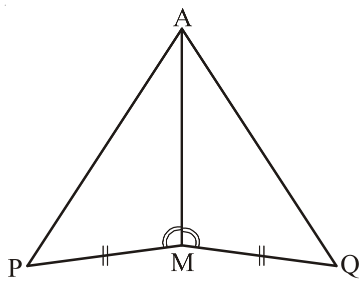
(iii) If it is given that AT = PN and you are to use ASA criterion, you need to have

(a) ? (b) ?

3. You have to show that $\Delta AMP \cong \Delta AMQ$.

In the following proof, supply the missing reasons.

Steps	Reasons
(i) PM = QM	(i) ....
(ii) $\angle PMA \cong \angle QMA$	(ii) ....
(iii) AM = AM	(iii) ....
(iv) $\Delta AMP \cong \Delta AMQ$	(iv) ....

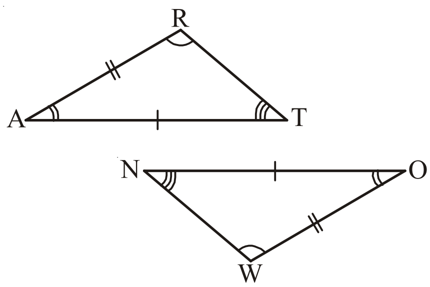


4. In ΔABC, $\angle A = 30^o$, $\angle B = 40^o$ and $\angle C = 110^o$

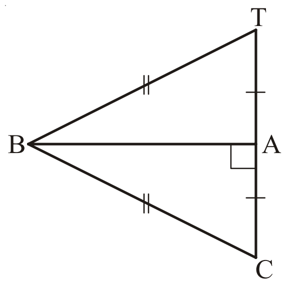
In ΔPQR, $\angle P = 30^o$, $\angle Q = 40^o$ and $\angle R = 110^o$

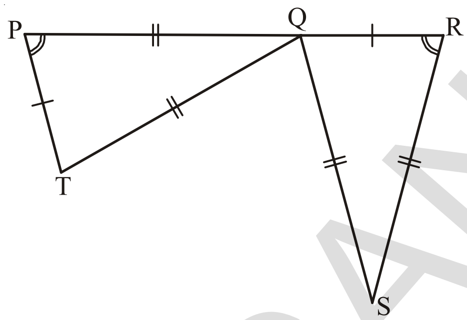
A student says that $\Delta ABC \cong \Delta PQR$ by AAA congruence criterion. Is he justified? Why or why not?

5. In the figure, the two triangles are congruent. The corresponding parts are marked. We can write $\Delta RAT \cong$?



6. Complete the congruence statement.





$\Delta ABC \cong$? $\Delta QRS \cong$?

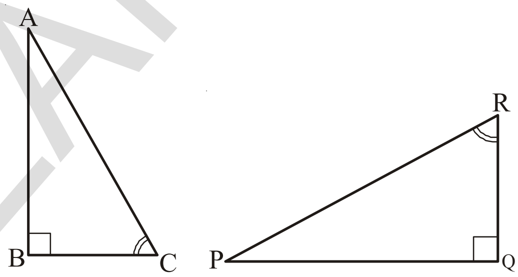
7. In a squared sheet, draw two triangles of equal areas such that

(i) the triangles are congruent.

(ii) the triangles are not congruent.

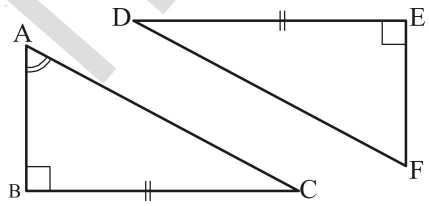
What can you say about their perimeters?

8. If ΔABC and ΔPQR are to be congruent, name one additional pair of corresponding parts. What criterion did you use?



9. Explain, why

$\Delta ABC \cong \Delta FED$.



Looking Back

1. Congruent objects are objects having the same shape and size.
2. The method of superimposition examines the congruence of plane figures.
3. Two line segments say, AB and CD are congruent if they have equal lengths. We write this as $AB \cong CD$. However, it is common to write it as $AB = CD$.
4. If all the parts of one triangle are equal to the corresponding parts of other triangle, then the triangles are congruent.

5. The necessary and sufficient conditions for two triangles to be congruent are as follows:

   (i) Side-Side-Side (SSS) criterion for congruence: If three sides of a triangle are equal to the corresponding three sides of another triangle, then the triangles are congruent.

   (ii) Side-Angle-Side(SAS) criterion for congruence: If two sides and the angle included between the two sides of a triangle are equal to the corresponding two sides and the included angle of another triangle, then the triangles are congruent.

   (iii) Angle-Side-Angle criterion of congruence: If two angles and the included side of a triangle are equal to the corresponding two angles and included side of another triangle then the triangles are congruent.

   (iv) Right-Angle Hypotenuse criterion of congruence: If the hypotenuse and one side of a right-angled triangle are equal to the corresponding hypotenuse and side of the other right-angled triangle, then the triangles are congruent.

CONSTRUCTION OF TRIANGLES 9

9.0 Introduction

You will learn how to construct triangles in this chapter. You do not need all the six elements i.e. the three angles and three sides of a triangle to construct a triangle. A triangle can be drawn if you know the elements that are required for two triangles to be congruent. Thus, a triangle can be drawn in any of the situations given below i.e., if we know the-

(i) Three sides of the triangle.

(ii) Two sides and the angle included between them.

(iii) Two angles and the side included between them.

(iv) Hypotenuse and one adjacent side of a right-angled triangle.

A triangle can also be drawn if two of its sides and a non-included angle are given. However, it is important to remember that this condition is not suffiecient to make two triangles, congruent.

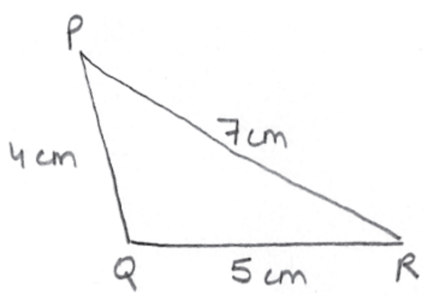
Let us learn to construct triangles in each of the above cases.

9.1 Construction of a triangle when measurements of the three sides are given.

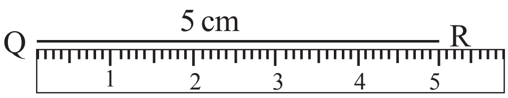
In the construction of any geometrical figure, drawing a rough sketch first, helps in indentifying the sides. So we should first draw a rough sketch of the triangle we want to construct and label it with the given measurements.

Example 1: Construct a Δ PQR with sides PQ = 4 cm, QR = 5 cm and RP = 7 cm.

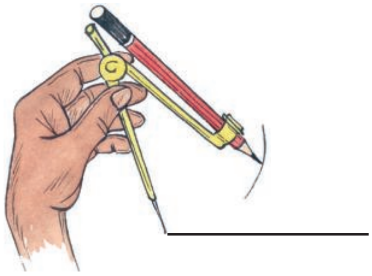
STEP 1 : Draw a rough sketch of the triangle and label it with the given measureents.

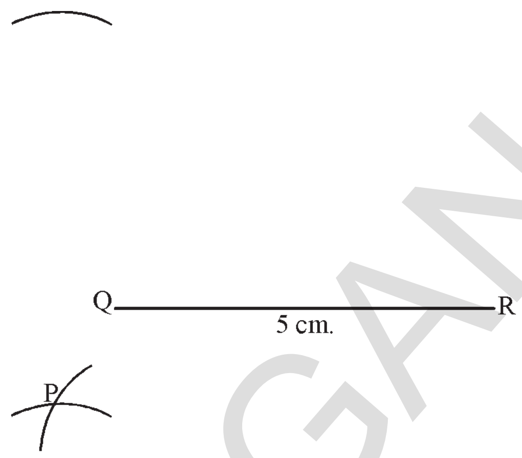


STEP 2 : Draw a line segment QR of length 5 cm.

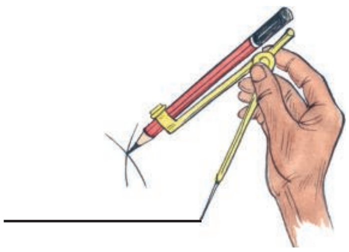


STEP 3 : With centre Q, draw an arc of radius 4 cm.

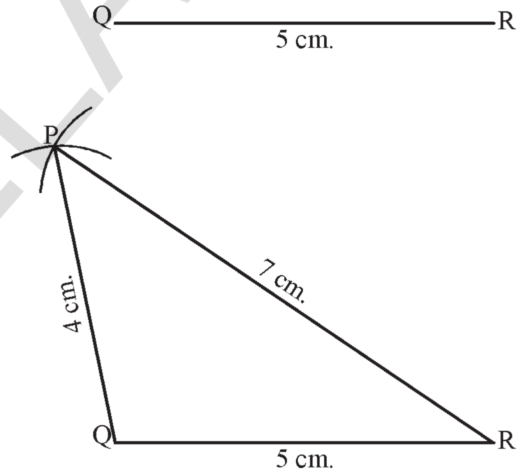




STEP 4 : Since P is at a distance of 7 cm from R, draw another arc from R with radius 7 cm such that it intersects first arc at P.



STEP 5 : Join Q,P and P,R. The required Δ PQR is constructed.



Try This

1. Construct a triangle with the same measurements given in above example, taking PQ as base. Are the triangles congruent?

2. Construct a Δ PET, PE = 4.5 cm, ET = 5.4 cm and TP = 6.5 cm in your notebook. Now construct Δ ABC, AB = 5.4 cm, BC = 4.5 cm and CA = 6.5 cm on a piece of paper. Cut it out and place it on the figure you have constructed in your note book. Are the triangles congruent? Write your answer using mathematical notation.

Exercise - 1

1. Construct Δ ABC in which AB = 5.5 cm, BC = 6.5 cm and CA = 7.5 cm.

2. Construct Δ NIB in which NI = 5.6 cm, IB = 6 cm and BN = 6 cm. What type of triangle is this?

3. Construct an equilateral Δ APE with side 6.5 cm.

4. Construct a Δ XYZ in which XY = 6 cm, YZ = 8 cm and ZX = 10 cm. Using protractor find the angle at X. What type of triangle is this?

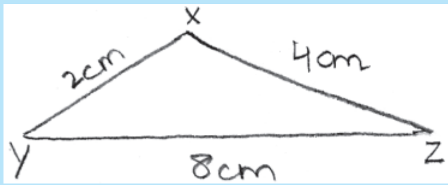
5. Construct Δ ABC in which AB = 4 cm, BC = 7 cm and CA= 3 cm. Which type of triangle is this?

6. Construct Δ PEN with PE = 4 cm, EN = 5 cm and NP = 3 cm. If you draw circles instead of arcs how many points of intersection do you get? How many triangles with given measurements are possible? Is this true in case of every triangle?

Try This

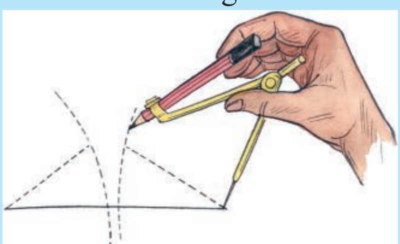
Sushanth prepared a problem: Construct Δ XYZ in which XY = 2 cm, YZ = 8 cm and XZ = 4 cm.

He also drew the rough sketch as shown in Figure 1.



Reading the problem, Srija told Sushanth that it would not be possible to draw a triangle with the given measurements.

However, Sushanth started to draw the diagram as shown in Figure 2.

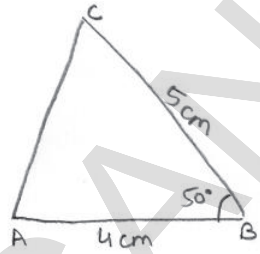


Check whether Sushanth can draw the triangle. If not why? Discuss with your friends. What property of triangles supports Srija's idea?

9.2 Construction of a triangle with two given sides and the included angle.

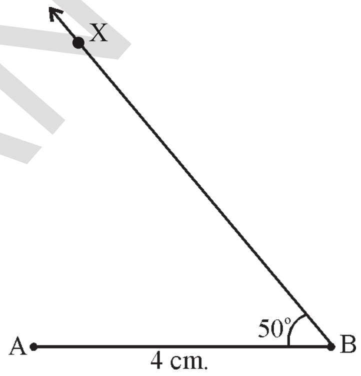
Example 2 : Construct Δ ABC in which AB = 4 cm, BC = 5cm and $\angle$B = 50°.

STEP 1 : Draw a rough sketch of a triangle and label it with the given measurements.

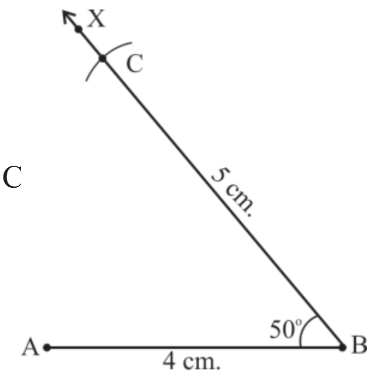


STEP 2 : Draw a line segment AB of length 4 cm.

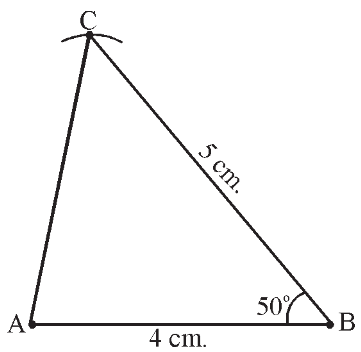
STEP 3 : Draw a ray $\overrightarrow{BX}$ making an angle 50° with AB.



STEP 4 : Draw an arc of radius 5 cm from B, which cuts $\overrightarrow{BX}$ at C



STEP 5 : Join C, A to get the required Δ ABC.



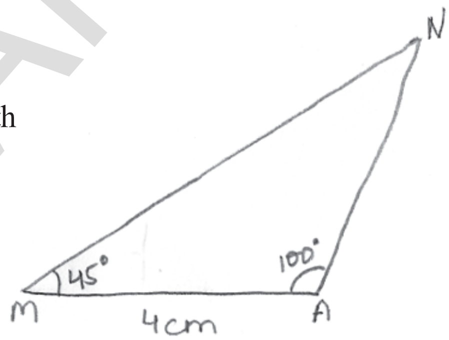
Exercise - 2

1. Draw ΔCAR in which CA = 8 cm, $\angle$A = 60° and AR = 8 cm. Measure CR, $\angle$R and $\angle$C. What kind of triangle is this?
2. Construct ΔABC in which AB = 5 cm, $\angle$B = 45° and BC = 6 cm.
3. Construct ΔPQR such that $\angle$R=100°, QR = RP = 5.4 cm.
4. Construct ΔTEN such that TE = 3 cm, $\angle$E = 90° and NE = 4 cm.

9.3 Construction of a triangle when two angles and the side between the angles is given

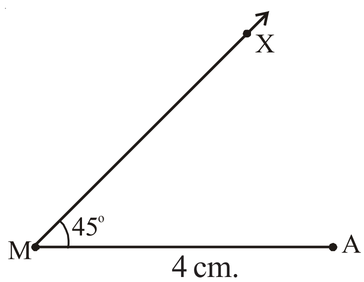
Example 3 : Construct ΔMAN with MA = 4 cm, $\angle$M = 45° and $\angle$A = 100°.

STEP 1 : Draw rough sketch of a triangle and label it with the given measurements.



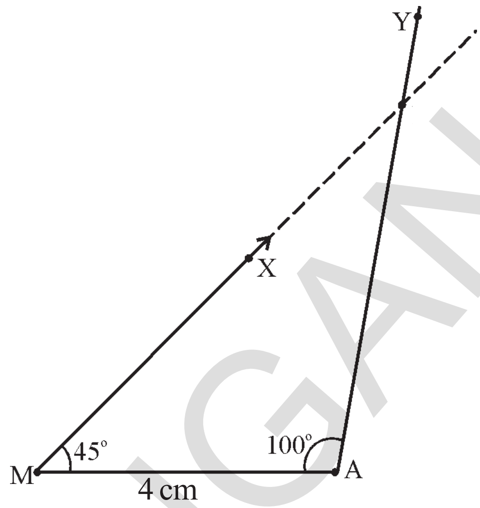
STEP 2 : Draw line segment MA of length 4 cm.

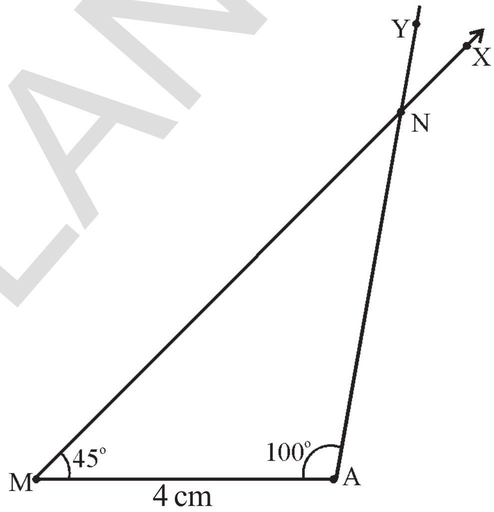
STEP 3 : Draw a ray $\overrightarrow{MX}$, making an angle 45° at M.



STEP 4 : Draw a ray $\overrightarrow{AY}$, making an angle 100° at A.

Extend the ray $\overrightarrow{MX}$ if necessary to intersect ray $\overrightarrow{AY}$.





STEP 5 : Mark the intersecting point of the two rays as N. You have the required Δ MAN

Try This

Construct a triangle with angles 105° and 95° and a side of length of your choice. Could you construct the triangle? Discuss and justify.

Exercise - 3

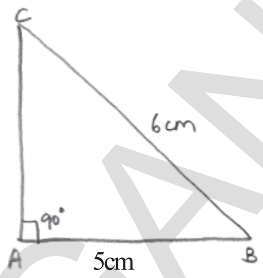
1. Construct Δ NET with measurement NE = 6.4 cm, $\angle$N = 50° and $\angle$E = 100°.
2. Construct Δ PQR such that QR = 6 cm, $\angle$Q = $\angle$R = 60°. Measure the other two sides of the triangle and name the triangle.
3. Construct Δ RUN in which RN = 5cm, $\angle$R = $\angle$N = 45°. Measure the other angle and other sides. Name the triangle.

9.4 Construction of right-angled triangle when the hypotenuse and a side are given.

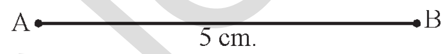
Example 4 : Construct Δ ABC, right-angled at A, and BC = 6 cm; AB = 5 cm.

STEP 1 : Draw a rough sketch of right-angled triangle and label it with given information.

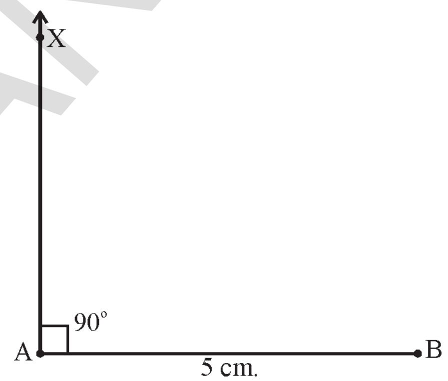
Note: side opposite to the right angle is called hypotenuse.



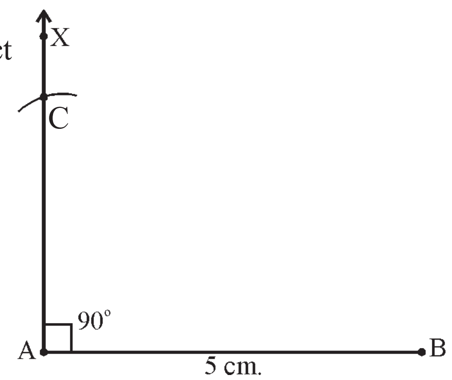
STEP 2 : Draw a line segment AB of length 5 cm.



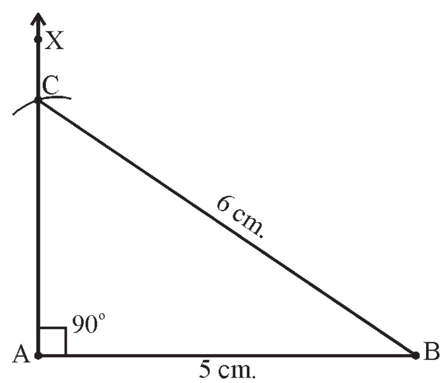
STEP 3 : Construct a ray $\overrightarrow{AX}$ perpendicular to $\overline{AB}$ at A.



STEP 4 : Draw an arc from B with radius 6cm to intersect $\overrightarrow{AX}$ at 'C'.



STEP 5 : Join B,C to get the required Δ ABC.



Exercise - 4

1. Construct a right-angled ΔABC such that $\angle B = 90°$, $AB = 8$ cm and $AC = 10$ cm.

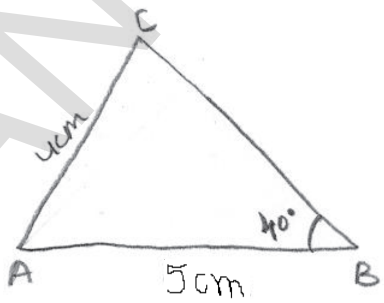
2. Construct a ΔPQR, right-angled at R, hypotenuse is 5 cm and one of its adjacent sides is 4 cm.

3. Construct an isosceles right-angled ΔXYZ in which $\angle Y = 90°$ and the two sides are 5 cm each.

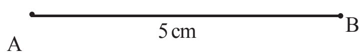
9.5 Construction of triangle when two sides and the non-included angle are given

Example 5 : Construct ΔABC such that $AB = 5$ cm, $AC = 4$ cm, $\angle B = 40°$.

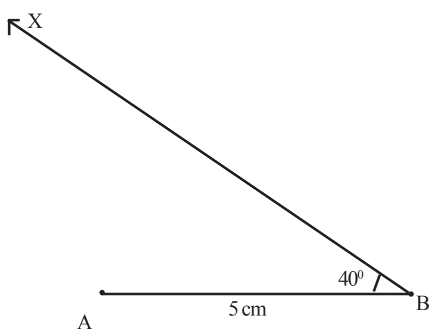
STEP 1 : Draw rough sketch of ΔABC and label it with the given measurements.



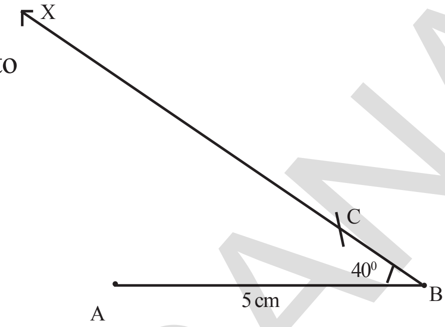
STEP 2 : Draw a line segment AB of length 5 cm.



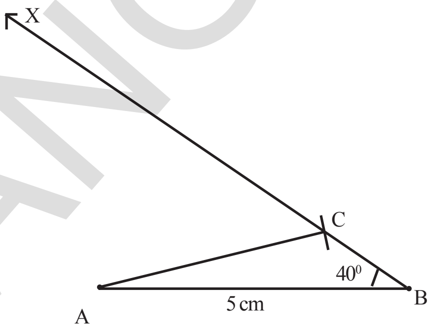
STEP 3 : Draw a ray $\overrightarrow{BX}$ making an angle 40° at B.



STEP 4 : With A as centre and radius 4 cm, draw an arc to cut ray $\overrightarrow{BX}$.

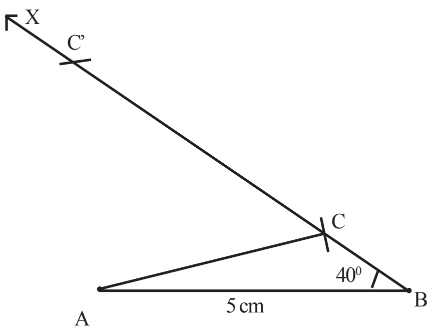


STEP 5 : Mark the intersecting point as C and join C, A to get the required Δ ABC.

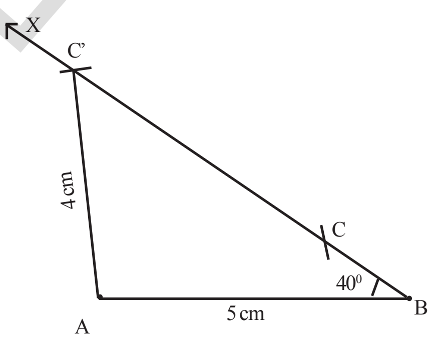


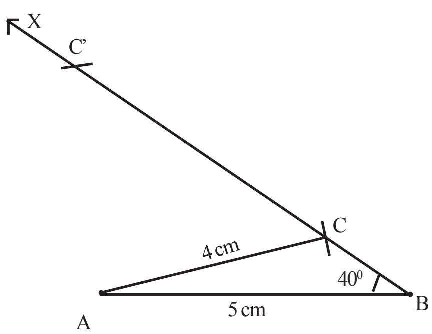
Can you cut the ray $\overrightarrow{BX}$ at any other point?

You will see that as $\angle$B is acute, the arc from A of radius 4 cm cuts the ray $\overrightarrow{BX}$ twice.



So we may have two triangles as given below:





Try This

Construct a triangle with two sides of length of your choice and the non-included angle as an obtuse angle. Can you draw two triangles in this solution?

Exercise - 5

1. Construct Δ ABC in which AB = 4.5 cm, AC = 4.5 cm and $\angle$B = 50°. Check whether you get two triangles.

2. Construct Δ XYZ such that XY = 4.5 cm, XZ = 3.5 cm and $\angle$Y = 70°. Check whether you get two triangles.

3. Construct Δ ANR with the sides AN and AR of lengths 5 cm and 6 cm respectively and $\angle$N is 100°. Check whether you get two triangles.

4. Construct Δ PQR in which QR = 5.5 cm, QP = 5.5 cm and $\angle$Q = 60°. Measure RP. What kind of triangle is this?

5. Construct the triangles with the measurement given in the following table.

Triangle	Measurements
Δ ABC	BC = 6.5 cm, CA = 6.3 cm, AB = 4.8 cm.
Δ PQR	PQ = 8 cm, QR = 7.5 cm, $\angle$PQR = 85°
Δ XYZ	XY = 6.2 cm, $\angle$Y = 130°, $\angle$Z = 70°
Δ ABC	AB = 4.8 cm, AC = 4.8 cm, $\angle$B = 35°
Δ MNP	$\angle$N = 90°, MP = 11.4 cm., MN = 7.3 cm.
Δ RKS	RK = KS = SR = 6.6 cm.
Δ PTR	$\angle$P = 65°, PT = PR = 5.7 cm.

Looking Back

A triangle can be constructed when.

(i) The three sides of the triangle are given.

(ii) Two sides and the angle included between them is given.

(iii) Two angles and their included side is given.

(iv) The hypotenuse and one adjacent side of a right angle triangle are given.

(v) Two sides and the not included angle are given.

ALGEBRAIC EXPRESSIONS 10

10.0 Introduction

In class VI you had already learnt that variables can take on different values and the value of constants is fixed. You had also learnt how to represent variables and constants using letters like x, y, z, a, b , p, m etc. You also came across simple algebraic expressions like $2x - 3$ and so on. You had also seen how these expressions are usefull in formulating and solving problems.

In this chapter, you will learn more about algebraic expressions and their addition and subtraction. However, before doing this we will get acquainted to words like 'terms', 'like terms','unlike terms' and 'coefficients'.

Let us first review what you had learnt in class VI.

Exercise - 1

1. Find the rule which gives the number of matchsticks required to make the following patterns-

   (i) A pattern of letter 'H' (ii) A pattern of letter 'V'

2. Given below is a pattern made from coloured tiles and white tiles.

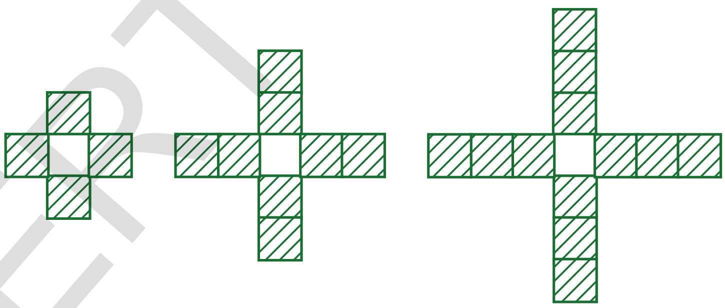


Figure 1 Figure 2 Figure 3

   (i) Draw the next two figures in the pattern above.

   (ii) Fill the table given below and express the pattern in the form of an algebraic expression.

Figure Number	1	2	3	4	5
Number of coloured tiles	4				

(iii) Fill the table given below and express the pattern in the form of an algebraic expression.

Figure Number	1	2	3	4	5
Number of total tiles	5				

3. Write the following statements using variables, constants and arithmetic operations.

(i) 6 more than p

(ii) 'x' is reduced by 4

(iii) 8 subtracted from y

(iv) q multiplied by '-5'

(v) y divided by 4

(vi) One-fourth of the product of 'p' and 'q'

(vii) 5 added to the three times of 'z'

(viii) x multiplied by 5 and added to '10'

(ix) 5 subtracted from two times of 'y'

(x) y multiplied by 10 and added to 13

4. Write the following expressions in statements.

(i) $x + 3$ (ii) $y - 7$ (iii) $10l$

(iv) $\frac{x}{5}$ (v) $3m + 11$ (vi) $2y - 5$

5. Some situations are given below. State the number in situations is a variable or constant?

Example : Our age - its value keeps on changing so it is an example of a variable quantity.

(i) The number of days in the month of January

(ii) The temperature of a day

(iii) Length of your classroom

(iv) Height of the growing plant

10.1 Algebraic Term and Numeric term

Consider the expression $2x + 9$.

Here 'x' is multiplied by 2 and then 9 is added to it. **Both '$2x$' and '9' are terms in the expression $2x + 9$.** Moreover $2x$ is called algebraic term and 9 is called numeric term.

Consider another expression $3x^2 - 11y$.

$3x^2$ is formed by multiplying 3, x and x. $11y$ is the product of 11 and y. $11y$ is then subtracted from $3x^2$ to get the expression $3x^2 - 11y$. **In the expression $3x^2 - 11y$, $3x^2$ is one term and $11y$ is the other term.**

> When we multiply x with x we can write this as x^2. This is similar to writing 4 multiplied by 4 as 4^2.
> Similarly when we multiply x three times i.e., $x \times x \times x$ we can write this as x^3. This is similar to writing $6 \times 6 \times 6$ as 6^3.

Do This

In the expressions given below identify all the terms.

(i) $5x^2 + 3y + 7$ (ii) $5x^2y + 3$ (iii) $3x^2y$

(iv) $5x - 7$ (v) $5x + 8 - 2(-y)$ (vi) $7x^2 - 2x$

10.1.1 Like and unlike terms

Let us observe the following examples.

(i) $5x$ and $8x$ (ii) $7a^2$ and $14a^2$

(iii) $3xy$ and $4xy$ (iv) $3xy^2$ and $4x^2y$

In the first example, both terms contain the same variable i.e. x and the exponent of the variable is also the same i.e. 1

In the second example, both terms contain the same variable i.e. a and the exponent of the variable is also the same i.e. 2

In the third example, both terms contain the same variables i.e. x and y and the exponent of variable x is 1 and the exponent of variable y is 1.

In the fourth example, both terms contain the same variables x and y. However, their exponents are not the same. In the first term, the exponent of x is 1 and in the second it is 2. Similarly, in the first term the exponent of y is 2 and in the second term it is 1.

The first three pairs of terms are examples of 'like terms' while the fourth is a pair of 'unlike terms'.

Like terms are terms which contain the same variables with the same exponents.

Do This

1. Group the like terms together.

   $12x$, 12, $25x$, -25, $25y$, 1, x, $12y$, y, $25xy$, $5x^2y$, $7xy^2$, $2xy$, $3xy^2$, $4x^2y$

2. State true or false and give reasons for your answer.

   (i) $7x^2$ and $2x$ are unlike terms

   (ii) pq^2 and $-4pq^2$ are like terms

   (iii) xy, $-12x^2y$ and $5xy^2$ are like terms

10.2 Co-efficient

In 9 xy ;

'9' is the co-efficient of 'xy' as $9\ (xy) = 9xy$

'x' is the co-efficient of '$9y$' as $x\ (9y) = 9xy$

'y' is the co-efficient of '$9x$' as $y\ (9x) = 9xy$

'$9x$' is the co-efficient of 'y' as $9x\ (y) = 9xy$

$9y$ is the co-efficient of 'x' as $9y\ (x) = 9xy$

xy is the co-efficient of '9' as $xy\ (9) = 9xy$

Since 9 has a numerical value it is called a numerical coefficient. x, y and xy are literal coefficients because they are variables.

Similarly in '$-5x$', '-5' is the numerical coefficient and 'x' is the literal coefficient.

Try This

(i) What is the numerical coefficient of 'x'?

(ii) What is the numerical coefficient of '$-y$'?

(iii) What is the literal coefficient of '$-3z$'?

(iv) Is a numerical coefficient a constant?

(v) Is a literal coefficient always a variable?

10.3 Expressions

An expression is a single term or a combination of terms connected by the symbols '+' (plus) or '–' (minus).

For example : $6x + 3y$, $3x^2 + 2x + y$, $10y^3 + 7y + 3$, $9a + 5$, $5a + 7b$, $9xy$, $5 + 7 - 2x$, $9 + 3 - 2$

Note: multiplication '×' and division '÷' do not separate terms. For example $2x \times 3y$ and $\dfrac{2x}{3y}$ are single terms.

Do This

1. How many terms are there in each of the following expressions?

(i) $x + y$ (ii) $11x - 3y - 5$ (iii) $6x^2 + 5x - 4$

(iv) $x^2z + 3$ (v) $5x^2y$ (vi) $x + 3 + y$

(vii) $x - \frac{11}{3}$ (viii) $\frac{3x}{7y}$ (ix) $2z - y$ (x) $3x + 5$

10.3.1 Numerical expressions and algebraic expressions

Consider the following examples.

(i) $1 + 2 - 9$ (ii) $-3 - 5$ (iii) $x - \frac{11}{3}$ (iv) $4y$

(v) $9 + (6-5)$ (vi) $3x + 5$ (vii) $(17-5) + 4$ (viii) $2x - y$

Do you find any algebraic terms in the examples (i), (ii), (v) and (vii)?

If every term of an expression is a constant term, then the expression is called numerical expression. If an expression has at least one algebraic term, then the expression is called an algebraic expression.

Which are the algebraic expressions in the above examples?

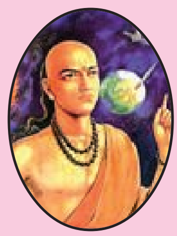
Try This

Write 3 algebraic expressions with 3 terms each.

Aryabhata (India)

475 - 550 AD

He wrote an astronomical treatise, Aryabhatiyam (499AD). He was the first Indian mathematician who used algebraic expressions. India's first satellite was named Aryabhata.



10.3.2 Types of algebraic expressions

Algebraic expressions are named according to the number of terms present in them.

No. of terms	Name of the Expression	Examples
One term	Monomial	(a) x (b) $7xyz$ (c) $3x^2y$ (d) qz^2
Two unlike terms	Binomial	(a) $a + 4x$ (b) $x^2 + 2y$ (c) $3x^2 - y^2$
Three unlike terms	Trinomial	(a) $ax^2 + 4x + 2$ (b) $7x^2 + 9y^2 + 10z^3$
More than one unlike terms	Multinomial	(a) $4x^2 + 2xy + cx + d$ (b) $9p^2 - 11q + 19r + t$

Note: Binomial, trinomials are also multinomial algebraic expressions.

Do This

1. Give two examples for each type of algebraic expression.
2. Identify the expressions given below as monomial, binomial, trinomial and multinomial.

   (i) $5x^2 + y + 6$ (ii) $3xy$

   (iii) $5x^2y + 6x$ (iv) $a + 4x - xy + xyz$

10.4 Degree of algebraic expressions

Before discussing the degree of algebraic expressions let us understand what we mean by the degree of a monomial.

10.4.1 Degree of a monomial

Consider the term $9x^2y^2$

1. What is the exponent of 'x' in the above term?
2. What is the exponent of 'y' in the above term?
3. What is the sum of these two exponents?

The sum of all exponents of the variables present in a monomial is called the degree of the term or degree of the monomial.

Study the following table.

S. No.	Monomial	Exponents			Degree of the monomial
		x	y	z	
1	x	1	-	-	1
2	$7x^2$	2	-	-	2
3	-3xyz	1	1	1	1 + 1 + 1 = 3
4	$8y^2z^2$	-	2	2	2 + 2 = 4

10.4.2 Degree of constant terms

Let us discuss the degree of the constant term 5.

Since $x^o = 1$, we can write 5 as $5x^o$. Therefore, the degree of 5 is '0'.

Degree of constant term is zero.

10.4.3 Degree of algebraic expressions

Study the following table.

S. No.	Algebraic Expression	Degree of each term				Highest Degree
		First term	Second term	Third term	Fourth term	
1.	$7xy^2$	3	-	-	-	3
2	$3y - x^2y^2$	1	4	-	-	4
3	$4x^2 + 3xyz + y$	2	3	1	-	3
4	$pq - 6p^2q^2 - p^2q + 9$	2	4	3	0	4

In the second example the highest degree of one of the terms is 4. Therefore, the degree of the expression is 4. Similarly, the degree of the third expression is 3 and the degree of the fourth expression is 4.

The highest of the degrees of all the terms of an expression is called the degree of the expression.

Exercise 2

1. Identify and write the like terms in each of the following groups.

   (i) $a^2, b^2, -2a^2, c^2, 4a$ (ii) $3a, 4xy, -yz, 2zy$

   (iii) $-2xy^2, x^2y, 5y^2x, x^2z$ (iv) $7p, 8pq, -5pq, -2p, 3p$

2. State whether the expression is a numerical expression or an algebraic expression.

   (i) $x + 1$ (ii) $3m^2$ (iii) $-30 + 16$

   (iv) $4p^2-5q^2$ (v) 96 (vi) x^2-5yz

   (vii) $215x^2yz$ (viii) $95 \div 5 \times 2$ (ix) $2 + m + n$

   (x) $310 + 15 + 62$ (xi) $11a^2+6b^2-5$

3. Which of the following multinomials is monomial, binomial or trinomial?

   (i) y^2 (ii) $4y - 7z$ (iii) $1 + x + x^2$

   (iv) $7mn$ (v) $a^2 + b^2$ (vi) 100 xyz

   (vii) $ax + 9$ (viii) $p^2 - 3pq + r$ (ix) $3y^2 - x^2 y^2 + 4x$

   (x) $7x^2 - 2xy + 9y^2 - 11$

4. What is the degree of each of the monomials.

   (i) $7y$ (ii) $-xy^2$ (iii) xy^2z^2

   (iv) $-11y^2z^2$ (v) $3mn$ (vi) $-5pq^2$

5. Find the degree of each algebraic expression.

   (i) $3x-15$ (ii) $xy + yz$ (iii) $2y^2z + 9yz - 7z - 11x^2y^2$

   (iv) $2y^2z + 10yz$ (v) $pq + p^2q - p^2q^2$ (vi) $ax^2 + bx + c$

6. Write any two Algebraic expressions with the same degree.

10.5 Addition and subtraction of like terms

Consider the following problems.

1. Number of pencils with Vinay is equal to 4 times the pencils with Siddu. What is the total number of pencils both have together?

2. Tony and Basha went to a store. Tony bought 7 books and Basha bought 2 books. All the books are of same cost. How much money did Tony spend more than Basha?

To find answers to such questions we have to know how to add and s[illegible]ct l[illegible] us learn how.

1. Number of pencils with Siddhu is not given in the problem, we shall take the number as 'x'.

   Vinay has 4 times of Siddu i.e., $4 \times x = 4x$

   To find the total number of pencils, we have to add x and $4x$

   Therefore, the total number of pencils $= x + 4x = (1 + 4)\,x = 5x$ (distributive law)

2. Since the cost of each book is not given, we shall take it as 'y'.

   Therefore, Tony spends $7 \times y =$ ₹$7y$

   Basha spends $2 \times y =$ ₹$2y$

   To find how much more Tony spends,we have to subtract 2y from $7y$

   Therefore, the amount spent more $= 7y - 2y = (7-2)y =$ ₹$5y$ (distributive law)

Thus, we can conclude that.

The sum of two or more like terms is a like term with a numerical coefficient equal to the sum of the numerical coefficients of all the like terms in addition.

The difference between two like terms is a like term with a numerical coefficient equal to the difference between the numerical coefficients of the two like terms.

Do This

1. Find the sum of the like terms.

(i) $5x, 7x$ (ii) $7x^2y, -6x^2y$ (iii) $2m, 11m$

(iv) $18ab, 5ab, 12ab$ (v) $3x^2, -7x^2, 8x^2$ (vi) $4m^2, 3m^2, -6m^2, m^2$

(vii) $18pq, -15pq, 3pq$

2. Subtract the first term from the second term.

(i) $2xy, 7xy$ (ii) $5a^2, 10a^2$ (iii) $12y, 3y$

(iv) $6x^2y, 4x^2y$ (v) $6xy, -12xy$

10.5.1 Addition and subtraction of unlike terms

$3x$ and $4y$ are unlike terms. Their sum can be wirtten as $3x + 4y$.

However, 'x' and 'y' are different variables so we can not apply distributive law and thus cannot add them.

10.6 Simplification of an algebraic expression

Consider the expression $9x^2 - 4xy + 5y^2 + 2xy - y^2 - 3x^2 + 6xy$

We can see that there are some like terms in the expression. These are $9x^2$ and $-3x^2$; $5y^2$ and y^2 and $2xy$ and $+6xy$. On adding the like terms we get an algebraic expression in its simplified form. Let us see how the expression given above is simplified.

S.No.	Steps	Process
1.	Write down the expression	$9x^2 - 4xy + 5y^2 + 2xy - y^2 - 3x^2 + 6xy$
2.	Group the like terms together	$(9x^2 - 3x^2) + (2xy - 4xy + 6xy) + (5y^2 - y^2)$
3.	Addding the like terms	$(9-3)x^2 + (2-4+6)xy + (5-1)y^2 = 6x^2 + 4xy + 4y^2$

Note : If no two terms of an expression are alike then it is said to be in the simplified form.

Let us study another example: $5x^2y + 2x^2y + 4 + 5xy^2 - 4x^2y - xy^2 - 9$

Step 1: $5x^2y + 2x^2y + 4 + 5xy^2 - 4x^2y - xy^2 - 9$

Step 2: $(5x^2y + 2x^2y - 4x^2y) + (5xy^2 - xy^2) + (4 - 9)$ (bringing the like terms together)

Step 3: $3x^2y + 4xy^2 - 5$

Do This

1. Simplify the following.

(i)	$3m + 12m - 5m$	(ii)	$25yz - 8yz - 6yz$
(iii)	$10m^2 - 9m + 7m - 3m^2 - 5m - 8$	(iv)	$9x^2 - 6 + 4x + 11 - 6x^2 - 2x + 3x^2 - 2$
(v)	$3a^2 - 4a^2b + 7a^2 - b^2 - ab$	(vi)	$5x^2 + 10 + 6x + 4 + 5x + 3x^2 + 8$

10.7 Standard form of an expression

Consider the expression $3x + 5x^2 - 9$. The degrees of first, second and third terms are 1, 2, and 0 respectively. Thus, the degrees of terms are not in the descending order.

By re-arranging the terms in such a way that their degrees are in descending order; we get the expression $5x^2 + 3x - 9$. Now the expression is said to be in standard form.

Let us consider 3c + 6a –2b. Degrees of all the terms in the expression are same. Thus the expression is said to be already in standard form. If we write it in alphabetical order as 6a –2b + 3c it looks more beautiful.

In an expression, if the terms are arranged in such a way that the degrees of the terms are in descending order then the expression is said to be in standard form.

Examples of expressions in standard form (i) $7x^2+2x+11$ (ii) $5y^2-6y-9$

Do This

1. Write the following expressions in standard form.

(i) $3x+18+4x^2$ (ii) $8-3x^2+4x$

(iii) $-2m+6-3m^2$ (iv) $y^3+1+y+3y^2$

2. Identify the expressions that are in standard form?

(i) $9x^2 + 6x + 8$ (ii) $9x^2 + 15 + 7x$

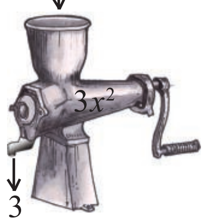
(iii) $9x^2 + 7$ (iv) $9x^3 + 15x + 3$

(v) $15x^2 + x^3 + 3x$ (vi) $x^2y + xy + 3$

(vii) $x^3 + x^2y^2 + 6xy$

3. Write 5 different expressions in standard form.

10.8 Finding the value of an expression



Example 1: Find the value of $3x^2$ if $x=-1$

Solution : Step 1: $3x^2$ (write the expression)

Step 2 : $3(-1)^2$ (substitute the value of variable)

Step 3 : $3(1) = 3$

Example 2 : Find the value of x^2-y+2 if $x=0$ and $y=-1$

Solution : Step 1 : x^2-y+2 (write the expression)

Step 2 : $0^2-(-1)+2$ (substitute the value of variable)

Step 3 : $1+2=3$

Example 3 : Area of a triangle is given by $A=\frac{1}{2}bh$. If $b=12$ cm and $h=7$cm find the area of the triangle.

Solution : Step 1: $A=\frac{1}{2}bh$

Step 2: $A=\frac{1}{2}\times 12\times 7$

Step 3: $A=42$ sq. cm.

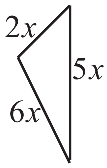
Try This

1. Find the value of the expression '$-9x$' if $x = -3$.
2. Write an expression whose value is equal to -9, when $x = -3$.

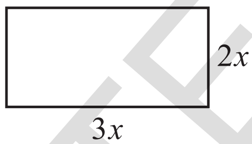
Exercise - 3

1. Find the length of the line segment PR in the following figure in terms of 'a'.

2. (i) Find the perimeter of the following triangle.



(ii) Find the perimeter of the following rectangle.



3. Subtract the second term from first term.

(i) $8x,\ 5x$ (ii) $5p,\ 11p$ (iii) $13m^2,\ 2m^2$

4. . Find the value of following monomials, if $x = 1$.

(i) $-x$ (ii) $4x$ (iii) $-2x^2$

5. Simplify and find the value of $4x + x - 2x^2 + x - 1$ when $x = -1$.

6. Write the expression $5x^2 - 4 - 3x^2 + 6x + 8 + 5x - 13$ in its simplified form. Find its value when $x = -2$

7. If $x = 1$; $y = 2$ find the values of the following expressions

(i) $4x - 3y + 5$ (ii) $x^2 + y^2$ (iii) $xy + 3y - 9$

8. Area of a rectangle is given by $A = l \times b$. If $l = 9$cm, $b = 6$cm, find its area?

9. Simple interest is given by $I = \dfrac{PTR}{100}$. If P = ₹. 900, T = 2 years; and R = 5%, find the simple interest.

10. The relationship between speed (s), distance (d) and time (t) is given by $s = \frac{d}{t}$. Find the value of s, if d = 135 meters and t = 10 seconds.

10.9 Addition of algebraic expressions

Consider the following problems.

1. Sameera has some mangoes. Padma has 9 more than Sameera. Mary says that she has 4 more mangoes than the number of mangoes Sameera and Padma have together. How many mangoes does Mary have?

Since we do not know the number of mangoes that Sameera has, we shall take them to be x mangoes.

Padma has 9 more mangoes than Sameera.

Therefore, the number of mangoes Padma has = $x + 9$ mangoes

Mary has 4 more mangoes than those Sameera and Padma have together.

Therefore, the number of mangoes Mary has = $x + (x + 9) + 4$ mangoes

$= 2x + 13$ mangoes

2. In a Mathematics test Raju got 11 marks more than Imran. Rahul got 4 marks less than what Raju and Imran got together. How much did Rahul score?

Since we do not know Imran's marks, we shall take them to be x marks.

Hint : Why are we taking Imran's marks as x?

Raju got 11 more marks than Imran = $x + 11$ marks

Rahul got 4 marks less than the marks Raju and Imran scored together = $x + x + 11 – 4$ marks

$= 2\,x + 7$ marks

In both the situations given above, we have to add and subtract algebraic expressions. There are number of real life situations in which we need to do this. Let us now learn how to add or subtract algebraic expressions.

10.9.1 Addition of Expressions

The sum of expressions can be obtained by adding like terms. This can be done in two ways.

(i) Column or Vertical Method

(ii) Row or Horizontal Method

(i) Column or Vertical Method

Example 4 : Add $3x^2 + 5x - 4$ and $6 + 6x^2$

Solution:

S. No.	Steps	Process
1	Write the expressions in standard form if necessary	(i) $3x^2 + 5x - 4 = 3x^2 + 5x - 4$ (ii) $6 + 6x^2 = 6x^2 + 6$
2	Write one expression below the other such that the like terms come in the same column	$3x^2 + 5x - 4$ $6x^2 \quad\quad + 6$
3.	Add the like terms column wise and write the result just below the concerned column	$3x^2 + 5x - 4$ $6x^2 \quad\quad + 6$ $9x^2 + 5x + 2$

Example 5 : Add $5x^2 + 9x + 6, 4x + 3x^2 - 8$ and $5 - 6x$

Solution: Step 1 :

$$5x^2 + 9x + 6 = 5x^2 + 9x + 6$$

$$4x + 3x^2 - 8 = 3x^2 + 4x - 8$$

$$5 - 6x = -6x + 5$$

Step 2 :

$$\begin{array}{r} 5x^2 + 9x + 6 \\ 3x^2 + 4x - 8 \\ -6x + 5 \end{array}$$

Step 3 :

$$\begin{array}{r} 5x^2 + 9x + 6 \\ 3x^2 + 4x - 8 \\ -6x + 5 \\ \hline 8x^2 + 7x + 3 \\ \hline \end{array}$$

(ii) Row or Horizontal Method

Example 6 : Add $3x^2 + 5x - 4$ and $6 + 6x^2$

S. No.	Steps	Process
1	Write all expressions with addition symbol in between them.	$3x^2 + 5x - 4 + 6 + 6x^2$
2	Re-arrange the term by grouping the like terms together.	$(3x^2 + 6x^2) + (5x) + (-4 + 6)$
3	Simplify the coefficients	$(3+6)\ x^2 + 5x + 2$
4	Write the resultant expression in standard form.	$9x^2 + 5x + 2$

Do This

1. Add the following expressions.

(i) $x - 2y,\ 3x + 4y$

(ii) $4m^2 - 7n^2 + 5mn,\ 3n^2 + 5m^2 - 2mn$

(iii) $3a - 4b,\ 5c - 7a + 2b$

10.9.2 Subtraction of algebraic expressions

10.9.2(a)Additive inverse of an expression

If we take a positive number '9' then there exists '–9' such that $9 + (-9) = 0$.

Here we say that '–9' is the additive inverse of '9' and '9' is the additive inverse of '–9'.

Thus, for every positive number, there exists a negative number such that their sum is zero. These two numbers are called the additive inverse of the each other.

Is this true for algebraic expressions also? Does every algebraic expression have an additive inverse?

If so, what is the additive inverse of '$3x$'?

For '$3x$' there also exists '$-3x$' such that $3x + (-3x) = 0$

Therefore, '$-3x$' is the additive inverse of '$3x$' and '$3x$' is the additive inverse of '$-3x$'.

Thus, for every algebraic expression there exists another algebraic expression such that their sum is zero. These two expressions are called the additive inverse of the each other.

Example 6 : Find the additive inverse of the expression ($6x^2 - 4x + 5$).

Solution: Additive inverse of $6x^2 - 4x + 5 = -(6x^2 - 4x + 5) = -6x^2 + 4x - 5$

10.9.2(b) Subtraction

Let A ad B be two expressions, then A – B = A + (–B)

i.e. to subtract the expression B from A, we can add the additive inverse of B to A.

Now, let us subtract algebraic expressions using both column and row methods-

(i) Column or Vertical Method

Example 7 : Subtract $3a + 4b - 2c$ from $3c + 6a - 2b$

Solution:

S. No.	Steps	Process
1	Write both expressions in standard form if necessary	$3c+6a-2b=6a-2b+3c$ $3a+4b-2c=3a+4b-2c$
2	Write the expressions one below the other such that the expression to be subtracted comes in the second row and the like terms come one below the other.	$6a-2b+3c$ $3a+4b-2c$
3	Change the sign of every term of the expression in the second row to get the additive inverse of the expression.	$6a-2b+3c$ $\underset{(-)}{3a}\underset{(-)}{+4b}\underset{(+)}{-2c}$
4	Add the like terms, column-wise and write the result below the concerned column.	$6a-2b+3c$ $\underset{(-)}{3a}\underset{(-)}{+4b}\underset{(+)}{-2c}$ $3a-6b+5c$

Example 8 : Subtract $4+3m^2$ from $4m^2+7m-3$

Solution: Step 1: $4m^2+7m-3 = 4m^2+7m-3$

$4+3m^2 = 3m^2+4$

Step 2: $4m^2+7m-3$

$3m^2 \quad +4$

Step 3: $4m^2 + 7m - 3$

$3m^2 \quad + 4$

$- \quad -$

Step 4: $4m^2 + 7m - 3$

$3m^2 \quad + 4$

$- \quad -$

$m^2 + 7m - 7$

(ii) Row or Horizontal Method

Example 9 : Subtract $3a + 4b - 2c$ from $3c + 6a - 2b$

Solution:

S. No.	Steps	Process
1	Write the expressions in one row with the expression to be subtracted in a bracket with assigning negative sign to it.	$3c + 6a - 2b - (3a + 4b - 2c)$
2	Add the additive inverse of the second expression to the first expression	$3c + 6a - 2b - 3a - 4b + 2c$
3	Group the like terms and add or subtract (as the case may be)	$(3c + 2c) + (6a - 3a) + (-2b - 4b)$ $= 5c + 3a - 6b$
4	Write in standard form.	$3a - 6b + 5c$

Example 10 : Subtract $3m^3 + 4$ from $6m^3 + 4m^2 + 7m - 3$

Solution: Step 1: $6m^3 + 4m^2 + 7m - 3 - (3m^3 + 4)$

Step 2: $6m^3 + 4m^2 + 7m - 3 - 3m^3 - 4$

Step 3: $(6m^3 - 3m^3) + 4m^2 + 7m - 3 - 4$

$= 3m^3 + 4m^2 + 7m - 7$

Step 4: $3m^3 + 4m^2 + 7m - 7$

Exercise - 4

1. Add the following algebraic expressions using both horizontal and vertical methods. Did you get the same answer with both methods.

   (i) $x^2 - 2xy + 3y^2$; $5y^2 + 3xy - 6x^2$

   (ii) $4a^2 + 5b^2 + 6ab$; $3ab$; $6a^2 - 2b^2$; $4b^2 - 5ab$

   (iii) $2x + 9y - 7z$; $3y + z + 3x$; 2x - 4y- z

   (iv) $2x^2 - 6x + 3$; $-3x^2 - x - 4$; $1 + 2x - 3x^2$

2. Simplify: $2x^2 + 5x - 1 + 8x + x^2 + 7 - 6x + 3 - 3x^2$

3. Find the perimeter of the following rectangle?

$6x + y$

$3x–2y$

4. Find the perimeter of a triangle whose sides are $2a + 3b$, $b–a$, $4a–2b$.

5. Subtract the second expression from the first expression

   (i) $2a+b$, $a–b$

   (ii) $x+2y+z$, $–x–y–3z$

   (iii) $3a^2–8ab–2b^2$, $3a^2–4ab+6b^2$

   (iv) $4pq–6p^2–2q^2$, $9p^2$

   (v) $7–2x–3x^2$, $2x^2–5x–3$

   (vi) $5x^2–3xy–7y^2$, $3x^2–xy–2y^2$

   (vii) $6m^3+4m^2+7m–3$, $3m^3+4$

6. Subtract the sum of $x^2–5xy+2y^2$ and $y^2–2xy–3x^2$ from the sum of $6x^2–8xy–y^2$ and $2xy–2y^2–x^2$.

7. What should be added to $1+2x–3x^2$ to get $x^2–x–1$?

8. What should be taken away from $3x^2 - 4y^2 + 5xy + 20$ to get $-x^2 - y^2 + 6xy + 20$.

9. The sum of 3 expressions is $8 + 13a + 7a^2$. Two of them are $2a^2 + 3a + 2$ and $3a^2 - 4a + 1$. Find the third expression.

10. If
$$A = 4x^2 + y^2 - 6xy;$$
$$B = 3y^2 + 12x^2 + 8xy;$$
$$C = 6x^2 + 8y^2 + 6xy$$

Find (i) A + B + C (ii) (A – B) –C (iii) 2A+B (iv) A-3B

Looking Back

- An algebraic expression is a single term or a combination of terms connected by the symbols '+' (plus) or '–' (minus).
- If every term of an expression is a constant term, then the expression is called a numerical expression. If an expression has at least one algebraic term, then the expression is called an algebraic expression.
- An algebraic expression contaning one term is called a monomial. An algebraic expression contaning two unlike terms is called a binomial. An algebraic expression contaning three unlike terms is called a trinomial. An algebraic expression contaning more than three unlike terms is called a multinomial.
- The sum of all the exponents of the variables in a monomial is called the degree of the term or degree of monomial.
- The degree of any constant term is zero.
- The highest of the degrees of all the terms of the expression is called the degree of the expression.
- If no two terms of an expression are alike then the expression is said to be in its simplified form.
- In an expression, if the terms are arranged in a manner such that the degrees of the terms are in descending order then the expression is said to be in standard form.
- The sum of two or more like terms is a like term with a numerical coefficient equal to the sum of the numerical coefficients of all the like terms.
- The difference between two like terms is a like term with a numerical coefficient equal to the difference between the numerical coefficients of the two like terms.

POWERS AND EXPONENTS

11

11.0 Introduction

The population of India according to 2011 census is about 120,00,00,000

The approximate distance between the sun and the earth is 15,00,00,000 km.

The speed of the light in vacuum is about 30,00,00,000 m/sec. Light travell a distance of 30,00,00,000 mts., approximately in 1 sec.,

The population of Andhra Pradesh according to 2011 census is about 8,50,00,000.

These are all very large numbers. Do you find it easy to read, write and understand such large numbers? No, certainly not.

Thus, we need a way in which we can represent such larger numbers in a simpler manner. Exponents help us in doing so. In this chapter you will learn more about exponents and the laws of exponents.

11.1 Exponential Form

Let us consider the following repeated additions:

$$4 + 4 + 4 + 4 + 4$$

$$5 + 5 + 5 + 5 + 5 + 5$$

$$7 + 7 + 7 + 7 + 7 + 7 + 7 + 7$$

We use multiplication to shorten the representation of repeated additions by writing 5×4, 6×5 and 8×7 respectively.

Now can we express repeated multiplication of a number by itself in a simpler way?

Let us consider the following illustrations.

The population of Bihar as per the 2011 Census is about 10,00,00,000.

Here 10 is multiplied by itself for 8 times i.e. $10\times10\times10\times10\times10\times10\times10\times10$.

So we can write the population of Bihar as 10^8. Here 10 is called the base and 8 is called the exponent. 10^8 is said to be in exponential form and it is read as 10 raised to the power of 8.

The speed of light in vacuum is 30,00,00,000 m/sec. This is expressed as 3×10^8 m/sec in exponential form. In 3×10^8, 10^8 is read as *'10 raised to the power of 8'*. 10 is the base and 8 is the exponent.

The approximate distance between the sun and the earth is 15,00,00,000 km. This is expressed as 15×10^7 km in exponential form. In 10^7, 10 is the base and 7 is the exponent.

The population of Andhra Pradesh according to 2011 census is about 8,50,00,000. This is expressed as 85×10^6 in exponential form. 10^6 is read as *'10 raised to the power of 6'*. Here 10 is the base and 6 is the exponent.

We can also use exponents in writing the expanded form of a given number for example the expanded form of 36584 = $(3 \times 10000) + (6 \times 1000) + (5 \times 100) + (8 \times 10) + (4 \times 1)$

$$= (3 \times 10^4) + (6 \times 10^3) + (5 \times 10^2) + (8 \times 10^1) + (4 \times 1)$$

Do This

1. Write the following in exponential form. (values are rounded off)
   (i) Total surface area of the Earth is 510,000,000 square kilometers.
   (ii) Population of Rajasthan is approximately 7,00,00,000
   (iii) The approximate age of the Earth is 4550 million years.
   (iv) 1000 km in meters
2. Express (i) 48951 (ii) 89325 in expanded form using exponents.

11.1.1 Exponents with other bases

So far we have seen numbers whose base is 10. However, the base can be any number.

For example $81 = 3 \times 3 \times 3 \times 3 = 3^4$

Here 3 is the base and 4 is the exponent.

Similarly, $125 = 5 \times 5 \times 5 = 5^3$

Here 5 is the base and 3 is the exponent.

Example 1: Which is greater 3^4 or 4^3 ?

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

$4^3 = 4 \times 4 \times 4 = 64$

$81 > 64$

Therefore, $3^4 > 4^3$

Do This

1. Is 3^2 equal to 2^3 ? Justity.
2. Write the following numbers in exponential form. Also state the

   (a) base (b)exponent and (c) how it is read.

   (i) 32 (ii) 64 (iii) 256 (iv) 243 (v) 49

Squared and cubed

When any base is raised to the power 2 or 3, it has a special name.

$10^2 = 10 \times 10$ and is read as **'10 raised to the power 2'** or **'10 squared'**. Similarly, $4^2 = 4 \times 4$ and can be read as **'4 raised to the power of 2'** or **'4 squared'**.

$10 \times 10 \times 10 = 10^3$ is read as **'10 raised to the power 3'** or **'10 cubed '**.

Similarly, $6 \times 6 \times 6 = 6^3$ and can be read as **'6 raised to the power 3'** or **'6 cubed'.**

In general, we can take any positive number 'a' as the base and write.

$a \times a = a^2$ (this is read as 'a raised to the power of 2' or 'a squared')

$a \times a \times a = a^3$ (this is read as 'a raised to the power of 3' or 'a cubed')

$a \times a \times a \times a = a^4$ (this is read as 'a raised to the power of 4')

__________ $= a^5$ (______________________________)

__________ $= a^6$ (______________________________) and so on.

Thus, we can say that $a \times a \times a \times a \times a \times a \times$..................... 'm' times = a^m where 'a' is the base and 'm' is the exponent.

Do This

1. Write the expanded form of the following.

   (i) p^7 (ii) l^4 (iii) s^9 (iv) d^6 (v) z^5

2. Write the following in exponential form.

   (i) $a \times a \times a \times$ .... 'l' times

   (ii) $5 \times 5 \times 5 \times 5$.................... '$n$' times

   (iii) $q \times q \times q \times q \times q$ .... 15 times

   (iv) $r \times r \times r \times$ .... 'b' times

11.2 Writing a number in exponential form through prime factorization.

Let us express the following numbers in the exponential form using prime factorization.

(i) 432 (ii) 450

Solution (i): $432 = 2 \times 216$

$= 2 \times 2 \times 108$

$= 2 \times 2 \times 2 \times 54$

$= 2 \times 2 \times 2 \times 2 \times 27$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 9$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= (2 \times 2 \times 2 \times 2) \times (3 \times 3 \times 3)$

$= 2^4 \times 3^3$

2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

Therefore, $432 = 2^4 \times 3^3$

(ii) 450 $= 2 \times 225$

$= 2 \times 3 \times 75$

$= 2 \times 3 \times 3 \times 25$

$= 2 \times 3 \times 3 \times 5 \times 5$

$= 2 \times 3^2 \times 5^2$

2	450
3	225
3	75
5	25
5	5
	1

Therefore, $450 = 2 \times 3^2 \times 5^2$

Do This

Write the following in exponential form using prime factorization.

(i) 2500 (ii) 1296 (iii) 8000 (iv) 6300

Exercise - 1

1. Write the base and the exponent in each case. Also, write the term in the expanded form.

   (i) 3^4 (ii) $(7x)^2$ (iii) $(5ab)^3$ (iv) $(4y)^5$

2. Write the exponential form of each expression.

   (i) $7 \times 7 \times 7 \times 7 \times 7$

   (ii) $3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5$

   (iii) $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$

3. Express the following as the product of exponents through prime factorization.

(i) 288 (ii)1250 (iii) 2250 (iv) 3600 (v) 2400

4. Identify the greater number in each of the following pairs.

(i) 2^3 or 3^2 (ii) 5^3 or 3^5 (iii) 2^8 or 8^2

5. If $a = 3, b = 2$ find the value of (i) $a^b + b^a$ (ii) $a^a + b^b$ (iii) $(a + b)^b$ (iv) $(a - b)^a$

11.3 Laws of exponents

When we multiply terms with exponents we use some rules to find the product easily. These rules have been discussed here.

11.3.1 Multiplying terms with the same base

Example 2 : $2^4 \times 2^3$

Solution : $2^4 \times 2^3 = \underbrace{(2 \times 2 \times 2 \times 2)}_{\text{4 times}} \times \underbrace{(2 \times 2 \times 2)}_{\text{3 times}}$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^7$ and this is same as 2^{4+3} (as $4 + 3 = 7$)

Therefore, $2^4 \times 2^3 = 2^{4+3}$

Example 3: $5^2 \times 5^3$

Solution : $5^2 \times 5^3 = \underbrace{(5 \times 5)}_{\text{2 times}} \times \underbrace{(5 \times 5 \times 5)}_{\text{3 times}}$

$= 5 \times 5 \times 5 \times 5 \times 5$

$= 5^5$ and this is same as 5^{2+3} (as $2 + 3 = 5$)

Therefore, $5^2 \times 5^3 = 5^{2+3}$

Do This

Find the values of 2^4, 2^3 and 2^7

verify whether $2^4 \times 2^3 = 2^7$

Find the values of 5^2, 5^3 and 5^5 and verify whether $5^2 \times 5^3 = 5^5$

Example 4 : $a^4 \times a^5$

Solution: $a^4 \times a^5 = (a \times a \times a \times a) \times (a \times a \times a \times a \times a)$

$= (a \times a \times a \times a \times a \times a \times a \times a \times a)$

$= a^9$ and this is same as a^{4+5} (as $4 + 5 = 9$)

Therefore, $a^4 \times a^5 = a^{4+5}$

Based on the above observations we can say that.

$\boldsymbol{a^m \times a^n = (a \times a \times a \ldots\ldots\ldots 'm' \text{ times}) \times (a \times a \times a \times \ldots\ldots 'n' \text{ times}) = a^{m+n}}$

For any non-zero integer '*a*', and integers '*m*' and '*n*'

$$a^m \times a^n = a^{m+n}$$

Do This

1. Simplify the following using the formula $a^m \times a^n = a^{m+n}$

   (i) $3^{11} \times 3^9$ (ii) $p^5 \times p^8$

2. Find the appropriate number in place of the symbol '?' in the following.

   Let 'k' be any non zero integer

   (i) $k^3 \times k^4 = k^?$ (ii) $k^{15} \times k^? = k^{31}$

11.3.2 Exponent of exponent

Example 5 : Consider $(3^2)^3$

Solution : Here ' 3^2 ' is the base and' 3' is the exponent

$(3^2)^3 = 3^2 \times 3^2 \times 3^2$

$= 3^{2+2+2}$ (multplying terms with the same base)

$= 3^6$ and this is the same as $3^{2\times3}$ (as $2 \times 3 = 6$)

Therefore, $(3^2)^3 = 3^{2\times3}$

Do This

Compute 3^6 , cube of 3^2 and verify whether $(3^2)^3 = 3^6$?

Example 6 : Let us consider $(4^5)^3$

Solution : $(4^5)^3 = 4^5 \times 4^5 \times 4^5$

$= 4^{5+5+5}$ (multplying terms with the same base)

$= 4^{15}$ and this is same as $4^{5\times 3}$ (as $5 \times 3 = 15$)

Therefore, $(4^5)^3 = 4^{5\times 3}$

Example 7: $(a^m)^4$

Solution : $(a^m)^4 = a^m \times a^m \times a^m \times a^m$

$= a^{m+m+m+m}$ (multplying terms with the same base)

$= a^{4m}$ and this is same as $a^{m\times 4}$ (as $4 \times m = 4m$)

Therefore, $(a^m)^4 = a^{m\times 4}$

Based on all the above we can say that $(a^m)^n = a^m \times a^m \times a^m ......$ ***n*** **times** $= a^{m+m+m+... n \text{ times}}$

$= a^{mn}$

For any non-zero integer '*a*' and integers '*m*' and '*n*'

$(a^m)^n = a^{mn}$

11.3.3 Exponent of a product

Example 8 : Consider $3^5 \times 4^5$

Solution : Here 3^5 and 4^5 have the same exponent 5 but different bases.

$3^5 \times 4^5 = (3\times3\times3\times3\times3) \times (4\times4\times4\times4\times4)$

$= (3\times4) \times (3\times4) \times (3\times4) \times (3\times4) \times (3\times4)$

$= (3\times4)^5$

Therefore, $3^5 \times 4^5 = (3 \times 4)^5$

Example 9: Consider $4^4 \times 5^4$

Solution : Here 4^4 and 5^4 have the same exponent 4 but have different bases.

$4^4 \times 5^4 = (4 \times 4 \times 4 \times 4) \times (5 \times 5 \times 5 \times 5)$

$= (4 \times 4 \times 4 \times 4 \times 5 \times 5 \times 5 \times 5)$

$= (4 \times 5)\times (4 \times 5) \times (4 \times 5) \times (4 \times 5)$

$= (4 \times 5)^4$

Therefore, $4^4 \times 5^4 = (4 \times 5)^4$

Example 10 : Consider $p^7 \times q^7$

Solution : Here p^7 and q^7 have the same exponent 7 but different bases.

$$p^7 \times q^7 = (p \times p \times p \times p \times p \times p \times p) \times (q \times q \times q \times q \times q \times q \times q)$$

$$= (p \times p \times p \times p \times p \times p \times p \times q \times q \times q \times q \times q \times q \times q)$$

$$= (p \times q) \times (p \times q) \times (p \times q) \times (p \times q) \times (p \times q) \times (p \times q) \times (p \times q)$$

$$= (p \times q)^7$$

Therefore, $p^7 \times q^7 = (p \times q)^7$

Based on all the above we can conclude that $a^m \times b^m = (a \times b)^m = (ab)^m$

For any two non-zero integers 'a', 'b' and any positive integer 'm'

$$a^m \times b^m = (ab)^m$$

Do This

Simplify the following using the law $a^m \times b^m = (a\,b)^m$

(i) $(2 \times 3)^4$ (ii) $x^p \times y^p$ (iii) $a^8 \times b^8$ (iv) $(5\times4)^{11}$

11.3.4 Division of exponents

Before discussing division of exponents we will now discuss about negative exponents.

11.3.4(a) Negative exponents

Observe the following pattern.

2^5	=	32	3^5	=	243
2^4	=	16	3^4	=	81
2^3	=	8	3^3	=	27
2^2	=	4	3^2	=	9
2^1	=	2	3^1	=	3
2^0	=	1	3^0	=	1
2^{-1}	=		3^{-1}	=	
(Hint: half of 1)			(Hint: one-third of 1)		
2^{-2}	=		3^{-2}	=	

What part of 32 is 16?

What is the difference between 2^6 and 2^4?

You will find that each time the exponent decreases by 1,the value becomes half of the previous.

From the above patterns we can say.

$2^{-1} = \frac{1}{2}$ and $2^{-2} = \frac{1}{4}$ and

$3^{-1} = \frac{1}{3}$ and $3^{-2} = \frac{1}{9}$

Furthermore, we can see that $2^{-2} = \frac{1}{4} = \frac{1}{2^2}$

similarly, $3^{-1} = \frac{1}{3}$ and $3^{-2} = \frac{1}{9} = \frac{1}{3^2}$

For any non -zero integer '*a*' and any integer '*n*'

$$a^{-n} = \frac{1}{a^n}$$

Do This

1. Write the following, by using $a^{-n} = \frac{1}{a^n}$, with positive exponants.

(i) x^{-7} (ii) a^{-5} (iii) 7^{-5} (iv) 9^{-6}

11.3.4(b) Zero exponents

In the earlier discussion we have seen that

$2^0 = 1$

$3^0 = 1$

Similarly we can say

$4^0 = 1$

$5^0 = 1$ and so on

Thus for non zero integer 'a'

$a^0 = 1$

11.3.4(c) Division of exponents having the same base

Example 11 : Consider $\frac{7^7}{7^3}$

Solution : $\frac{7^7}{7^3}=\frac{7\times7\times7\times7\times\cancel{7}\times\cancel{7}\times\cancel{7}}{\cancel{7}\times\cancel{7}\times\cancel{7}} = 7\times7\times7\times7$

$= 7^4$ which is same as 7^{7-3} (as $7 - 3 = 4$)

Therefore, $\frac{7^7}{7^3} = 7^{7-3}$

Example 12: Consider $\frac{3^8}{3^3}$

Solution : $\frac{3^8}{3^3}=\frac{3\times3\times3\times3\times3\times\cancel{3}\times\cancel{3}\times\cancel{3}}{\cancel{3}\times\cancel{3}\times\cancel{3}}=3\times3\times3\times3\times3$

$= 3^5$ which is same as 3^{8-3} (as $8 - 3 = 5$)

Therefore, $\frac{3^8}{3^3} = 3^{8-3}$

Example 13: Consider $\frac{5^5}{5^8}$

Solution : $\frac{5^5}{5^8}=\frac{\cancel{5}\times\cancel{5}\times\cancel{5}\times\cancel{5}\times\cancel{5}}{5\times5\times5\times\cancel{5}\times\cancel{5}\times\cancel{5}\times\cancel{5}\times\cancel{5}} = \frac{1}{5\times5\times5} = \frac{1}{5^3}$

$\frac{1}{5^3}$ which is same as $\frac{1}{5^{8-5}}$ (as $8 - 5 = 3$)

Therefore, $\frac{5^5}{5^8} = \frac{1}{5^{8-5}}$

Example 14: Consider $\frac{a^2}{a^7}$

Solution : $\frac{a^2}{a^7}=\frac{\cancel{a}\times\cancel{a}}{a\times a\times a\times a\times a\times\cancel{a}\times\cancel{a}}=\frac{1}{a\times a\times a\times a\times a}$

$=\frac{1}{a^5}$ which is the same as $\frac{1}{a^{7-2}}$ (as $7 - 2 = 5$)

Therefore, $\dfrac{a^2}{a^7} = \dfrac{1}{a^{7-2}}$

Based on all the above examples we can say that-

$$\frac{a^m}{a^n} = a^{m-n} \text{ if } m > n \text{ and } \frac{a^m}{a^n} = \frac{1}{a^{n-m}} \text{ if } m<n$$

> **For '*d*' a non-zero integer '*a*' and integers '*m*' and '*n*'**
>
> $$\frac{a^m}{a^n} = a^{m-n} \quad \text{if } m > n \quad \text{and} \quad \frac{a^m}{a^n} = \frac{1}{a^{n-m}} \quad \text{if } n > m$$

What happens *when m = n?* Give your answer.

Example 15 : Consider $\dfrac{4^3}{4^3}$

Solution : $\dfrac{4^3}{4^3} = \dfrac{4 \times 4 \times 4}{4 \times 4 \times 4} = \dfrac{1}{1} = 1 \quad .....(1)$

Also we know that $\dfrac{a^m}{a^n} = a^{m-n}$

$\therefore \dfrac{4^3}{4^3} = 4^{3-3} = 4^0 = 1$ from (1)

Similarly find $\dfrac{7^4}{7^4} = ?$

What do you observe from above?

Also consider $\dfrac{a^4}{a^4} = \dfrac{a \times a \times a \times a}{a \times a \times a \times a} = 1$

But from $\dfrac{a^m}{a^n} = a^{m-n}$

We have $\dfrac{a^4}{a^4} = a^{4-4} = a^0 = 1$

For any non zero number 'a' we have $a^0 = 1$.
Observe here m, n (m = n)

Thus if m = n $\quad \boxed{\dfrac{a^m}{a^n} = 1}$

Do This

1. Simplify and write in the form of a^{m-n} or $\frac{1}{a^{n-m}}$.

(i) $\frac{13^8}{13^5}$ (ii) $\frac{3^4}{3^{14}}$

2. Fill the appropriate number in the box.

Ex : $\frac{8^8}{8^3} = 8^{\boxed{8-3}} = 8^{\boxed{5}}$

(i) $\frac{12^{12}}{12^7} = 12^{\square} = 12^{\square}$ (ii) $\frac{a^{18}}{a^{\square}} = a^{\square} = a^{\boxed{10}}$

11.3.4(c) Dividing terms with the same exponents

Example 16: Consider $\left(\frac{7}{4}\right)^5$

Solution : $\left(\frac{7}{4}\right)^5 = \frac{7}{4} \times \frac{7}{4} \times \frac{7}{4} \times \frac{7}{4} \times \frac{7}{4}$

$= \frac{7 \times 7 \times 7 \times 7 \times 7}{4 \times 4 \times 4 \times 4 \times 4}$

$= \frac{7^5}{4^5}$ (by the definition of exponent)

Therefore, $\left(\frac{7}{4}\right)^5 = \frac{7^5}{4^5}$

Example 17: Consider $\left(\frac{p}{q}\right)^6$

Solution : $\left(\frac{p}{q}\right)^6 = \left(\frac{p}{q}\right) \times \left(\frac{p}{q}\right) \times \left(\frac{p}{q}\right) \times \left(\frac{p}{q}\right) \times \left(\frac{p}{q}\right) \times \left(\frac{p}{q}\right)$

$= \frac{p \times p \times p \times p \times p \times p}{q \times q \times q \times q \times q \times q}$

$$= \frac{p^6}{q^6} \qquad \text{(By the definition of exponent)}$$

$$\text{Therefore, } \left(\frac{p}{q}\right)^6 = \frac{p^6}{q^6}$$

Based on the above observations we can say that.

$$\left(\frac{a}{b}\right)^m = \frac{a \times a \times a \times a \times ....\times a \text{ 'm' times}}{b \times b \times b \times b \times ....\times b \text{ 'm' times}} = \frac{a^m}{b^m}$$

For any non-zero integers a, b and integer '*m*' $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Do This

1. Complete the following

(i) $\left(\frac{5}{7}\right)^3 = \frac{5^3}{\boxed{}}$ (ii) $\left(\frac{3}{2}\right)^{\boxed{}} = \frac{3^5}{2^5}$

(iii) $\left(\frac{8}{3}\right)^4 = \frac{\boxed{}}{\boxed{}}$ (iv) $\left(\frac{x}{y}\right)^{11} = \frac{\boxed{}}{y^{11}}$

11.3.5 Terms with negative base

Example 18 : Evaluate $(1)^4$, $(1)^5$, $(1)^7$, $(-1)^2$, $(-1)^3$, $(-1)^4$, $(-1)^5$

Solution : $(1)^4 = 1 \times 1 \times 1 \times 1 = 1$

$(1)^5 = 1 \times 1 \times 1 \times 1 \times 1 = 1$

$(1)^7 = 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$

$(-1)^2 = (-1) \times (-1) = 1$

$(-1)^3 = (-1) \times (-1) \times (-1) = -1$

$(-1)^4 = (-1) \times (-1) \times (-1) \times (-1) = 1$

$(-1)^5 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) = -1$

From the above illustrations we observe that:

(i) 1 raised to any power is 1.

(ii) (–1) raised to odd power is (–1) and (–1) raised to even power is (+1).

Thus $(-a)^m = -a^m$ If 'm' is odd

$(-a)^m = a^m$ If 'm' is even

Now, let us look at some more examples.

$(-3)^4 = (-3)\ (-3)\ (-3)\ (-3) = 81$

$(-a)^4 = (-a)\ (-a)\ (-a)\ (-a) = a^4$

$$(-a)^{-3} = \frac{1}{(-a)} \times \frac{1}{(-a)} \times \frac{1}{(-a)} = \frac{1}{-a^3} = \frac{-1}{a^3}$$

Example 19 : Express $\frac{-27}{125}$ in exponential form

Solution : $-27 = (-3)\ (-3)\ (-3) = (-3)^3$

$125 = 5 \times 5 \times 5 = (5)^3$

Therefore, $\frac{-27}{125} = \frac{(-3)^3}{(5)^3}$ as $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

Thus, $\frac{-27}{125} = \left(\frac{-3}{5}\right)^3$

Do This

1. Write in expanded form.

(i) $(a)^{-5}$ (ii) $(-a)^4$ (iii) $(-7)^{-5}$ (iv) $(-a)^m$

2. Write in exponential form

(i) $(-3) \times (-3) \times (-3)$ (ii) $(-b) \times (-b) \times (-b) \times (-b)$

(iii) $\frac{1}{(-2)} \times \frac{1}{(-2)} \times \frac{1}{(-2)}$ ...... 'm' times

Exercise 2

1. Simplify the following using laws of exponents.

(i) $2^{10} \times 2^4$ (ii) $(3^2) \times (3^2)^4$ (iii) $\frac{5^7}{5^2}$

(iv) $9^2 \times 9^{18} \times 9^{10}$ (v) $\left(\frac{3}{5}\right)^4 \times \left(\frac{3}{5}\right)^3 \times \left(\frac{3}{5}\right)^8$ (vi) $(-3)^3 \times (-3)^{10} \times (-3)^7$

(vii) $(3^2)^2$ (viii) $2^4 \times 3^4$ (ix) $2^{4a} \times 2^{5a}$

(x) $(10^2)^3$ (xi) $\left[\left(\frac{-5}{6}\right)^2\right]^5$ (xii) $2^{3a+7} \times 2^{7a+3}$

(xiii) $\left(\frac{2}{3}\right)^5$ (xiv) $(-3)^3 \times (-5)^3$ (xv) $\frac{(-4)^6}{(-4)^3}$

(xvi) $\frac{9^7}{9^{15}}$ (xvii) $\frac{(-6)^5}{(-6)^9}$ (xiii) $(-7)^7 \times (-7)^8$

(xix) $(-6^4)^4$ (xx) $a^x \times a^y \times a^z$

2. By what number should 3^{-4} be multiplied so that the product is 729?

3. If $5^6 \times 5^{2x} = 5^{10}$, then find x.

4. Evaluate $2^0 + 3^0$

5. Simplify $\left(\frac{x^a}{x^b}\right)^a \times \left(\frac{x^b}{x^a}\right)^a \times \left(\frac{x^a}{x^a}\right)^b$

6. State true or false and justify your answer.

(i) $100 \times 10^{11} = 10^{13}$ (ii) $3^2 \times 4^3 = 12^5$ (iii)) $5^0 = (100000)^0$

(iv) $4^3 = 8^2$ (v) $2^3 > 3^2$ (vi) $(-2)^4 > (-3)^4$

(vii) $(-2)^5 > (-3)^5$

Classroom Project

Collect the annual income particulars of any ten families in your locality and round it to the nearest thousands / lakhs and express the income of each family in the exponential form.

11.3.6 Expressing large numbers in standard form

The mass of the Earth is about 5976 x 10^{21} kg.

The width of the Milky Way Galaxy from one edge to the other edge is about 946×10^5 km.

These numbers are still not very easy to comprehend. Thus, they are often expressed in standard form. In standard form the:

Mass of the Earth is about 5.976×10^{24} kg

Width of the Milky Way Galaxy from one edge to the other edge is about 9.46×10^7 km.

Thus, in standard form (Scientific notation) a number is expressed as the product of largest integer exponent of 10 and a decimal number between 1 and 10.

Exercise 3

Express the number appearing in the following statements in standard form.

(i) The distance between the Earth and the Moon is approximately 384,000,000m.

(ii) The universe is estimated to be about 12,000,000,000 years old.

(iii) The distance of the sun from the center of the Milky Way Galaxy is estimated to be 300,000,000,000,000,000,000 m.

(iv) The earth has approximately 1,353,000,000 cubic km of sea water.

Looking Back

- Very large numbers are easier to read, write and understand when expressed in exponential form.
- 10,000 = 10^4 (10 raised to the power of 4); 243 = 3^5 (3 raised to the power of 5); 64 = 2^6 (2 raised to the power of 6). In these examples 10, 3, 2 are the respective bases and 4, 5, 6 are the respective exponents.
- Laws of Exponents: For any non-zero integers 'a' and 'b' and integers 'm' and 'n'

(i) $a^m \times a^n = a^{m+n}$ (ii) $(a^m)^n = a^{mn}$ (iii) $a^m \times b^m = (ab)^m$

(iv) $a^{-n} = \dfrac{1}{a^n}$ (v) $\dfrac{a^m}{a^n} = a^{m-n}$ if $m > n$

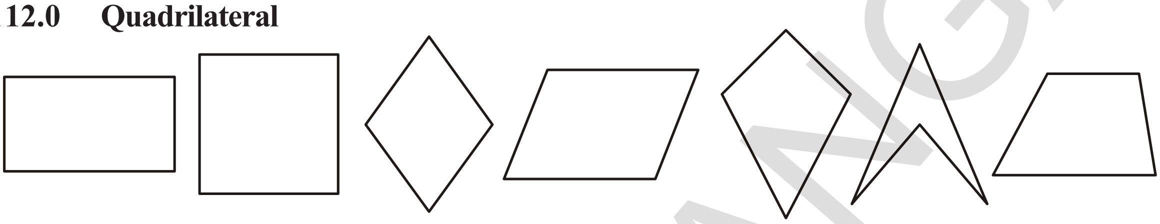
(vi) $\dfrac{a^m}{b^n} = \dfrac{1}{a^{n-m}}$ if $n > m$ (vii) $\dfrac{a^m}{b^m} = \left(\dfrac{a}{b}\right)^m$

(viii) $a^0 = 1$ (where $a \neq 0$)

12 QUADRILATERALS

In Class VI, we have been introduced to quadrilaterals. In this unit you will learn about the different types of quadrilaterals and their properties in detail.

12.0 Quadrilateral



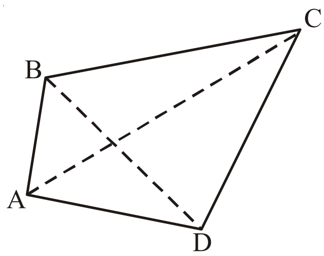
What is common among all these pictures?

(Hints: Number of sides, angles, vertices. Is it an open or closed figure?)

Thus, a quadrilateral is a closed figure with four sides, four angles and four vertices.

Quadrilateral ABCD has

(i) Four sides, namely $\overline{AB}, \overline{BC}, \overline{CD}$ and $\overline{DA}$



(ii) Four vertices, namely A, B, C and D.

(iii) Four angles, namely $\angle$ABC, $\angle$BCD, $\angle$CDA and $\angle$DAC.

(iv) The line segments joining the opposite vertices of a quadrilateral are called the diagonals of the quadrilateral. $\overline{AC}$ and $\overline{BD}$ are the diagonals of quadrilateral ABCD.

(v) The two sides of a quadrilateral which have a common vertex are called the 'adjacent sides' of the quadrilateral. In quadrilateral ABCD, $\overline{AB}$ is adjacent to $\overline{BC}$ and B is their common vertex.

(vi) The two angles of a quadrilateral having a common side are called the pair of 'adjacent angles' of the quadrilateral. Thus, $\angle$ABC and $\angle$BCD are a pair of adjacent angles and $\overline{BC}$ is the common side.

Do This :

1. Find the other adjacent sides and common vertices.

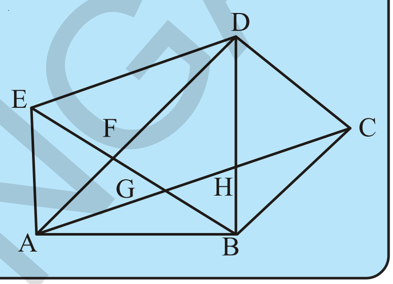
2. Find the other pairs of adjacent angles and sides.

(vii) The two sides of a quadrilateral, which do not have a common vertex, are called a pair of 'opposite sides' of the quadrilateral. Thus $\overline{AB}$, $\overline{CD}$ and $\overline{AD}$, $\overline{BC}$ are the two pairs of 'opposite sides' of the quadrilateral.

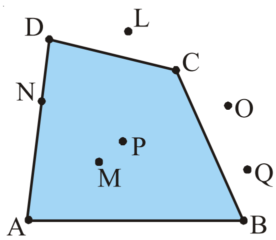
(viii) The two angles of a quadrilateral which do not have a common side are known as a pair of 'opposite angles' of the quadrilateral. Thus $\angle$BAD, $\angle$DCB and $\angle$ADC, $\angle$CBA are the two pairs of opposite angles of the quadrilateral.

Try This

How many different quadrilaterals can be obtained from the adjacent figure? Name them.



12.1 Interior-Exterior of a quadrilateral



In quadrilateral ABCD which points lie inside the quadrilateral?

Which points lie outside the quadrilateral?

Which points lie on the quadrilateral?

Points P and M lie in the interior of the quadrilateral. Points L, O and Q lie in the exterior of the quadrilateral. Points N, A, B, C and D lie on the quadrilateral.

Mark as many points as you can in the interior of the quadrilateral.

Mark as many points as you can in the exterior of the quadrilateral.

How many points, do you think will be there in the interior of the quadrilateral?

12.2 Convex and Concave quadrilateral

Mark any two points L and M in the interior of quadrilateral ABCD and join them with a line segment.

Does the line segment or a part of it joining these points lie in the exterior of the quadrilateral? Can you find any two points in the interior of the quadrilateral ABCD for which the line segment joining them falls in the exterior of the quadrilateral?

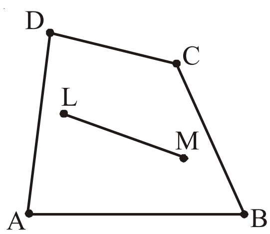
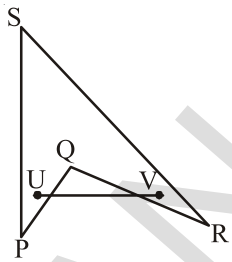


Figure1

You will see that this is not possible.

Now let us do similar work in quadrilateral PQRS.

Mark any two points U and V in the interior of quadrilateral PQRS and join them. Does the line segment joining these two points fall in the exterior of the quadrilateral? Can you make more line segments like these in quadrilateral PQRS.



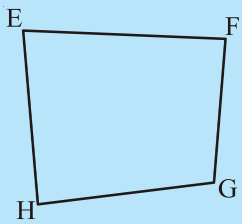
Can you also make line segments, joining two points, which lie in the interior of the quadrilateral. You will find that this is possible too.

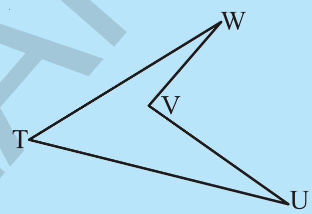
Quadrilateral ABCD is said to be a convex quadrilateral if all line segments joining points in the interior of the quadrilateral also lie in interior of the quadrilateral.

Quadrilateral PQRS is said to be a concave quadrilateral if all line segment joining points in the interior of the quadrilateral do not necessarily lie in the interior of the quadrilateral.

Try This

1.





(i) Is quadrilateral EFGH a convex quadrilateral?

(ii) Is quadrilateral TUVW a concave quadrilateral?

(iii) Draw both the diagonals for quadrilateral EFGH. Do they intersect each other?

(iv) Draw both the diagonals for quadrilateral TUVW. Do they intersect each other?

You will find that the digonals of a convex quadrilateral intersect each other in the interior of the quadrilateral and the diagonals of a concave quadrialteral intersect each other in the exterior of the quadrilateral.

12.3 Angle-sum property of a quadrilateral

Activity 1

Take a piece of cardboard. Draw a quadrilateral ABCD on it. Make a cut of it. Then cut quadrilateral into four pieces (Figure 1) and arrange them as shown in the Figure 2, so that all angles $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$ meet at a point.

Figure 1

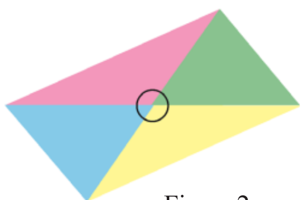


Figure 2

Is the sum of the angles $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$ equal to 360^o? (sum of angles at a point)

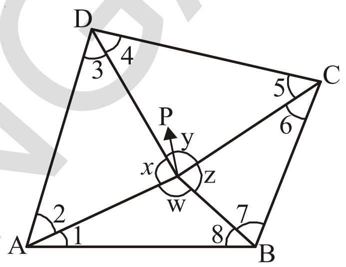
The sum of the four angles of a quadrilateral is 360°.

[Note: We can denote the angles by $\angle 1$, $\angle 2$, $\angle 3$, etc., as their respective measures i.e. $m\angle 1$, $m\angle 2$, $m\angle 3$, etc.]

You may arrive at this result in several other ways also.

1. Let P be any point in the interior of quadrilateral ABCD. Join P to vertices A, B, C and D.

    In the figure, consider ΔPAD.



    $m\angle 2 + m\angle 3 = 180° - x$ .... (1)

    Similarly, in ΔPDC, $m\angle 4 + m\angle 5 = 180° - y$ .... (2)

    in ΔPCB, $m\angle 6 + m\angle 7 = 180^o - z$ and .... (3)

    in ΔPBA, $m\angle 8 + m\angle 1 = 180^o - w$. .... (4)

    (angle-sum property of a triangle)

    Adding (1), (2), (3) and (4) we get

    $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6 + m\angle 7 + m\angle 8$

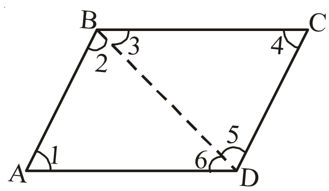
    $= 180° - x + 180° - y + 180° - z + 180° - w$

    $= 720° - (x + y + z + w)$

    ($x + y + z + w = 360^o$; sum of angles at a point)

    $=720° - 360°$ $= 360°$

    Thus, the sum of the angles of the quadrilateral is 360^o.

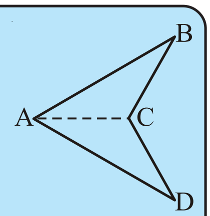


2. Take any quadrilateral, say ABCD. Divide it into two triangles, by drawing a diagonal. You get six angles 1, 2, 3, 4, 5 and 6.

    Using the angle-sum property of a triangle and you can easily find how the sum of the measures of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ amounts to 360°.

Try This

What would happen if the quadrilateral is not convex? Consider quadrilateral ABCD. Split it into two triangles and find the sum of the interior angles. What is the sum of interior angles of a concave quadrilateral?



Example 1 : The three angles of a quadrilateral are 55°, 65° and 105°. What is the fourth angle?

Solution : The sum of the four angles of a quadrilateral = 360°.

The sum of the given three angles $= 55° + 65° + 105° = 225°$

Therefore, the fourth angle $= 360° – 225° = 135°$

Example 2 : In a quadrilateral, two angles are 80° and 120°. The remaining two angles are equal. What is the measure of each of these angles?

Solution : The sum of the four angles of the quadrilateral = 360°.

Sum of the given two angles $= 80° + 120° = 200°$

Therefore, the sum of the remaining two angles $= 360° – 200° = 160°$

Both these angles are equal.

Therefore, each angle $= 160° \div 2 = 80°$

Example 3 : The angles of a quadrilateral are $x°$, $(x – 10)°$, $(x + 30)°$ and $2x°$. Find the angles.

Solution: The sum of the four angles of a quadrilateral $= 360°$

Therefore, $x + (x – 10) + (x + 30) + 2x = 360°$

Solving, $5x + 20 = 360°$

$x = 68°$

Thus, the four angles are $= 68°$; $(68–10)°$; $(68+30)°$; $(2\times68)°$

$= 68°, 58°, 98°$ and $136°$.

Example 4 : The angles of a quadrilateral are in the ratio 3 : 4 : 5 : 6. Find the angles.

Solution : The sum of four angles of a quadrilateral = 360°

The ratio of the angles is 3 : 4 : 5 : 6

Thus, the angles are $3x$, $4x$, $5x$ and $6x$.

$$3x + 4x + 5x + 6x = 360$$

$$18x = 360$$

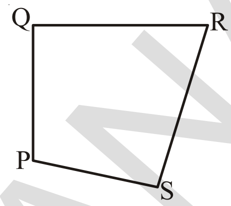
$$x = \frac{360}{18} = 20$$

Thus, the angles are $= 3 \times 20; 4 \times 20; 5 \times 20; 6 \times 20$

$= 60°, 80°, 100°$ and $120°$

Exercise - 1

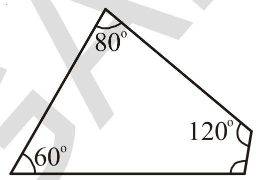
1. In quadrilateral PQRS



   (i) Name the sides, angles, vertices and diagonals.

   (ii) Also name all the pairs of adjacent sides, adjacent angles, opposite sides and opposite angles.

2. The three angles of a quadrilateral are 60°, 80° and 120°. Find the fourth angle?



3. The angles of a quadrilateral are in the ratio 2 : 3 : 4 : 6. Find the measure of each of the four angles.

4. The four angles of a quadrilateral are equal. Draw this quadrilateral in your notebook. Find each of them.

5. In a quadrilateral, the angles are $x°$, $(x + 10)°$, $(x + 20)°$, $(x + 30)°$. Find the angles.

6. The angles of a quadrilateral cannot be in the ratio 1 : 2 : 3 : 6. Why? Give reasons.

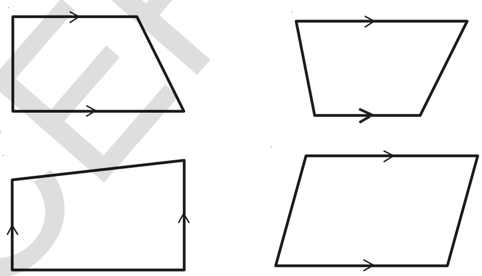
   (Hint: Try to draw a rough diagram of this quadrilateral)

12.4 Types of quadrilaterals

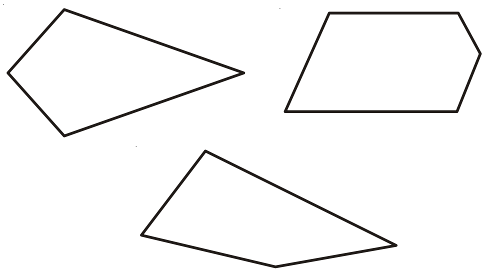
Based on the nature of the sides and angles, quadrilaterals have different names.

12.4.1 Trapezium

Trapezium is a quadrilateral with one pair of parallel sides.



These are trapeziums



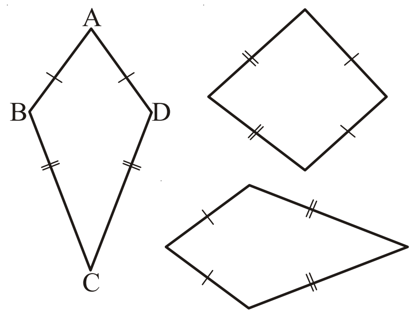
These are not trapeziums

(Note: The arrow marks indicate parallel lines).

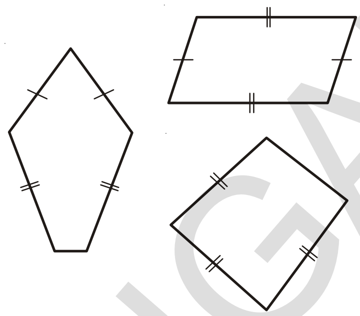
Why the second set of figures not trapeziums?

12.4.2 Kite

A Kite is a special type of quadrilateral. The sides with the same markings in each figure are equal in length. For example AB = AD and BC = CD.



These are kites



These are not kites

Why the second set of figures are not kites?

Observe that:

(i) **A kite has 4 sides (It is a convex quadrilateral).**

(ii) **There are exactly two distinct, consecutive pairs of sides of equal length.**

Activity 2

Take a thick sheet of paper. Fold the paper at the centre. Draw two line segments of different lengths as shown in Figure 1. Cut along the line segments and open up the piece of paper as shown in Figure 2. You have the shape of a kite.

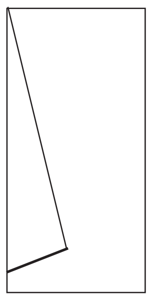


Figure1

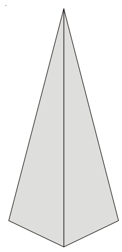


Figure2

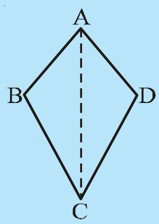
Does the kite have line symmetry?

Fold both the diagonals of the kite. Use the set-square to check if they cut at right angles.

Are the diagonals of the kite equal in length? Verify (by paper-folding or measurement) if the diagonals bisect each other.

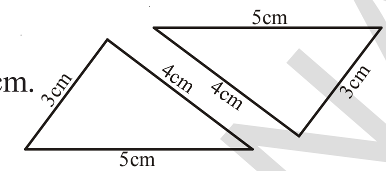
Try This

Prove that in a kite ABCD, ΔABC and ΔADC are congruent.



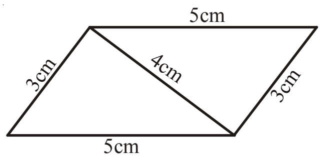
12.4.3 Parallelogram

Activity 3



Take two identical cut-outs of a triangle of sides 3 cm, 4 cm, 5 cm.

Arrange them as shown in the figure given below:



You get a parallelogram. Which are the parallel sides here? Are the parallel sides equal? You can get two more parallelograms using the same set of triangles. Find them out.

A parallelogram is a quadrilateral with two pairs of opposite sides arallel.

Activity 4

Take a ruler. Place it on a paper and draw two lines along its two sides as shown in Figure1. Then place the ruler over the lines as shown in Figure2 and draw two more lines along its edges again.

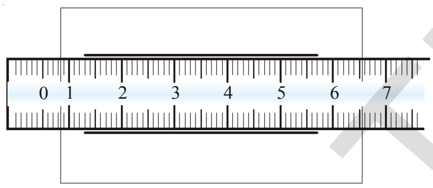


Figure 1

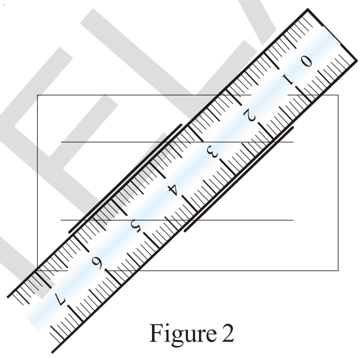


Figure 2

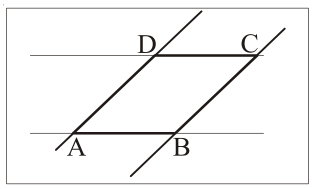


Figure 3

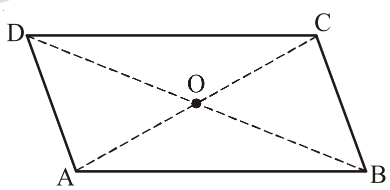
These four lines enclose a quadrilateral which is made up of two pairs of parallel lines. It is a parallelogram.

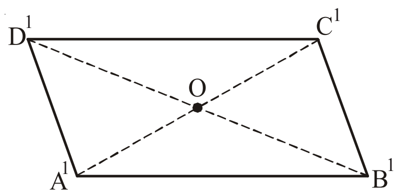
12.4.3(a) Properties of a parallelogram

Sides of parallelogram

Activity 5

Take cut-outs of two identical parallelograms, say ABCD and $A^1B^1C^1D^1$.





Here $\overline{AB}$ is same as $\overline{A'B'}$ except for the name. Similarly, the other corresponding sides are equal too. Place $\overline{A'B'}$ over $\overline{DC}$. Do they coincide? Are the lengths $\overline{A'B'}$ and $\overline{DC}$ equal?

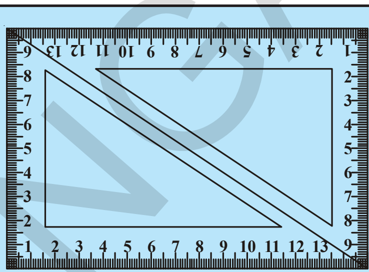
Similarly examine the lengths $\overline{AD}$ and $\overline{B'C'}$. What do you find?

You will find that the sides are equal in both cases. Thus, **the opposite sides of a parallelogram are of equal length.**

You will also find the same results by measuring the side of the parallelogram with a scale.

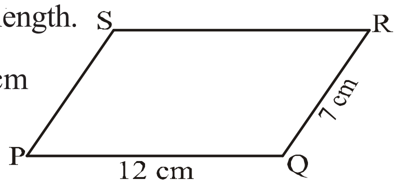
Try This

Take two identical set squares with angles 30° – 60° – 90° and place them adjacently as shown in the adjacent figure. Does this help you to verify the above property? Can we say every rectangle is a parallelogram?



Example 5 : Find the perimeter of the parallelogram PQRS.

Solution : In a parallelogram, the opposite sides have same length.



According to the question, PQ= SR = 12 cm and QR = PS = 7 cm

Thus, Perimeter = PQ + QR + RS + SP

= 12 cm + 7 cm + 12 cm + 7 cm = 38 cm

Angles of a parallelogram

Activity 6

Let ABCD be a parallelogram. Copy it on a tracing sheet. Name this copy as A′, B′, C′, D′. Place A′, B′, C′, D′ on ABCD as shown in Figure 1. Pin them together at the point where the diagonals meet. Rotate the transparent sheet by 90° as shown in Figure 2. Then rotate the parallelogram again by 90° in the same direction. You will find that the parallelograms coincide as shown in Figure 3. You now find A′ lying exactly on C and C′ lying on A. Similarly B′ lies on D and D′ lies on B as shown in Figure 3.

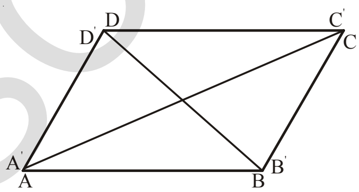


Figure 1

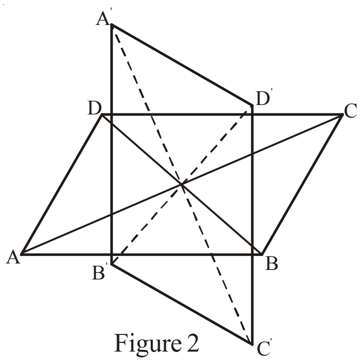


Figure 2

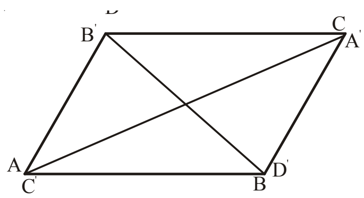


Figure 3

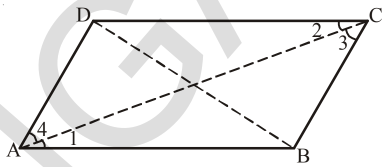
Does this tell you anything about the measures of the angles A and C? Examine the same for angles B and D. State your findings.

You will conclude that the opposite angles of a parallelogram are of equal measure.

Try This

Take two identical $30° - 60° - 90°$ set squares and form a parallelogram as before. Does the figure obtained help you confirm the above property?

You can justify this idea through logical arguments-



If $\overline{AC}$ and $\overline{BD}$ are the diagonals of the parallelogram ABCD you find that $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ (alternate angles property)

ΔABC and ΔCDA are congruent $\Delta ABC \cong \Delta CDA$ (ASA congruency).

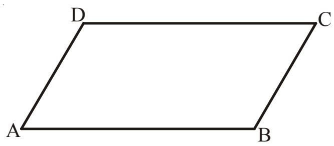
Therefore, $m\angle B = m\ \angle D$ (c.p.c.t.).

Similarly, $\Delta ABD \cong \Delta CDB$, therefore, $m\ \angle A = m\ \angle C$. (c.p.c.t.).

Thus, the opposite angles of a parallelogram are of equal measure.

We now turn our attention to adjacent angles of a parallelogram.

In parallelogram ABCD, $\overline{DC} \parallel \overline{AB}$ and $\overline{DA}$ is the transversal.



Therefore, $\angle A$ and $\angle D$ are the interior angles on the same side of the transversal.thus are supplementary.

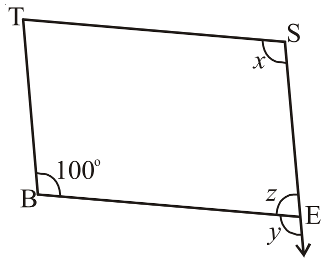
$\angle A$ and $\angle B$ are also supplementary. Can you say 'why'?

$\overline{AD} \parallel \overline{BC}$ and $\overline{BA}$ is a transversal, making $\angle A$ and $\angle B$ interior angles.

Do This

Identify two more pairs of supplementary angles from the parallelogram ABCD given above.

Example 6 : BEST is a parallelogram. Find the values x, y and z.



Solution : $\angle S$ is opposite to $\angle B$.

So, $x = 100°$ (opposite angles property)

$y = 100°$ (corresponding angles)

$z = 80°$ (since $\angle y, \angle z$ is a linear pair)

The adjacent angles in a parallelogram are supplementary. You have observed the same result in the previous example.

Example 7 : In parallelogram RING if m $\angle R = 70°$, find all the other angles.

Solution : According to the question, $m\angle R = 70°$

Then $m\angle N = 70°$

(opposite angles of a parallelogram)

Since $\angle R$ and $\angle I$ are supplementary angles,

$m\angle I = 180° - 70° = 110°$

Also, $m\angle G = 110°$ since $\angle G$ and $\angle I$ are opposite angles of a parallelogram.

Thus, $m\angle R = m\angle N = 70°$ and $m\angle I = m\angle G = 110°$

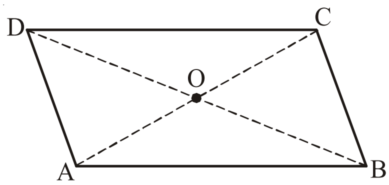
Try this

For the above example, can you find m $\angle I$ and m $\angle G$ by any other method?

Hint : angle-sum property of a quadrilateral

12.4.3 (b) Diagonals of parallelogarm

Activity 7

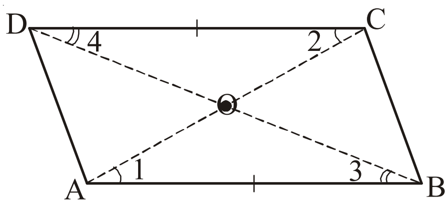


Take a cut-out of a parallelogram, say, ABCD. Let its diagonals $\overline{AC}$ and $\overline{DB}$ meet at O.

Find the mid-point of $\overline{AC}$ by folding and placing C on A. Is the mid-point same as O?

Find the mid-point of $\overline{DB}$ by folding and placing D on B. Is the mid-point same as O?

Does this show that diagonal $\overline{DB}$ bisects the diagonal AC at the point O? Discuss it with your friends. Repeat the activity to find where the mid point of DB could lie.



The diagonals of a parallelogram bisect each other

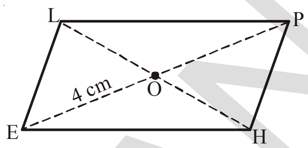
It is not very difficult to justify this property using ASA congurency:

$\Delta AOB \cong \Delta COD$ (How is ASA used here?)

This gives AO = CO and BO = DO

Example 8 : HELP is a parallelogram. Given that OE = 4 cm, where O is the point of intersection of the diagonals and HL is 5 cm more than PE? Find OH.

Solution : If OE = 4 cm then OP also is 4 cm (Why?)



So PE = 8 cm (Why?)

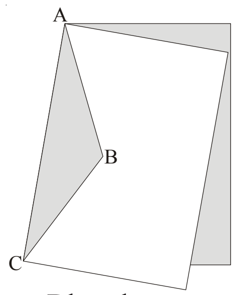
HL is 5 cm more than PE

Therefore, HL = 8 + 5 = 13 cm

Thus, $OH = \frac{1}{2} \times 13 = 6.5$ cms

12.4.4 Rhombus

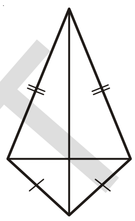
Recall the paper-cut kite you made earlier. When you cut along ABC and opened up, you got a kite. Here lengths AB and BC were different. If you draw AB = BC, then the kite you obtain is called a rhombus.

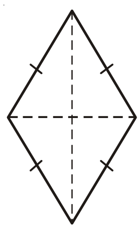


Rhombus-cut

Note that all the sides of rhombus are of same length; this is not the case with the kite.

Since the opposite sides of a rhombus are parallel, it is also a parallelogram. So, a rhombus has all the properties of a parallelogram and also that of a kite. Try to list them out. You can then verify your list with the check list at the end of the chapter.


Kite


Rhombus

The diagonals of a rhombus are perpendicular bisectors of one another

Activity 8

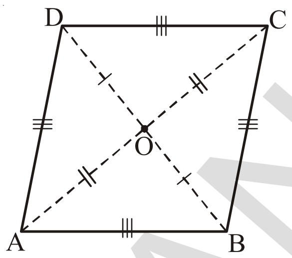
Take a copy of a rhombus. By paper-folding verify if the point of intersection is the mid-point of each diagonal. You may also check if they intersect at right angles, using the corner of a set-square.

Now let us justify this property using logical steps.

ABCD is a rhombus. It is a parallelogram too, so diagonals bisect each other.

Therefore, OA = OC and OB = OD.

We now have to show that m$\angle$AOD = m$\angle$COD = 90°.



It can be seen that by SSS congruency criterion.

ΔAOD $\cong$ ΔCOD

Therefore, m $\angle$AOD = m$\angle$COD

Since $\angle$AOD and $\angle$COD are a linear pair,

m $\angle$AOD = m $\angle$COD = 90°

We conclude, the diagonals of a rhombus are perpendicular bisectors of each other.

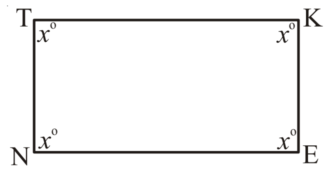
12.4.5 Rectangle

A rectangle is a parallelogram with equal angles.

What is the full meaning of this definition? Discuss with your friends.

If the rectangle is to be equiangular, what could be the measure of each angle?

Let the measure of each angle be $x°$.



Then $4x° = 360°$ (Why)?

Therefore, $x° = 90°$

Thus, each angle of a rectangle is a right angle.

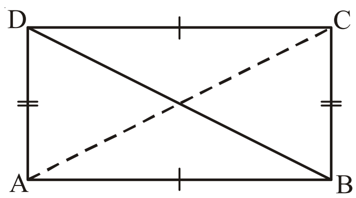
So, a rectangle is a parallelogram in which every angle is a right angle.

Being a parallelogram, the rectangle has opposite sides of equal length and its diagonals bisect each other.

In a parallelogram, the diagonals can be of different lengths. (Check this); but surprisingly the rectangle (being a special case) has diagonals of equal length.

This is easy to justify:

If ABCD is a rectangle,



ΔABC $\cong$ ΔABD

This is because AB = AB (Common)

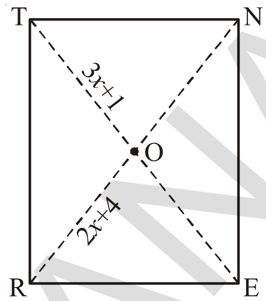
BC = AD (Why?)

m $\angle$A = m $\angle$B = 90° (Why?)

Thus, by SAS criterion ΔABC $\cong$ ΔABD and AC = BD (c.p.c.t.)

Thus, in a rectangle the diagonals are of equal length.

Example 9 : RENT is a rectangle. Its diagonals intersect at O. Find x, if OR = $2x + 4$ and OT = $3x + 1$.



Solution : OT is half of the diagonal TE and OR is half of the diagonal RN .

Diagonals are equal here. (Why?)

So, their halves are also equal.

Therefore $3x + 1 = 2x + 4$

or $x = 3$

12.4.6 Square

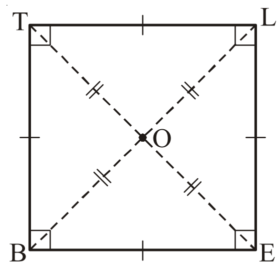
A square is a rectangle with equal adjacent sides.

This means a square has all the properties of a rectangle with an additional property that all the sides have equal length.

The square, like the rectangle, has diagonals of equal length.

In a rectangle, there is no requirement for the diagonals to be perpendicular to one another (Check this). However, this is not true for a square.

Let us justify this-



BELT is a square, therefore, BE = EL = LT = TB

Now, let us consider ΔBOE and ΔLOE

OB=OL (why?)

OE is common

Thus, by SSS congruency $\Delta BOE \cong LOE$

So $\angle BOE = \angle LOE$

but $\angle BOE + \angle LOE = 180^\circ$ (why?)

$$\angle BOE = \angle LOE = \frac{180}{2} = 90^\circ$$

Thus, the diagonals of a square are perpendicular bisectors of each other.

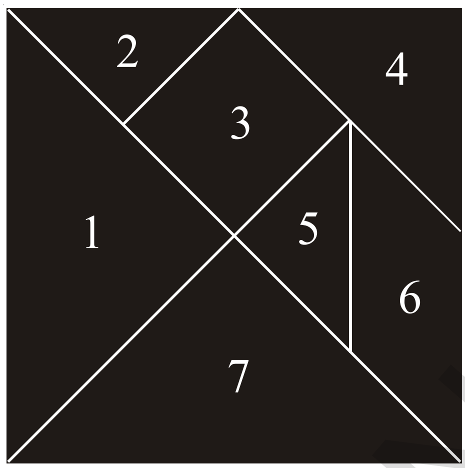
In a square the diagonals.

(i) bisect one another (square being a parallelogram)

(ii) are of equal length (square being a rectangle) and

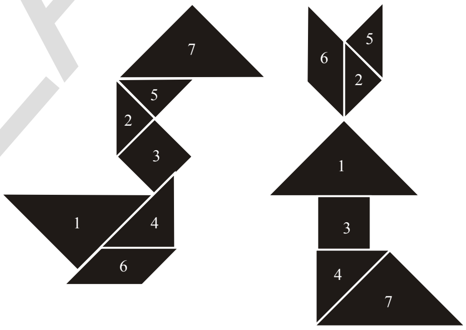
(iii) are perpendicular to one another.

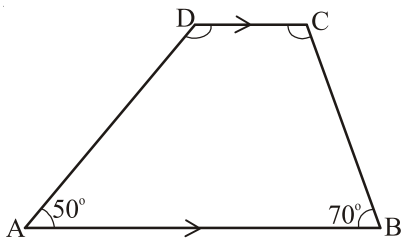
12.5 Making figures with a tangram.



Use all the pieces of tangarm to form a trapezium, a parallelogram, a rectangle and a square.

Also make as many different kinds of figures as you can by using all the pieces. Two examples have been given for you.





Example 10 : In trapezium ABCD, $\overline{AB}$ is parallel to $\overline{CD}$. If $\angle A = 50°$, $\angle B = 70°$. Find $\angle C$ and $\angle D$.

Solution : Since $\overline{AB}$ is parallel to $\overline{CD}$

$\angle A + \angle D = 180°$ (interior angles on the same side of the transversal)

So $\angle D = 180° - 50° = 130°$

Similarly, $\angle B + \angle C = 180°$

So $\angle C = 180° - 70° = 110°$

Example 11 : The measures of two adjacent angles of a parallelogram are in the ratio 3 : 2. Find the angles of the parallelogram.

Solution : The adjacent angles of a parallelogram are supplementary.

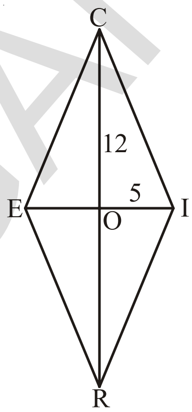
i.e. their sum = 180°

Ratio of adjacent angles = 3:2

So, each of the angles is $180 \times \frac{3}{5} = 108^{\circ}$ and

$180 \times \frac{2}{5} = 72^{\circ}$

Example 12 : RICE is a rhombus. Find OE and OR. Justify your findings.



Solution : Diagonals of a rhombus bisect each other

i.e., OE = OI and OR = OC

Therefore, OE = 5 and OR = 12

Exercise - 2

1. State whether true or false-

(i)	All rectangles are squares	( )
(ii)	All rhombuses are parallelogram	( )
(iii)	All squares are rhombuses and also rectangles	( )
(iv)	All squares are not parallelograms	( )
(v)	All kites are rhombuses	( )
(vi)	All rhombuses are kites	( )
(vii)	All parallelograms are trapeziums	( )
(viii)	All squares are trapeziums	( )

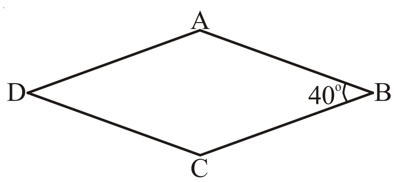
2. Explain how a square is a-

(i) quadrilateral (ii) parallelogram

(iii) rhombus (iv) rectangle.

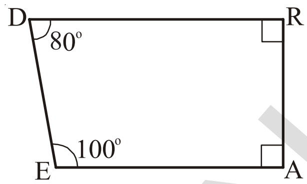
3. In a rhombus ABCD, $\angle CBA = 40°$.

Find the other angles.

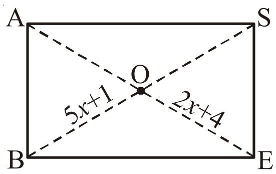


4. The adjacent angels of a parallelogram are $x°$ and $(2x + 30)°$. Find all the angles of the parallelogram.

5. Explain how DEAR is a trapezium. Which of its two sides are parallel?



6. BASE is a rectangle. Its diagonals intersect at O. Find x, if OB = $5x+1$ and OE = $2x+4$.



7. Is quadrilateral ABCD a parallelogram, if $\angle A = 70°$ and $\angle C = 65°$? Give reason.

8. Two adjacent sides of a parallelogram are in the ratio 5:3 the perimeter of the parallelogram is 48cm. Find the length of each of its sides.

9. The diagonals of the quadrilateral are perpendicular to each other. Is such a quadrilateral always a rhombus? Draw a rough figure to justify your answer.

10. ABCD is a trapezium in which $\overline{AB} \parallel \overline{DC}$. If $\angle A = \angle B = 30°$, what are the measures of the other two angles?

11. Fill in the blanks.

(i) A parallelogram in which two adjacent sides are equal is a ____________.

(ii) A parallelogram in which one angle is 90° and two adjacent sides are equal is a ____________.

(iii) In trapezium ABCD, $\overline{AB} \parallel \overline{DC}$. If $\angle D = x°$ then $\angle A =$ __________.

(iv) Every diagonal in a parallelogram divides it in to ________ triangles.

(v) In parallelogram ABCD, its diagonals $\overline{AC}$ and $\overline{BD}$ intersect at O. If AO = 5cm then AC = ________ cm.

(vi) In a rhombus ABCD, its diagonals intersect at 'O'. Then $\angle AOB =$ ________ degrees.

(vii) ABCD is a parallelogram then $\angle A - \angle C =$ ________ degrees.

(viii) In a rectangle ABCD, the diagonal AC = 10cm then the diagonal BD = _______ cm.

(ix) In a square ABCD, the diagonal $\overline{AC}$ is drawn. Then $\angle BAC =$ __________ degrees.

Looking back

1. A simple closed figure bounded by four line segments is called a quadrilateral.
2. Every quadrilateral divides a plane into three parts interior, exterior and the quadrilateral.
3. Every quadrilateral has a pair of diagonals.
4. If the diagonals lie in the interior of the quadrilateral it is called convex quadrilateral. If any one of the diagonals is not in the interior of the quadrilateral it is called a concave Quadrilateral.
5. The sum of interior angles of a quadrilateral is equal to 360°.
6. Properties of Quadrilateral

Quadrilateral	Properties
Parallelogram : A quadrilateral with both pair, of opposite sides parallel	(1) Opposite sides are equal. (2) Opposite angles are equal. (3) Diagonals bisect one another.
Rhombus : A parallelogram with all sides of equal length.	(1) All the properties of a parallelogram. (2) Diagonals are perpendicular to each other.
Rectangle : A parallelogram with all right angles.	(1) All the properties of a parallelogram. (2) Each of the angles is a right angle. (3) Diagonals are equal.
Square : A rectangle with sides of equal length.	All the properties of a parallelogram, rhombus and a rectangle
Kite : A quadrilateral with exactly two pairs of equal consecutive sides.	(1) The diagonals are perpendicular to one another. (2) The diagonals are not of equal length. (3) One of the diagonals bisects the other.
Trapezium: A quadrilateral with one pair sides parallel.	1) One pair of opposite sides are parallel

AREA AND PERIMETER

13

13.0 Introduction

Ira wants to find the area of her agricultural land, which is irregular in shape (Figure 1). So she divided her land into some regular shapes- triangles, rectangle, parallelogram, rhombus and square (Figure 2). She thought, 'if I know the area of all these parts, I will know the area of my land.'

Figure 1

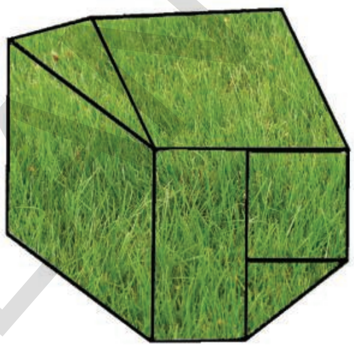
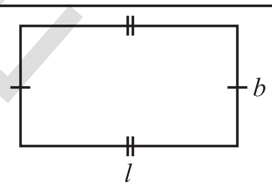
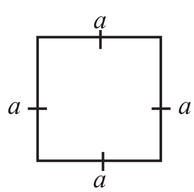


Figure 2

We have learnt how to find the area of a rectangle and square in earlier classes. In this chapter we will learn how to find the area of a parallelogram, triangle, rhombus. First let us review what we have learnt about the area and perimeter of a square and rectangle in earlier classes.

Exercise - 1

1. Complete the table given below.

Diagram	Shape	Area	Perimeter
	Rectangle	$l \times b = lb$	______
	Square	______	4a

2. The measurements of some squares are given in the table below. However, they are incomplete. Find the missing information.

Side of a square	Area	Perimeter
15 cm	225 cm^2	
		88 cm

3. The measurements of some rectangles are given in the table below. However, they are incomplete. Find the missing information.

Length	Breadth	Area	Perimeter
20 cm	14 cm		
	12 cm		60 cm
15 cm		150 cm	

13.1 Area of a parallelogram

Look at the shape in Figure 1. It is a parallelogram. Now let us learn how to find its area-

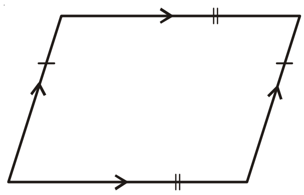


Figure 1

Activity 1

- Draw a parallelogram on a sheet of paper.
- Cut out the parallelogram.

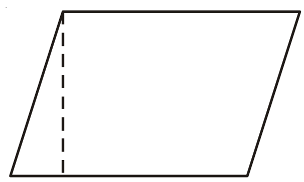


Figure 2

- Now cut the parallelogram along the dotted line as shown in Figure 2 and separate the triangular shaped piece of paper.

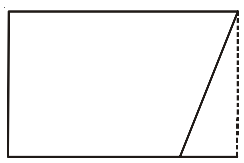


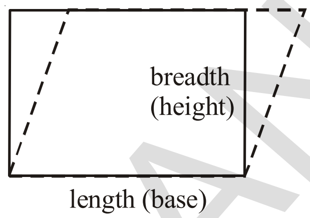
Figure 3

- Keep the triangle on the other side as shown in Figure 3 and see if both the pieces together form a rectangle.

Can we say that the area of the parallelogram in Figure 2 equal to the area of the rectangle in Figure 3? You will find this to be true.

As you can see from the above activity the area of the parallelogram is equal to the area of the rectangle.

We know that the area of the rectangle is equal to length × breadth. We also know that the length of the rectangle is equal to the base of the parallelogram and the breadth of the rectangle is equal to its height.

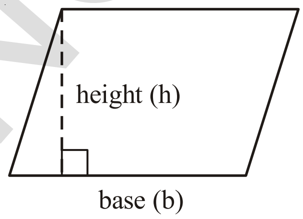


Therefore, Area of parallelogram = Area of rectangle

= length × breadth

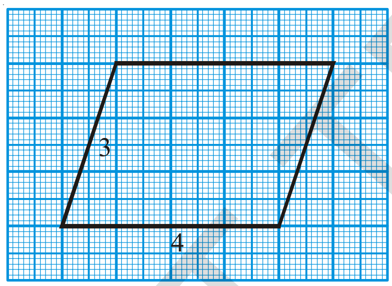
= base × height (length = base ; breadth = height)

Thus, the area of the parallelogram is equal to the product of its base (b) and corresponding height (h) i.e., A = bh



Example 1 : Find the area of each parallelogram.

(i)



Solution :

Base (b) of a parallelogram = 4 units

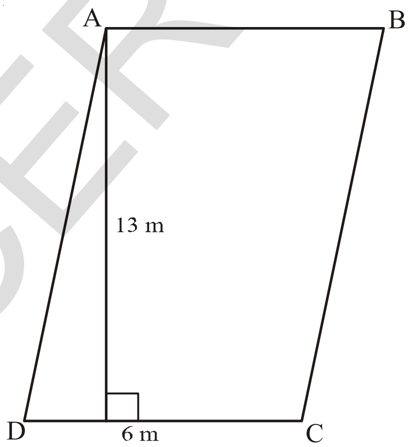
Height (h) of a parallelogram = 3 units

Area (A) of a parallelogram = bh

Therefore, A =4 × 3 = 12 sq. units

Thus, area of the parallelogram is 12 sq. units.

(ii)



Solution :

Base of a parallelogram (b) = 6 m.

Height of a parallelogram (h) = 13 m.

Area of a parallelogram (A) = bh

Therefore, A = 6×13 = 78 m^2

Thus, area of parallelogram ABCD is 78 m^2

Try This

ABCD is a parallelogram with sides 8 cm and 6 cm. In Figure 1, what is the base of the parallelogram? What is the height? What is the area of the parallelogram?

In Figure 2, what is the base of the parallelogram? What is the height? What is the area of the parallelogram? Is the area of Figure 1 and Figure 2 the same?

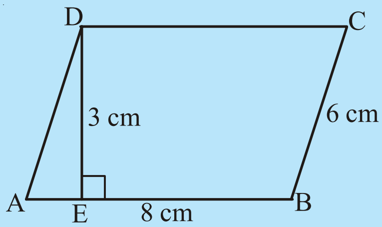


Figure 1

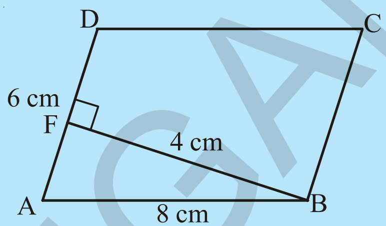
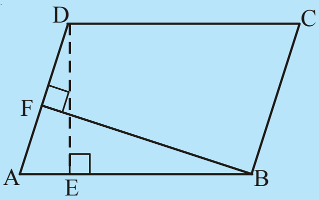


Figure 2

Any side of a parallelogram can be chosen as base of the parallelogram. In Figure 1, DE is the perpendicular falling on AB. Hence AB is the base and DE is the height of the parallelogram. In Figure 2, BF is the perpendicular falling on side AD. Hence, AD is the base and BF is the height.

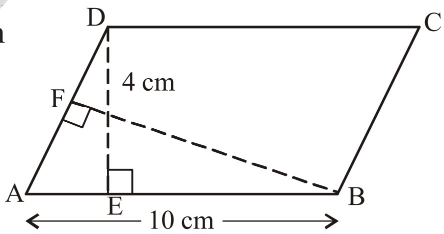


Do This

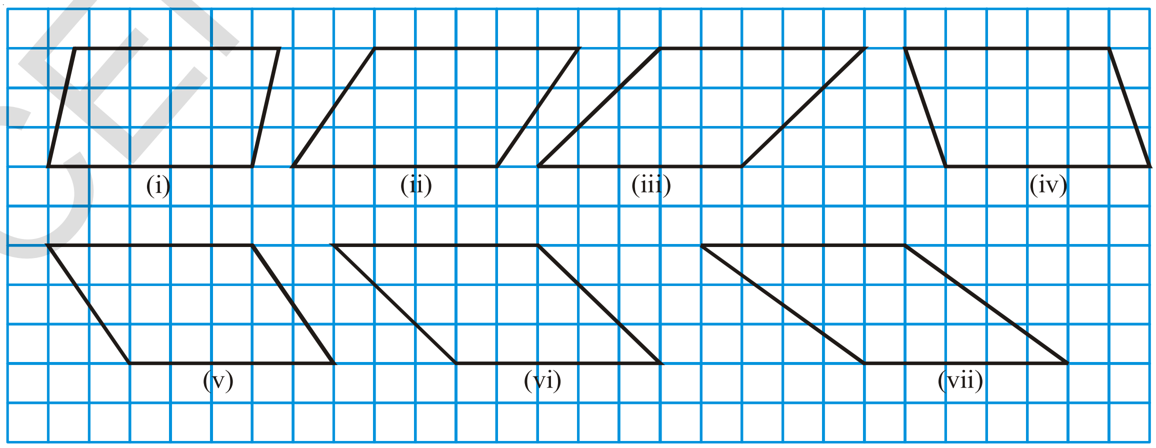
1. In parallelogram ABCD, AB = 10 cm and DE = 4 cm

Find (i) The area of ABCD.

(ii) The length of BF, if AD = 6 cm



2. Carefully study the following parallelograms.



(i) Find the area of each parallelogram by counting the squares enclosed in it. For counting incomplete squares check whether two incomplete squares make a complete square in each parallelogram.

Complete the following table accordingly-

Parallelogram	Base	Height	Area	Area by counting squares		
				No. of full squares	No. of incomplete squares	Total squares (full)
(i)	5 units	3 units	$5 \times 3 = 15$ sq. units	12	6	15
(ii)						
(iii)						
(iv)						
(v)						
(vi)						
(vii)						

(ii) Do parallelograms with equal bases and equal heights have the same area?

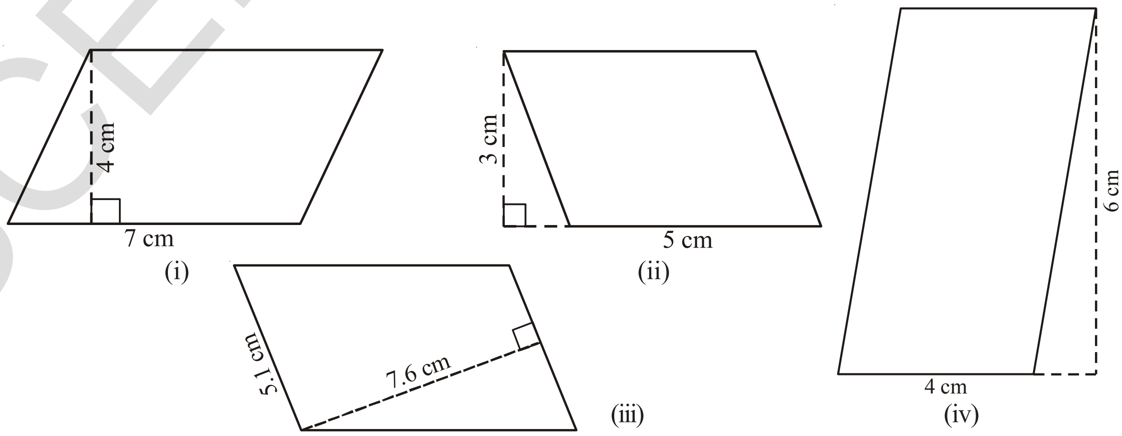
Try This

(i) Why is the formula for finding the area of a rectangle related to the formula for finding the area of a parallelogram?

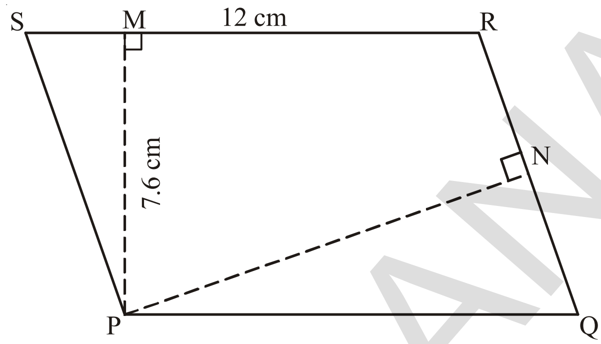
(ii) Explain why a rectangle is a parallelogram but a parallelogram may not be a rectangle.

Exercise - 2

1. Find the area of the each of the following parallelograms.



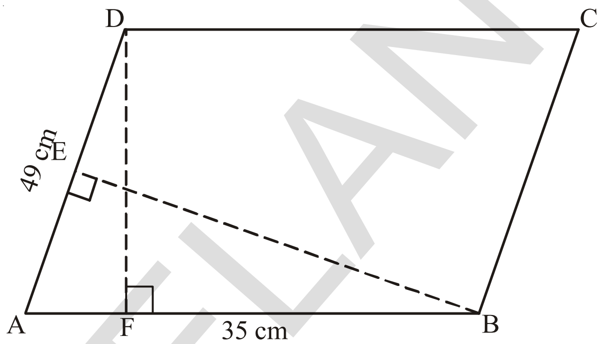
2. PQRS is a parallelogram. PM is the height from P to $\overline{SR}$ and PN is the height from P to $\overline{QR}$. If SR = 12 cm and PM = 7.6 cm.



   (i) Find the area of the parallelogram PQRS

   (ii) Find PN, if QR = 8 cm.

3. DF and BE are the height on sides AB and AD respectively in parallelogram ABCD. If the area of the parallelogram is 1470 cm^2, AB = 35 cm and AD = 49 cm, find the length of BE and DF.



4. The height of a parallelogram is one third of its base. If the area of the parallelogram is 192cm^2, find its height and base.

5. In a parallelogram the base and height are in the ratio of 5:2. If the area of the parallelogram is 360m^2, find its base and height.

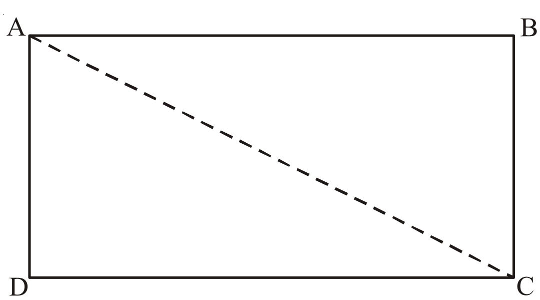
6. A square and a parallelogram have the same area. If a side of the square is 40m and the height of the parallelogram is 20m, find the base of the parallelogram.

13.2 Area of triangle

13.2.1 Triangles are parts of rectangles

Draw a rectangle Cut the rectangle along its diagonal to get two triangles.

Superimpose one triangle over the other. Are they exactly the same in area? Can we say that the triangles are congruent?



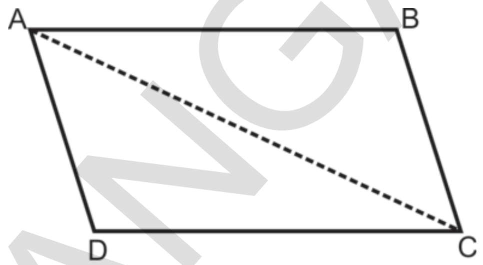
You will find that both the triangles are congruent. Thus, the area of the rectangle is equal to the sum of the area of the two triangles.

$$\text{Therefore, the area of each triangle} = \frac{1}{2} \times (\text{area of rectangle})$$

$$= \frac{1}{2} \times (l \times b) = \frac{1}{2}\, lb$$

13.2.2 Triangles are parts of parallelograms

Make a parallelogram as shown in the Figure. Cut the parallelogram along its diagonal. You will get two triangles. Place the triangles one on top of each other. Are they exactly the same size (area)?



You will find that the area of the parallelogram is equal to the area of both the triangles.

We know that area of parallelogram is equal to product of its base and height. Therefore,

$$\text{Area of each triangle} = \frac{1}{2} \times (\text{area of parallelogram})$$

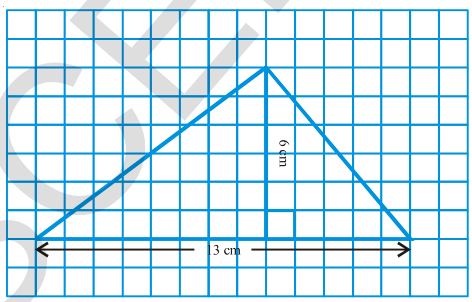
$$\text{Area of triangle} = \frac{1}{2} \times (\text{base} \times \text{height})$$

$$= \frac{1}{2} \times \text{b} \times \text{h} = \frac{1}{2}\,\text{bh}$$

Thus, the area of a triangle is equal to half the product of its base (b) and height (h) i.e.,

$$\mathbf{A = \frac{1}{2}\, bh}$$

Example 2 : Find the area of the triangle.



Solution :

Base of triangle (b) = 13 cm

Height of triangle (h) = 6 cm

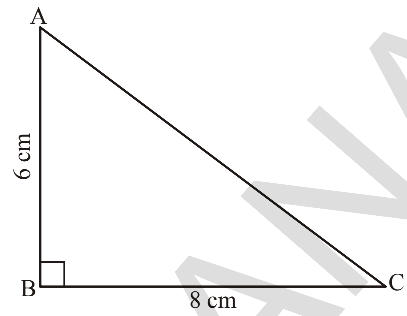
$$\text{Area of a triangle (A)} = \frac{1}{2}\,(\text{base} \times \text{height}) \text{ or } \frac{1}{2}\, bh$$

$$\text{Therefore,} \quad A = \frac{1}{2} \times 13 \times 6$$

$$= 13 \times 3 = 39 \text{ cm}^2$$

Thus the area of the triangle is 39 cm^2

Example 3 : Find the area of ΔABC.



Solution: Base of the triangle (b) $= 8$ cm

Height of the triangle (h) $= 6$ cm

Area of the triangle (A) $= \frac{1}{2}$ bh

Therefore, $A = \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$

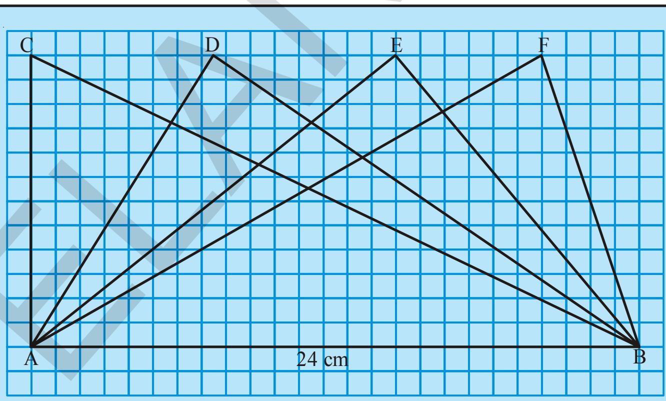
Thus, the area of $\Delta ABC = 24 \text{ cm}^2$

Notice that in a right angle triangle two of its sides can be the height.

Try This

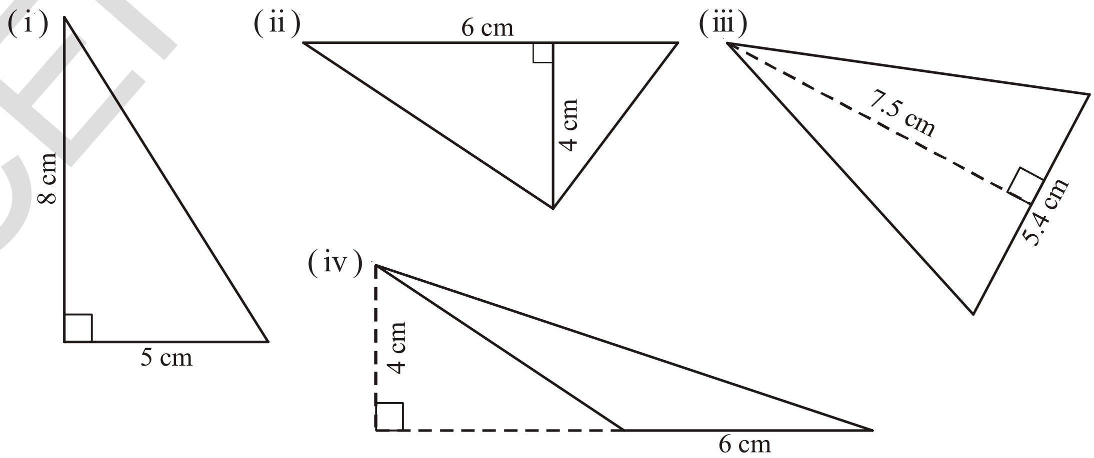
In Figure all the triangles are on the base AB = 24 cm. Is the height of each of the triangles drawn on base AB, the same?

Will all the triangles have equal area? Give reasons to support your answer. Are the triangles congruent also?

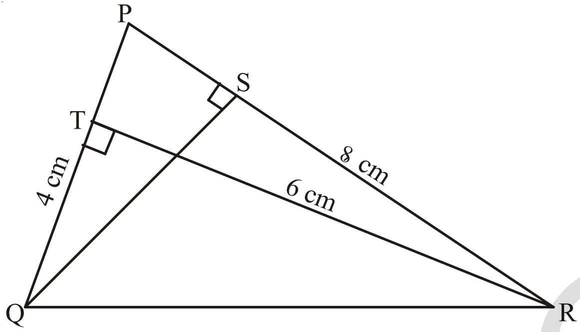


Exercise - 3

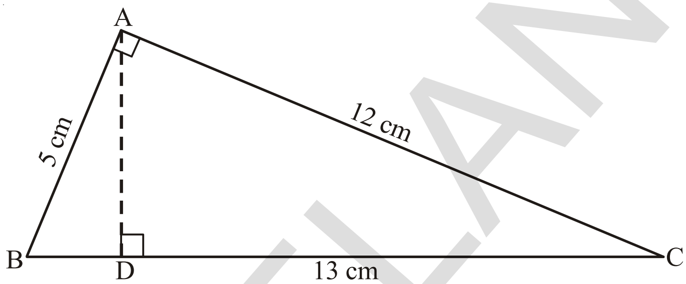
1. Find the area of each of the following triangles.



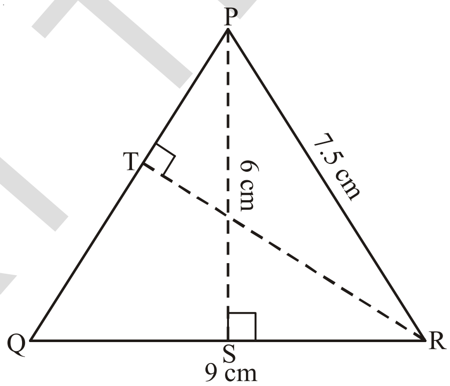
2. In ΔPQR, PQ = 4 cm, PR = 8 cm and RT = 6 cm. Find (i) the area of ΔPQR (ii) the length of QS.



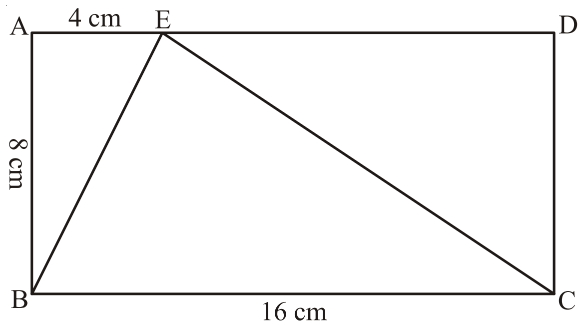
3. ΔABC is right-angled at A. AD is perpendicular to BC, AB = 5 cm, BC = 13 cm and AC = 12 cm. Find the area of ΔABC. Also, find the length of AD.



4. ΔPQR is isosceles with PQ = PR = 7.5 cm and QR = 9 cm. The height PS from P to QR, is 6 cm. Find the area of ΔPQR. What will be the height from R to PQ i.e. RT?

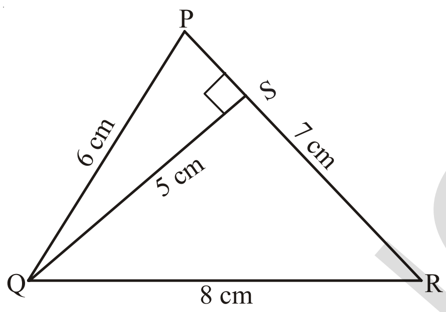


5. ABCD rectangle with AB = 8 cm, BC = 16 cm and AE=4 cm. Find the area of ΔBCE. Is the area of ΔBEC equal to the sum of the area of ΔBAE and ΔCDE. Why?



6. Ramu says that the area of ΔPQR is, $A = \frac{1}{2} \times 7 \times 5\ cm^2$

Gopi says that it is, $A = \frac{1}{2} \times 8 \times 5\ cm^2$. Who is correct? Why?

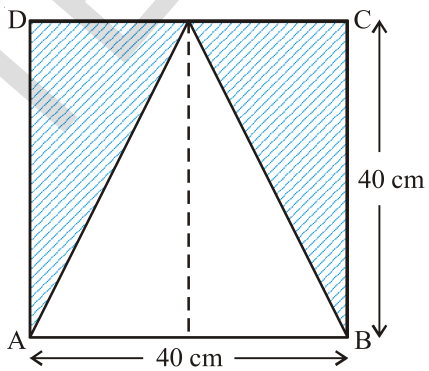


7. Find the base of a triangle whose area is 220 cm^2 and height is 11cm.

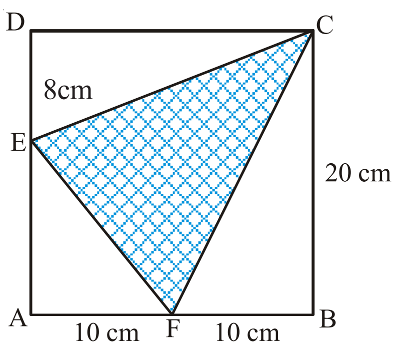
8. In a triangle the height is double the base and the area is 400 cm^2. Find the length of the base and height.

9. The area of triangle is equal to the area of a rectangle whose length and breadth are 20 cm and 15 cm respectively. Calculate the height of the triangle if its base measures 30 cm.

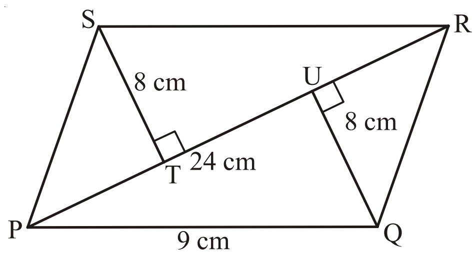
10. In Figure ABCD find the area of the shaded region.



11. In Figure ABCD, find the area of the shaded region.



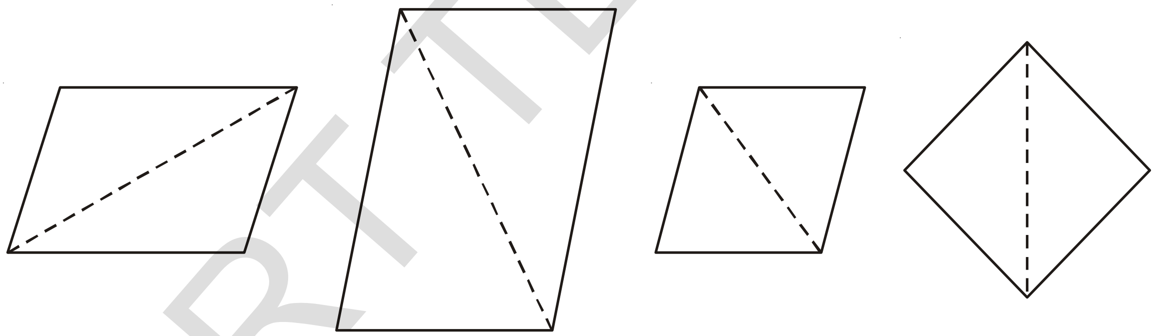
12. Find the area of a parallelogram PQRS, if PR = 24 cm and QU = ST = 8 cm.



13. The base and height of the triangle are in the ratio 3:2 and its area is 108 cm^2. Find its base and height.

13.3 Area of a rhombus

Santosh and Akhila are good friends. They are fond of playing with paper cut-outs. One day, Santosh gave different triangle shapes to Akhila. From these she made different shapes of parallelograms. These parellelograms are given below-



Santosh asked Akhila, 'which parallelograms has 4 equal sides?"

Akhila said, 'the last two have equal sides."

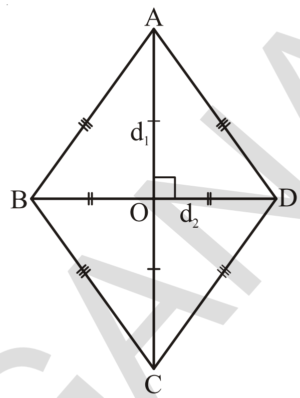
Santhosh said, **'If all the sides of a parallelogram are equal, it is called a Rhombus.'**

Now let us learn how to calculate the area of a Rhombus.

Like in the case of a parallelogram and triangle, we can use the method of splitting into congruent triangles to find the area of a rhombus.

ABCD is a rhombus.

Area of rhombus ABCD $= (\text{area of } \Delta ACD) + (\text{area of } \Delta ACB)$



$$= \left(\frac{1}{2} \times AC \times OD\right) + \left(\frac{1}{2} \times AC \times OB\right)$$

diagonals bisect perpendicularly

$$= \frac{1}{2} AC \times (OD + OB)$$

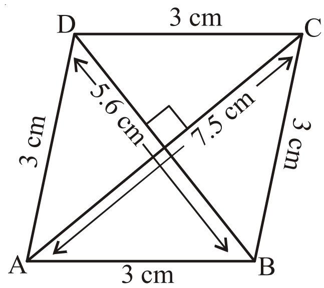
$$= \frac{1}{2} AC \times BD$$

$$= \frac{1}{2} d_1 \times d_2 \ \ (\text{as } AC = d_1 \text{ and } BD = d_2)$$

In other words, the area of a rhombus is equal to half the product of its diagonals i.e.,

$$\mathbf{A} = \frac{1}{2}\, d_1 d_2$$

Example 4 : Find the area of rhombus ABCD



Solution : Length of the diagonal (d_1) $= 7.5$ cm

Length of the other diagonal (d_2) $= 5.6$ cm

Area of the rhombus (A) $= \frac{1}{2}\, d_1\, d_2$

Therefore, A $= \frac{1}{2} \times 7.5 \times 5.6 = 21 \text{ cm}^2$

Thus, area of rhombus ABCD = 21 cm^2

Example 5 : The area of a rhombus is 60 cm^2 and one of its diagonals is 8 cm. Find the other diagonal.

Solution : Length of one diagonal (d_1) $= 8$ cm

Length of the other diagonal $= d_2$

Area of rhombus $= \frac{1}{2} \times d_1 \times d_2$

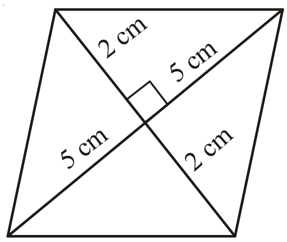
Therefore, $60 = \frac{1}{2} \times 8 \times d_2$

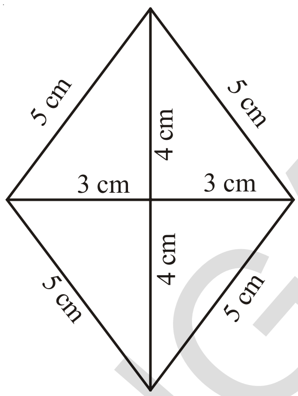
$d_2 = 15$ cm.

Thus, length of the other diagonal is 15 cm.

Exercise 4

1. Find the area of the following rhombuses.





2. Find the missing values.

Diagonal-1 (d_1)	Diagonal-2 (d_2)	Area of rhombus
12 cm	16 cm	
27 mm		2025 mm^2
24 m	57.6 m	

3. If length of diagonal of a rhombus whose area 216 sq. cm. is 24 cm. Then find the length of second diagonal.

4. The floor of a building consists of 3000 tiles which are rhombus shaped. The diagonals of each of the tiles are 45 cm and 30 cm. Find the total cost of polishing the floor, if cost per m^2 is ₹ 2.50.

13.4 Circumference of a circle

Nazia is playing with a cycle tyre. She is rotating the tyre with a stick and running along with it.

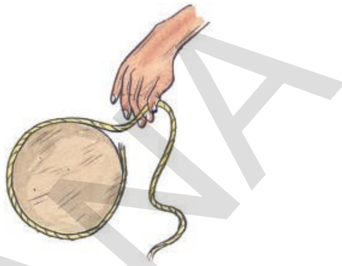
What is the distance covered by tyre in one rotation?

The distance covered by the tyre in one rotation is equal to the length around the wheel. The length around the tyre is also called the circumference of the tyre .

What is the relation between the total distance covered by the tyre and number of rotations?

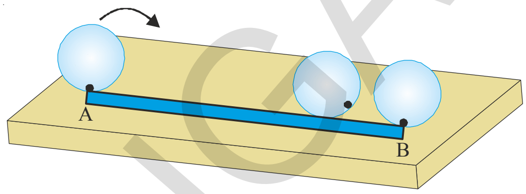
Total distance covered by the tyre = number of rotations × length around the tyre.

Activity 2



Jaya cut a circular shape from a cardboard. She wants to stick lace around the card to decorate it. Thus, the length of the lace required by her is equal to the circumference of the card. Can she measure the circumference of the card with the help of a ruler?

Let us see what Jaya did?



Jaya drew a line on the table and marked its starting point A. She then made a point on the edge of the card. She placed the circular card on the line, such that the point on the card coincided with point A. She then rolled the card along the line, till the point on the card touched the line again. She marked this point B. The length of line AB is the circumference of the circular card. The length of the lace required around the circular card is the distance AB.

Try This

Take a bottle cap, a bangle or any other circular object and find its circumference using a string.

It is not easy to find the circumference of every circular shape using the above method. So we need another way for doing this. Let us see if there is any relationship between the diameter and the circumference of circles.

A man made six circles of different radii with cardboard and found their circumference using a string. He also found the ratio between the circumference and diameter of each circle.

He recorded his observations in the following table-

Circle	Radius	Diameter	Circumference	Ratio of circumference and diameter
1.	3.5 cm	7.0 cm	22.0 cm	$\frac{22}{7} = 3.14$
2.	7.0 cm	14.0 cm	44.0 cm	$\frac{44}{14} = 3.14$
3.	10.5 cm	21.0 cm	66.0 cm	
4.	21.0 cm	42.0 cm	132.0 cm	
5.	5.0 cm	10.0 cm	32.0 cm	
6.	15.0cm	30.0 cm	94.0 cm	

What can you infer from the above table? Is the ratio between the circumference and the diameter of each circle approximately the same? Can we say that the circumference of a circle is always about three times its diameter?

The approximate value of the ratio of the circumference to the diameter of a circle is $\frac{22}{7}$ or 3.14.

Thus it is a constant and is denoted by π (pi).

Therefore, $\frac{c}{d} = \pi$ where 'c' is the circumference of the circle and 'd' its diameter.

Since, $\frac{c}{d} = \pi$

$c = \pi d$

Since, diameter of a circle is twice the radius i.e. $d = 2r$

$c = \pi \times 2r$ or $c = 2\pi r$

Thus, circumference of a circle = πd or 2πr

Example 6 : Find the circumference of a circle with diameter 10 cm. (Take $\pi = 3.14$)

Solution : Diameter of the circle (d) = 10 cm.

Circumference of circle (c) $= \pi d$

$= 3.14 \times 10$

$c = 31.4$ cm

Thus, the circumference of the circle is 31.4 cm.

Example 7 : Find the circumference of a circle with radius 14 cm. (Take $\pi = \frac{22}{7}$)

Radius of the circle (r) = 14 cm

Circumference of a circle (c) $= 2\pi r$

Therefore, $c = 2 \times \frac{22}{7} \times 14$

$c = 88$ cm

Thus, the circumference of the circle is 88 cm.

Exercise - 5

1. Find the circumference of a circle whose radius is-

   (i) 35 cm (ii) 4.2 cm (iii) 15.4 cm

2. Find the circumference of circle whose diameter is-

   (i) 17.5 cm (ii) 5.6 cm (iii) 4.9 cm

Note : take $\pi = \frac{22}{7}$ in the above two questions.

3. (i) Taking $\pi = 3.14$, find the circumference of a circle whose radius is

   (a) 8 cm (b) 15 cm (c) 20 cm

   (ii) Calculate the radius of a circle whose circumference is 44cm?

4. If the circumference of a circle is 264 cm, find its radius. Take $\pi = \frac{22}{7}$.

5. If the circumference of a circle is 33 cm, find its diameter.

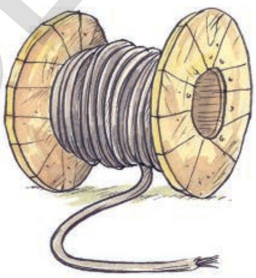
6. How many times will a wheel of radius 35cm be rotated to travel 660 cm?

(Take $\pi = \frac{22}{7}$).

7. The ratio of the diameters of two circles is 3 : 4. Find the ratio of their circumferences.

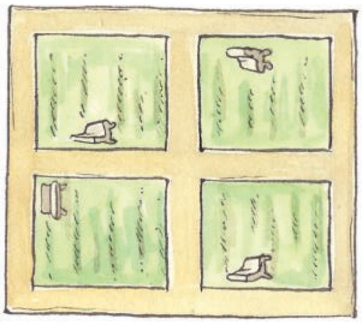
8. A road roller makes 200 rotations in covering 2200 m. Find the radius of the roller.

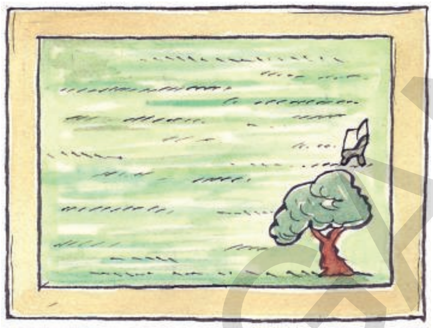
9. The minute hand of a circular clock is 15 cm. How far does the tip of the minute hand move in 1 hour? (Take $\pi = 3.14$)



10. A wire is bent in the form of a circle with radius 25 cm. It is straightened and made into a square. What is the length of the side of the square?

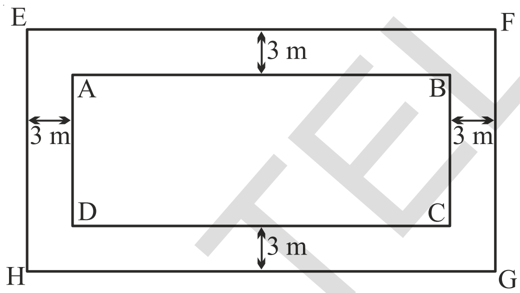
13.5 Rectangular Paths





We often come across such walking paths in garden areas. Now we shall learn how to measure the areas of such paths as this often useful in calculating their costs of construction.

Example 8 : A plot is 60m long and 40m wide. A path 3m wide is to be constructed around the plot. Find the area of the path.



Solution : Let ABCD be the given plot. A 3m wide path is running all around it. To find the area of this path we have to subtract the area of the smaller rectangle ABCD from the area of the bigger rectangle EFGH.

Length of inner rectangle	=	60m
Breadth of inner rectangle	=	40m
Area of the plot ABCD	=	(60 x 40) m^2
	=	2400 m^2
Width of the path	=	3m
Length of outer rectangle	=	60 m + (3+3) m
	=	66 m
Breadth of outer rectangle	=	40 m + (3+3) m
	=	46 m

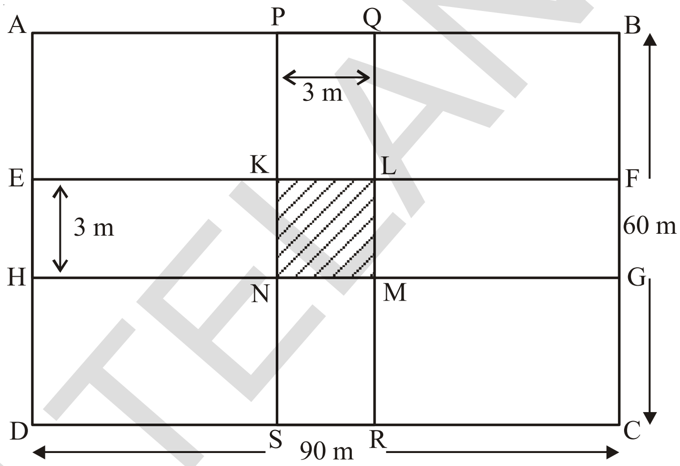
Area of the outer rectangle	=	66 x 46 m^2
	=	3036 m^2
Therefore, area of the path	=	(3036 - 2400) m^2
	=	636 m^2

Example 9 : The dimensions of a rectangular field are 90 m and 60 m. Two roads are constructed such that they cut each other at the centre of the field and are parallel to its sides. If the width of each road is 3 m, find-

(i) The area covered by the roads.

(ii) The cost of constructing the roads at the rate of ₹110 per m^2.

Solution : Let ABCD be the rectangular field. PQRS and EFGH are the 3m roads.



(i) Area of the crossroads is the area of the rectangle PQRS and the area of the rectangle EFGH. As is clear from the picture, the area of the square KLMN will be taken twice in this calculation thus needs to be subtracted once.

From the question we know that,

PQ = 3 m, and	PS = 60 m
EH = 3 m, and	EF = 90 m
KL = 3 m	KN = 3 m

Area of the roads = Area of the rectangle PQRS + area of the rectangle EFGH – Area of the square KLMN

$= (PS \times PQ) + (EF \times EH) - (KL \times KN)$

$= (60 \times 3) + (90 \times 3) - (3 \times 3)$

$= (180 + 270 - 9)\ m^2$

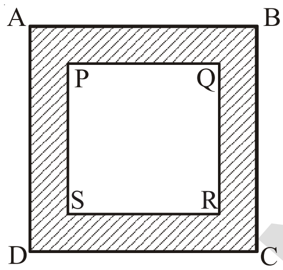
$= 441\ m^2$

(ii) Cost of construction = ₹110 × m^2

Cost of constructing the roads = 110 × 441

= ₹48,510

Example 10 : A path of 5m wide runs around a square park of side 100m. Find the area of the path. Also find the cost of cementing it at the rate of ₹250 per $10m^2$



Solution : Let PQRS be the square park of the side 100 m. The shaded region represents the 5m wide path.

Length of AB = 100 + (5 + 5) = 110 m

Area of the square PQRS = $(\text{side})^2 = (100\text{ m})^2 = 10000\text{ m}^2$

Area of the square ABCD = $(\text{side})^2 = (110\text{ m})^2 = 12100\text{ m}^2$

Therefore, area of the path = $(12100 - 10000) = 2100\text{ m}^2$

Cost of the cementing per 10 m^2 = ₹250

Therefore, cost of the cementing 1 m^2 = $\frac{250}{10}$

Thus, cost of cementing 2100 m^2 = $\frac{250}{10} \times 2100$

= ₹52,500

Exercise - 6

1. A path 2.5 m wide is running around a square field whose side is 45 m. Determine the area of the path.

2. The central hall of a school is 18m long and 12.5 m wide. A carpet is to be laid on the floor leaving a strip 50 cm wide near the walls, uncovered. Find the area of the carpet and also the uncovered portion?

3. The length of the side of a grassy square plot is 80 m. Two walking paths each 4 m wide are constructed parallel to the sides of the plot such that they cut each other at the centre of the plot. Determine the area of the paths.

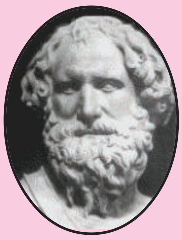
4. A verandah 2 m wide is constructed all around a room of dimensions 8 m × 5 m. Find the area of the verandah

5. The length of a rectangular park is 700 m and its breadth is 300 m. Two crossroads, each of width 10 m, cut the centre of a rectangular park and are parallel to its sides. Find the area of the roads. Also, find the area of the park excluding the area of the crossroads.

Looking Back

- The area of the parallelogram is equal to the product of its base (b) and corresponding height (h) i.e., A = bh. Any side of the parallelogram can be taken as the base.
- The area of a triangle is equal to half the product of its base (b) and height (h) i.e., $A = \frac{1}{2}bh$.
- The area of a rhombus is equal to half the product of its diagonals i.e., $\mathbf{A} = \frac{1}{2}d_1d_2$.
- The circumference of a circle = $2\pi r$ where r is the radius of the circle and $\pi = \frac{22}{7}$ or 3.14.

Archimedes (Greece)
287 - 212 BC
He calculated the value of π first time.
He also evolved the mathematical formulae for finding out the circumference and area of a circle.



UNDERSTANDING 3D AND 2D SHAPES

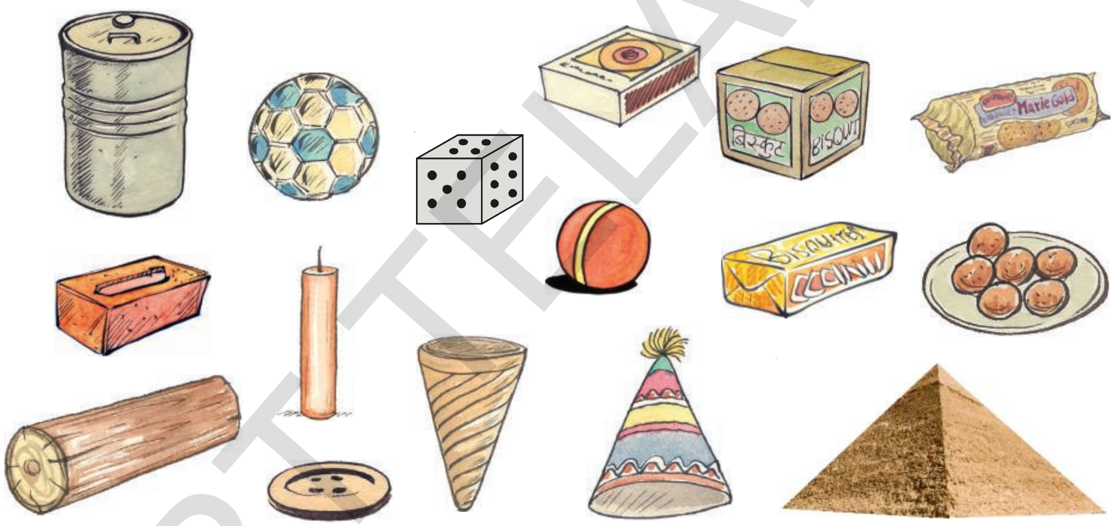
14

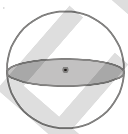
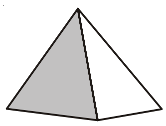
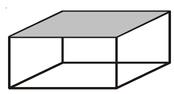
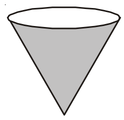
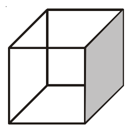
14.0 Introduction

We have been introduced to various three-dimensional shapes in class VI. We have also identified their faces, edges and vertices. Let us first review what we have learnt in class VI.

Exerciese - 1

1. Given below are the pictures of some objects. Categorise and fill write their names according to their shape and fill the table with name of it.



 Sphere	 Cylinder	 Pyramid	 Cuboid	 Cone	 Cube

2. Write names of at least 2 objects from day-to-day life, which are in the shape of the basic 3D shapes given below:

(i) Cone ----------- ----------- ----------- -----------

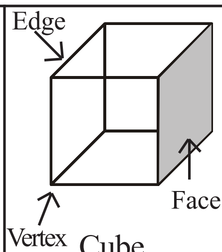
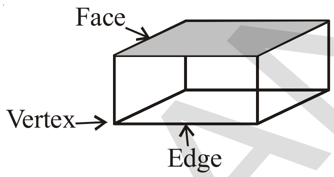
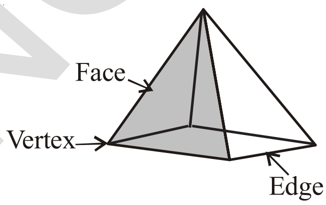
(ii) Cube ----------- ----------- ----------- -----------

(iii) Cuboid ----------- ----------- ----------- -----------

(iv) Sphere ----------- ----------- ----------- -----------

(v) Cylinder ----------- ----------- ----------- -----------

3. Identify and state the number of faces, edges and vertices of the figures given below:

	 Cube	 Cuboid	 Pyramid
Faces			
Edges			
Vertices			

14.1 Nets of 3-D shapes

We now visualise 3-D shapes on 2-D surfaces, that is on a plain paper. This can be done by drawing the 'nets' of the various 3-D figures.

Take a cardboard box (cartoon of tooth paste or shoes etc.,). Cut the edges to lay the box flat. You have now a net for that box. A net is a sort of skeleton-outline in 2-D (Figure 1), which, when folded (Figure 2), results in a 3-D shape (Figure 3).

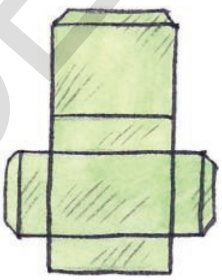


Figure 1

Figure 2

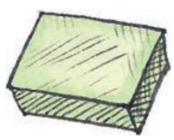
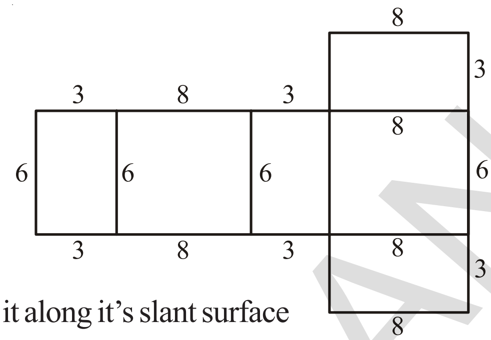
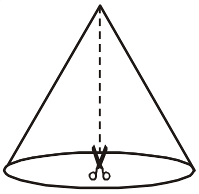


Figure 3

Here is a net pattern for a box. Trace it and paste it on a thick paper and try to make the box by suitably folding and gluing together. What is the shape of the box?





Figure

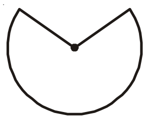


Figure 2

Similarly, take a cover of an ice-cream cone (or any like shape). Cut it along it's slant surface as shown in Figure 1. You will get the net for the cone (Figure 2).

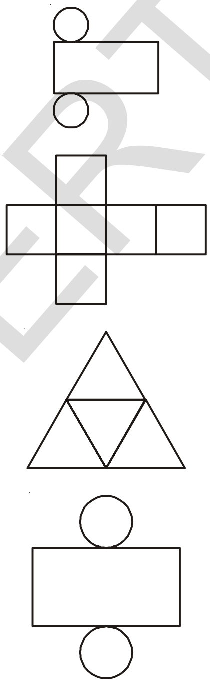
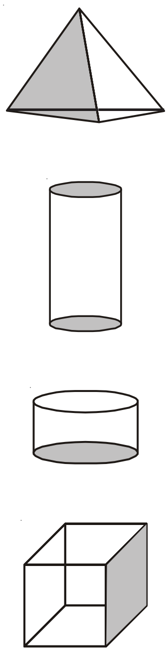
Try This

Take objects having different shapes (cylinder, cube, cuboid and cone) and cut them to get their nets with help of your teachers or friends.

You will come to know by the above activity that you have different nets for different shapes. Also, each shape can also have more than one net according to the way we cut it.

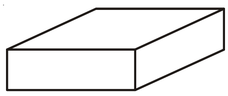
Exercise - 2

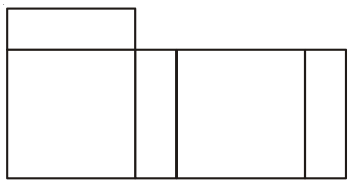
1. Some nets are given below. Trace them and paste them on a thick paper. Try to make 3-D shapes by suitably folding them and gluing together. Match the net with it's 3-D shape.

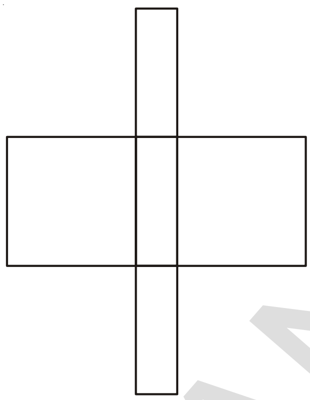
Nets	3D shapes
	

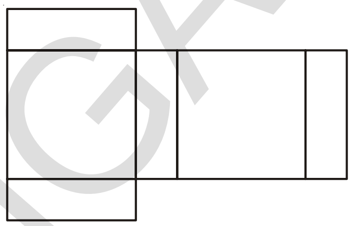
2. Three nets for each shape are given here. Match the net with its 3D-shape.

(i)



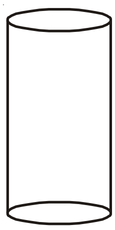


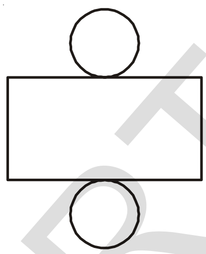


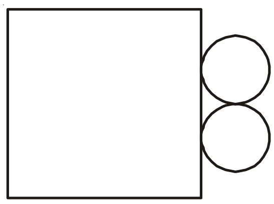


(a) (b) (c)

(ii)

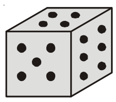




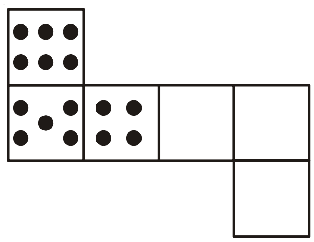


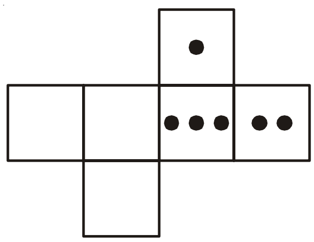
(a) (b) (c)

3. A dice is a cube with dots on each face. The opposite faces of a dice always have a total of seven dots on them.



Here are two nets to make dice. Insert the suitable number of dots in blanks.





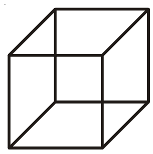
Play This

You and your friend sit back to back. One of you read out a net to make a 3-D shape, while the other copies it and sketches or builds the described object.

14.2 Drawing solids on a flat surface

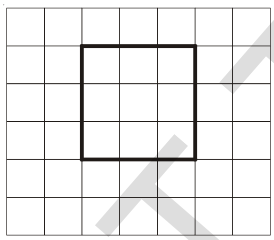
Our drawing surface is a paper, which is a flat surface. When you draw a solid shape, the images are somewhat distorted. It is a visual illusion. You will find here two techniques to help you to draw the 3-D shapes on a plane surface.

14.2.1 Oblique Sketches

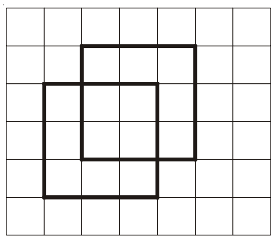


Here is a picture of a cube. It gives a clear idea of how the cube looks, when seen from the front. You do not see all the faces as we see in reality. In the picture, all the lengths are not equal, as they are in a real cube. Still, you are able to recognise it as a cube. Such a sketch of a solid is called an oblique sketch.

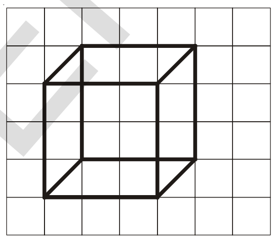
How can you draw such sketches? Let us attempt to learn the technique. You need a squared (lines or dots) paper. Initially practice to draw on these sheets and later on a plain sheet (without the aid of squared lines or dots!) Let us attempt to draw an oblique sketch of a $3 \times 3 \times 3$ cube (each edge is 3 units).



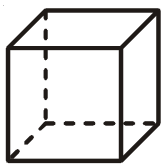
Step 1
Draw the front face.



Step 2
Draw the opposite face. Sizes of the faces have to be same, but the sketch is somewhat off-set from step 1.



Step 3
Join the corresponding corners



Step 4
Redraw using doted lines for hidden edges. (It is a convention) The sketch is ready now.

In the above oblique sketch did you notice the following?

(i) The sizes of the front face and its opposite face are same.

(ii) The edges, which are all equal in a cube, appear so in the sketch, though the actual measures of edges are not taken so.

You could now try to make an oblique sketch of a cuboid (remember the faces in this case are rectangles).

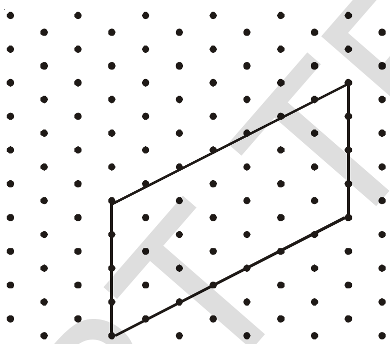
You can draw sketches in which measurements also agree with those of a given solid. To do this we need what is known as an isometric sheet.

Let us try to make a cuboid with dimensions 7 cm length, 3 cm breadth and 4 cm height on an isometric sheet.

14.2.2 Isometric Sketches

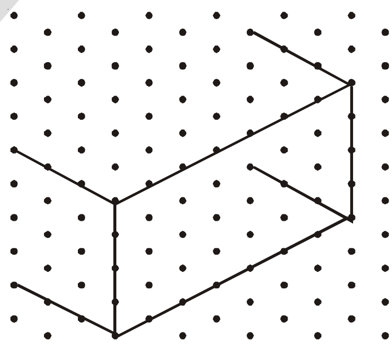
To draw sketches in which measurements also agree with those of the given solid, we can use isometric dot sheets. In such a sheet the paper is divided into small equilateral triangles made up of dots or lines.

Let us attempt to draw an isometric sketch of a cuboid of dimensions $7 \times 3 \times 4$ (which means the edges forming length, breadth and height are 7, 3, 4 units respectively).



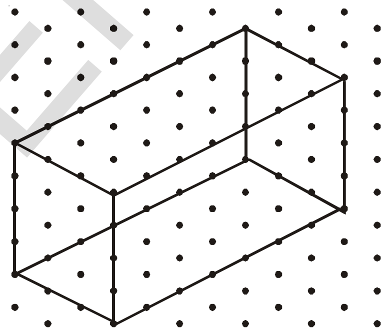
Step 1

Draw a rectangle to show the front face



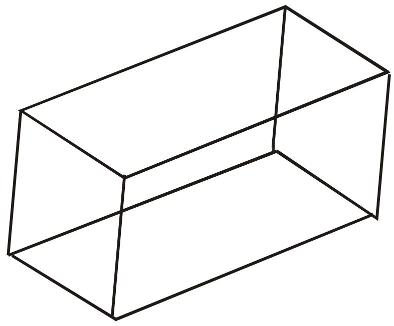
Step 2

Draw four parallel line segments of length 3 units starting from the four corners of the rectangle.



Step 3

Connect the matching corners with appropriate line segments.

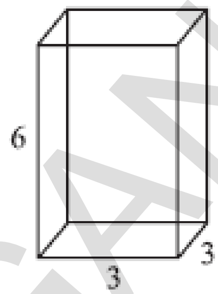


Step 4

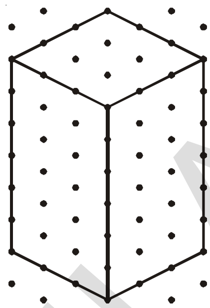
This is an isometric sketch of a cuboid.

Note that the measurements of the solid are of exact size in an isometric sketch; this is not so in the case of an oblique sketch.

Example 1 : Here is an oblique sketch of a cuboid. Draw an isometric sketch that matches this drawing.



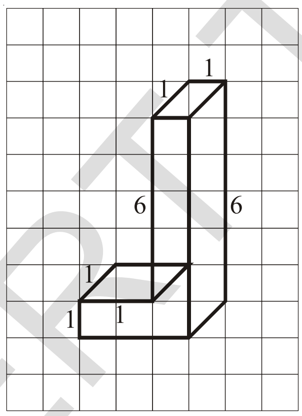
Solution : The length, breadth and height are 3, 3 and 6 units respectively



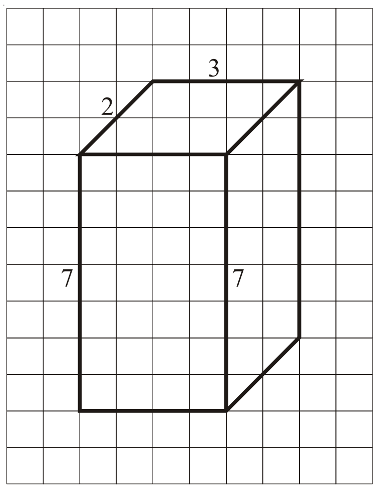
Exercise - 3

1. Use an isometric dot paper and make an isometric sketch for each one of the given shapes.

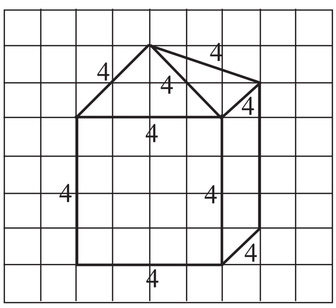
(i)



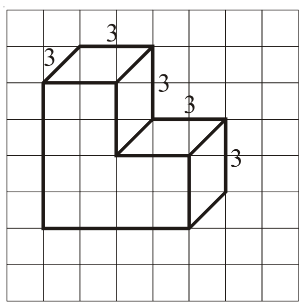
(ii)



(iii)



(iv)

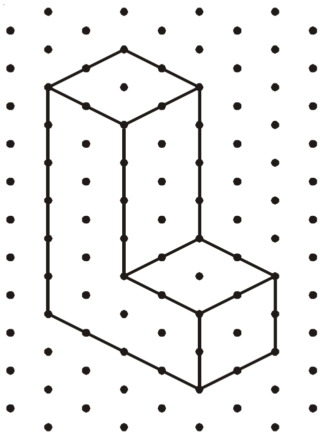


2. The dimensions of a cuboid are 5 cm, 3 cm and 2 cm. Draw three different isometric sketches of this cuboid.

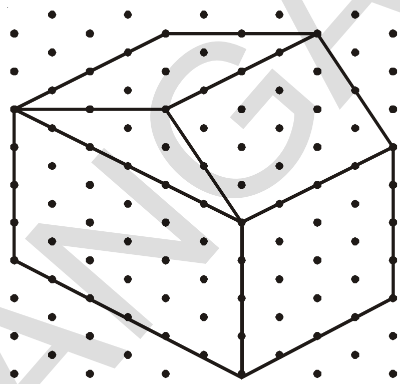
3. Three cubes each with 2 cm edge are placed side by side to form a cuboid. Draw an oblique or isometric sketch of this cuboid.

4. Make an oblique sketch for each of the given isometric shapes.

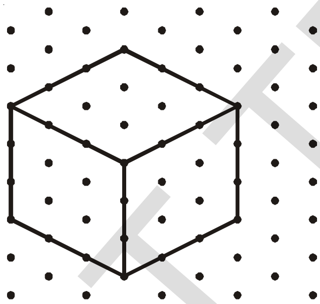
(i)



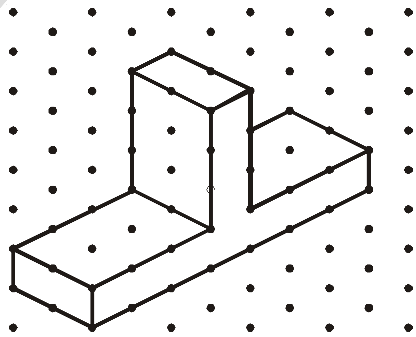
(ii)



(iii)



(iv)



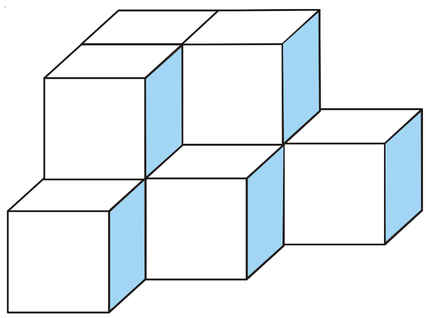
5. Give (i) an oblique sketch and (ii) an isometric sketch for each of the following:

(a) A cuboid of dimensions 5 cm, 3 cm and 2 cm. (Is your sketch unique?)

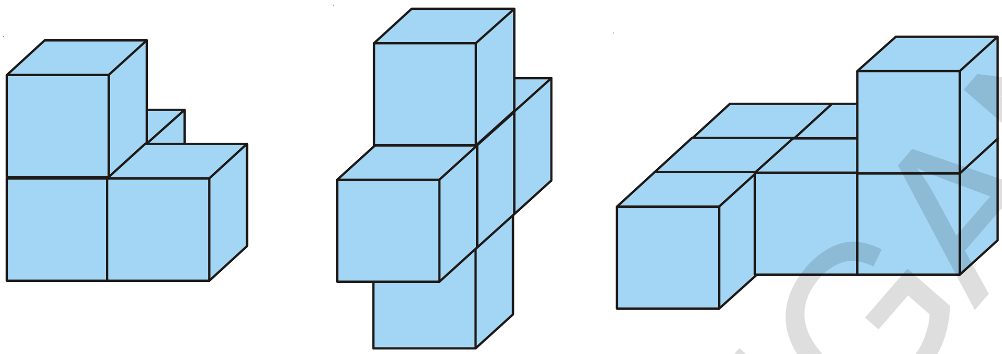
(b) A cube with an edge 4 cm long.

14.3 Visualising solid objects

Sometimes when you look at combined shapes, some of them may be hidden from your view.



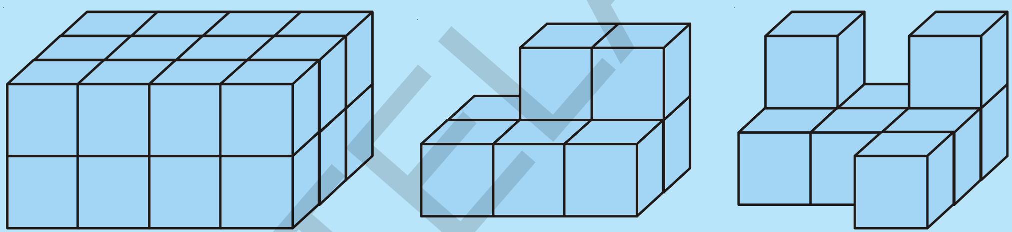
Here are some activities to help you visualise some solid objects and how they look. Take some cubes and arrange them as shown below.



Now ask your friend to guess the total number of cubes in the following arrangements.

Try This

Estimate the number of cubes in the following arrangements.



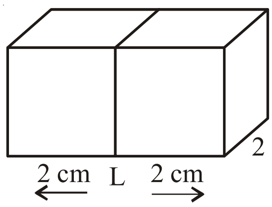
Such visualisations are very helpful.

Suppose you form a cuboid by joining cubes. You will be able to estimate what the length, breadth and height of the cuboid would be.

Example 2 : If two cubes of dimensions 2 cm by 2cm by 2cm are placed side by side, what would the dimensions of the resulting cuboid be?

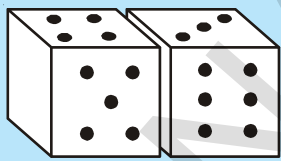
Solution : As you can see when kept side by side, the length is the only measurement which increases, it becomes 2 + 2 = 4 cm.

The breadth = 2 cm and the height = 2 cm.



Try This

1. Two dice are placed side by side as shown. Can you say what the total would be on the faces opposite to them? (i) 5 + 6 (ii) 4 + 3



(Remember that in a dice the sum of numbers on opposite faces is 7)

2. Three cubes each with 2 cm edge are placed side by side to form a cuboid. Try to make an oblique sketch and say what could be its length, breadth and height.

14.3.1 Viewing different sections of a solid

Now let us see how an object which is in 3-D can be viewed in different ways.

14.3.1a) One way to view an object is by cutting or slicing the object

Slicing game

Here is a loaf of bread. It is like a cuboid with a square faces. You 'slice' it with a knife.

When you give a 'horizontal' cut, you get several pieces, as shown in the figure. Each face of the piece is a square! We call this face a 'cross-section' of the whole bread. The cross section is nearly a square in this case.

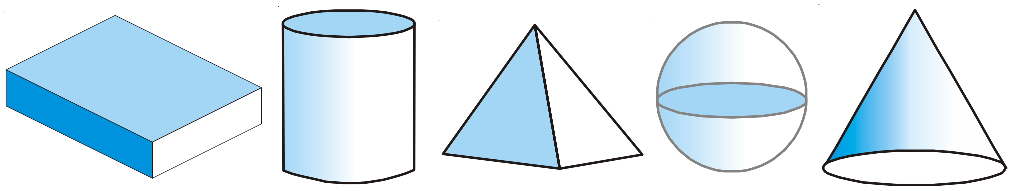
Beware! If your cut is 'vertical' you may get a different cross section! Think about it. The boundary of the cross-section you obtain is a plane curve. Do you notice it?

A kitchen play

Have you noticed cross-sections of some vegetables when they are cut for the purposes of cooking in the kitchen? Observe the various slices and get aware of the shapes that results as cross-sections.

Do This

1. Make clay (or plasticine) models of the following solids and make vertical or horizontal cuts. Draw rough sketches of the cross-sections you obtain. Name them if possible.



2. What cross-sections do you get when you give a (i) vertical cut (ii) horizontal cut to the following solids?

   (a) A brick (b) A round apple (c) A die (d) A circular pipe (e) An ice cream cone

14.3.1b) Another Way is by Shadow Play

A shadow play

Shadows are a good way to illustrate how three-dimensional objects can be viewed in two dimensions. Have you seen a shadow play? It is a form of entertainment using solid articulated figures in front of an illuminated backdrop to create the illusion of moving images. It makes some indirect use of ideas of Mathematics.

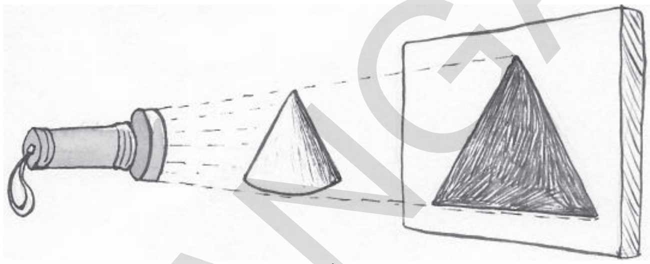


Figure 1

You will need a source of light and a few solid shapes for this activity. If you have an overhead projector, place the solid under the lamp and do these investigations.

Keep a torchlight, right in front of a cone. What type of shadow does it cast on the screen? (Figure 1).

The solid is three-dimensional; what about the shadow?

If, instead of a cone, you place a cube in the above game, what type of shadow will you get?

Experiment with different positions of the source of light and with different positions of the solid object. Study their effects on the shapes and sizes of the shadows you get.

Here is another funny experiment that you might have tried already:

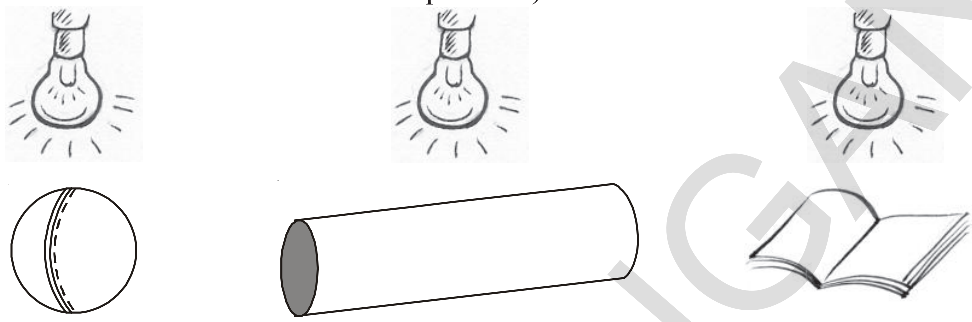
Place a circular tumbler in the open when the sun at the noon time is just right above it as shown in the figure below. What is the shadow that you obtain?

Will it be same during (a) afternoon? (b) evening?

Study the shadows in relation to the position of the sun and the time of observation.

Exerciese - 4

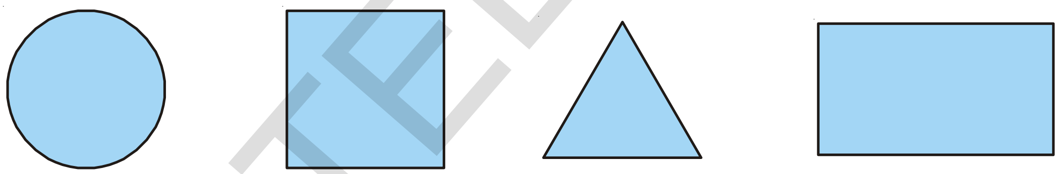
1. A bulb is kept burning just right above the following solids. Name the shape of the shadows obtained in each case. Attempt to give a rough sketch of the shadow. (You may try to experiment first and then answer these questions).



A ball　　A cylindrical pipe　　A book

2. Here are the shadows of some 3D objects, when seen under the lamp of an overhead projector. Identify the solid(s) that match each shadow. (There may be many answers for these!)

A circle　　A square　　A triangle　　A rectangle



(i)　　(ii)　　(iii)　　(iv)

Looking Back

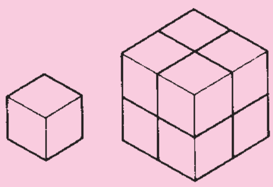
3D shapes can be visualised on 2D surfaces, that is on paper by drawing their nets.

Oblique sketches and isometric sketches help in visualising 3D shapes on a plane surface.

Fun with a cube

A unit cube can fitted together with 7 other identical cubes to make a larger cube with an edge of 2 units as shown in figure.

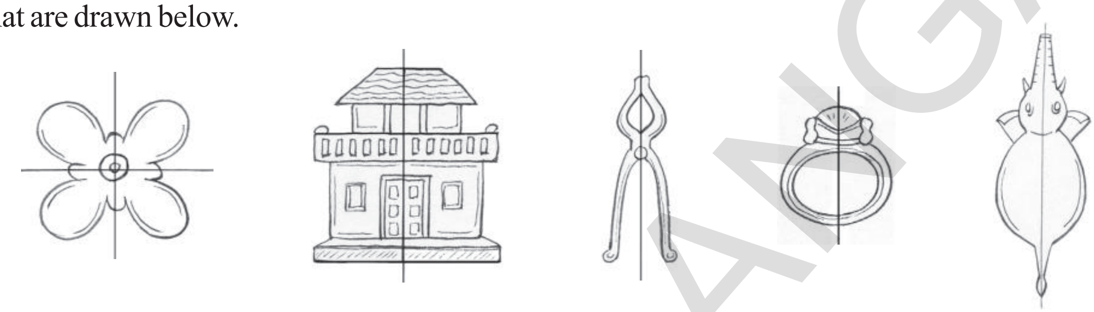
How many unit cubes are needed to make a cube with an edge of 3 units?



SYMMETRY 15

15.0 Introduction

Look around you. You will find that many objects around you are symmetrical. So are the objects that are drawn below.



All these objects are symmetrical as they can be divided in such a way that their two parts coincide with each other.

15.1 Line Symmetry

Let us take some more examples and understand what we mean. Trace the following figures on a tracing paper.

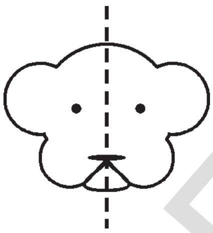


Figure 1

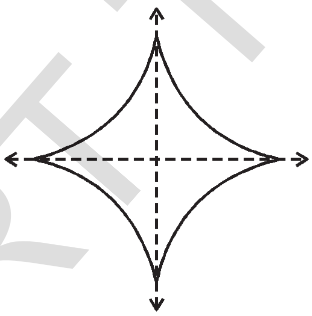


Figure 2

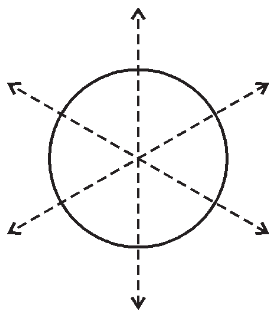


Figure 3

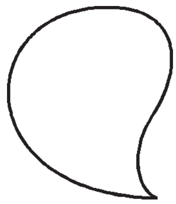


Figure 4

Fold Figure 1 along the dotted line. What do you observe?

You will find that the two parts coincide with each other. Is this true in Figure 2 and 3?

You will observe that in Figure 2, this is true along two lines and in Figure 3 along many lines. Can Figure 4 be divided in the same manner?

Figure 1, 2 and 3 have line symmetry as they can be divided in such a manner that two parts of the figure coincide with each other when they are folded along the line of symmetry. The dotted line which divides the figures into two equal parts is the line of symmetry or axis of symmetry. As you have seen, an object can have one or more than one lines of symmetry or axes of symmetry.

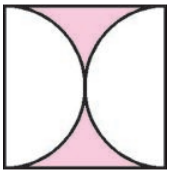
Try This

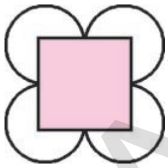
1. Name a few things in nature, that are symmetric.
2. Name 5 man-made things that are symmetric.

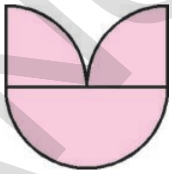
Exercise - 1

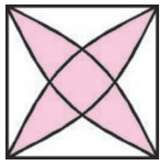
1. Given below are some fiugres. Which of them are symmetric? Draw the axes of symmetry for the symmetric figures.

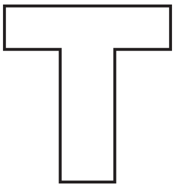

(i)

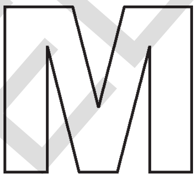

(ii)

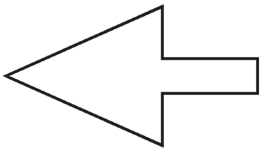

(iii)

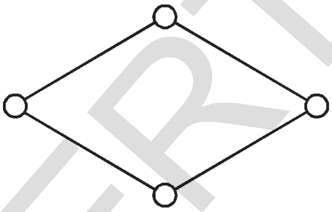

(iv)

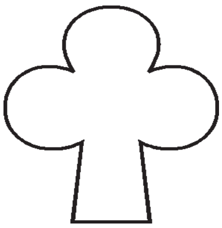

(v)


(vi)

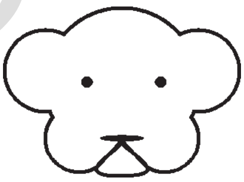

(vii)

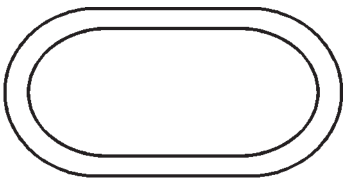

(viii)

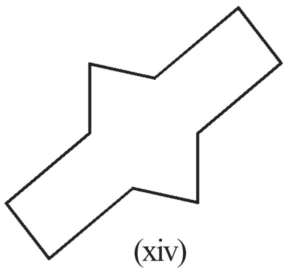

(ix)

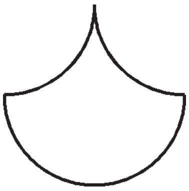

(x)

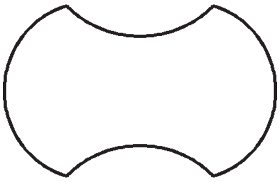

(xi)

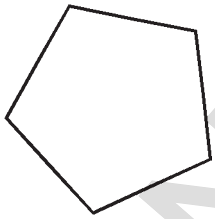

(xii)

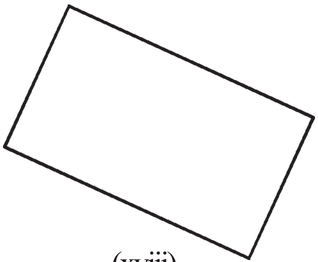

(xiii)

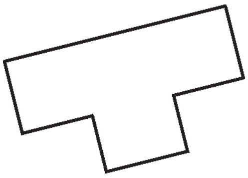

(xiv)

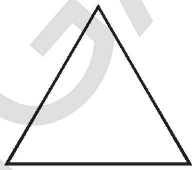

(xv)

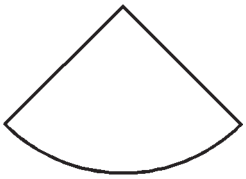

(xvi)

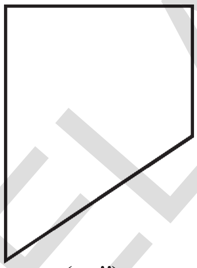

(xvii)

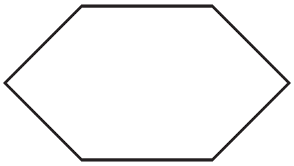

(xviii)


(xix)


(xx)

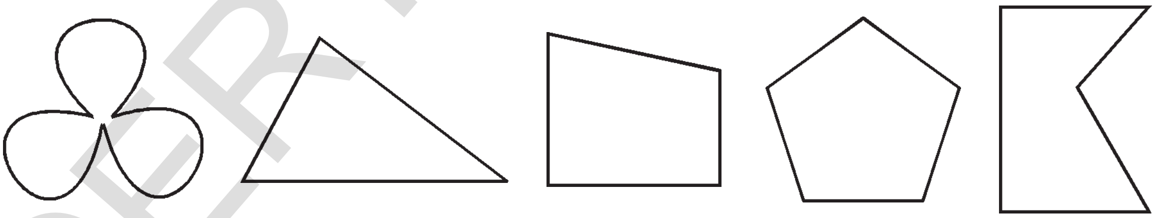

(xxi)


(xxii)


(xxiii)

15.1.1 Lines of symmetry for regular polygons

Look at the following closed figures.



A closed figure made from several line segments is called a 'Polygon'. Which of the above figures are polygons?

Try This

1. Can we make a polygon with less than three line segments?
2. What is the minimum number of sides of a polygon?

Observe the different triangles below.

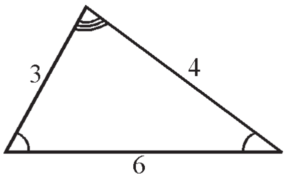


Figure 1

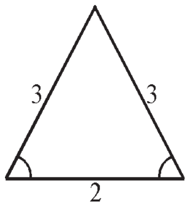


Figure 2

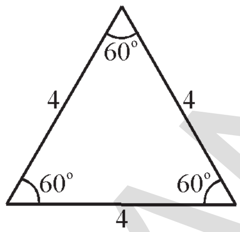
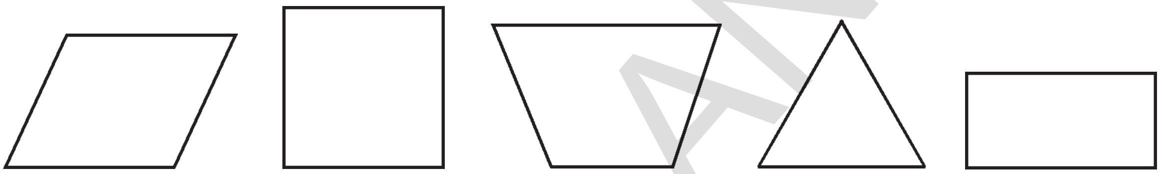


Figure 3

In Figure 3, the triangle has equal sides and congruent angles. It is thus called a regular polygon.

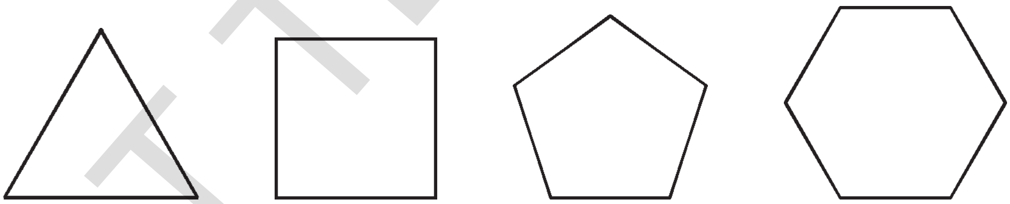
A polygon, with all sides and all angles equal is called a 'Regular Polygon'.

Which of the following polygons are regular polygons?



Parallelogram Square Trapezium Equilateral triangle Rectangle

Now draw axes of symmetry for the following regular polygons.



Equilateral Triangle Square Regular Pentagon Regular Hexagon

Write down your conclusions in the table below.

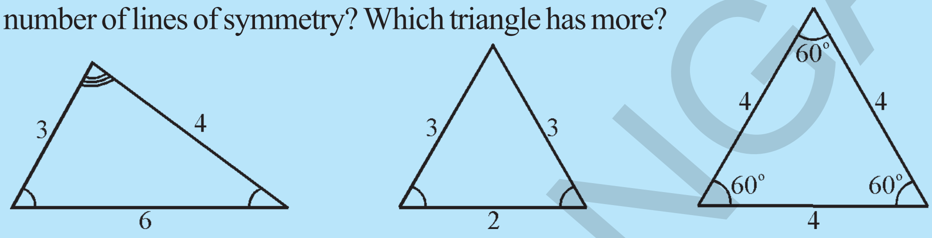
Regular Polygon	No. of sides	No. of axes of symmetry
Equilateral Traingle		3 3
Square		
Pentagon		
Hexagon		

Did you find any relationship between the number of sides of a regular polygon and number of axes symmetry? You will find that the number of sides is equal to number of axes of symmetry.

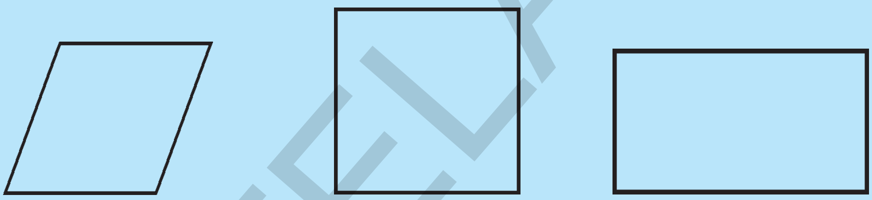
You can verify your results by tracing out all the four figures on a paper, cuting them out and actually folding each figure to find the axes of symmetry.

Try This

1. Given below are three types of triangles. Do all the triangles have the same number of lines of symmetry? Which triangle has more?



2. Given below are different types of quadrilaterals. Do all of them have the same number of lines of symmetry? Which quadrilateral has the most?

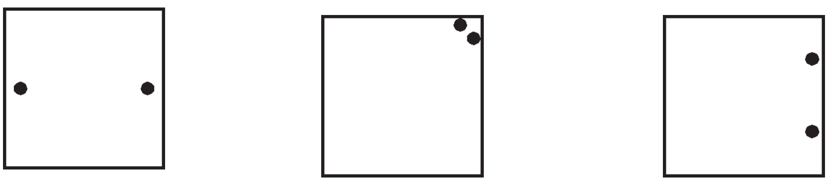


Hint: You can trace the triangles and quadrilaterals on a tracing paper and actually fold each figure to find the axes of symmetry.

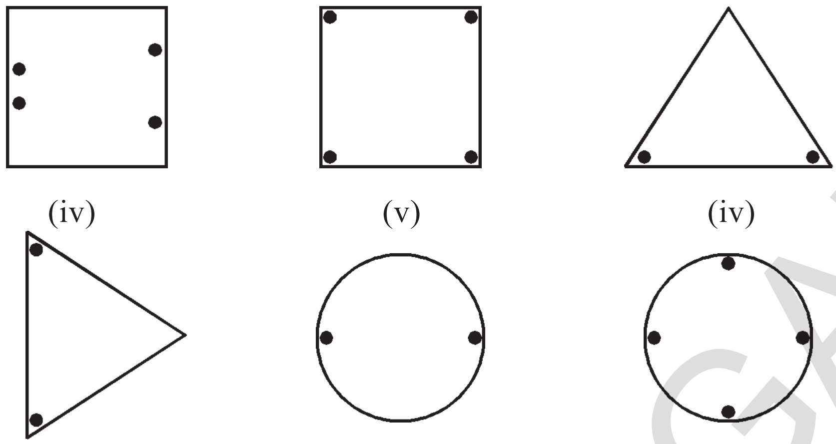
On the basis of (i) and (ii) can we say that a regular polygon has the maximum number of axes of symmetry.

Exercise - 2

1. In the figures given below find the axes of symmetry such that on folding along the axis the two dots fall on each other.



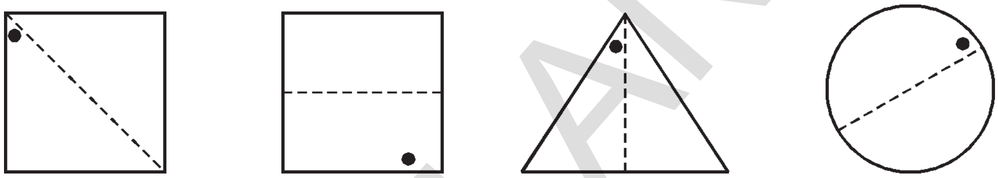
(i) (ii) (iii)



(iv) (v) (iv)

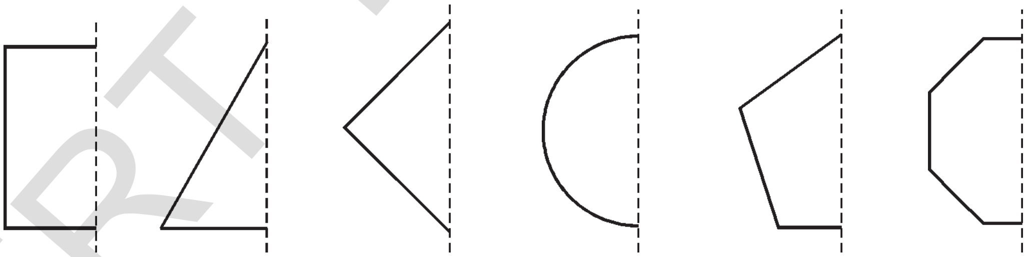
(vii) (viii) (ix)

2. Given the line of symmetry, find the other dot.



(i) (ii) (iii) (iv)

3. In the following incomplete figures, the mirror line (i.e. the line of symmetry) is given as a dotted line. Complete each figure, performing reflection on the dotted (mirror) line. (You might perhaps place a mirror along the dotted line and look into the mirror for the image). Are you able to recall the name of the figure you complete?



(i) (ii) (iii) (iv) (v) (vi)

4. State whether the following statements are true or false.

(i) Every closed figure has an axis of symmetry. ( )

(ii) A figure with at least one axis of symmetry is called a symmetric figure. ( )

(iii) A regular polygon of 10 sides will have 12 axes of symmetry. ( )

5. Draw a square and construct all its axes of symmetry. Measure the angles between each pair of successive axes of symmetry. What do you notice? Does the same rule apply for other regular polygons?

15.2 Rotational Symmetry

Activity 1 : Trace the following diagram onto a tracing paper.

Try to fold the diagram so that its two parts coincide. Is this diagram symmetric?

Now, let us try to match the different positions of the diagram in another way. Draw the above diagram on a piece of paper. Mark a point 'o'at the centre and name the four edges of the paper A,B,C,D as shown in Figure 1.

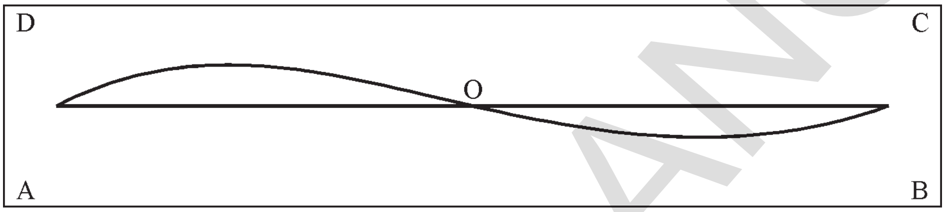


Figure 1

Rotate the paper around the marked point for 180^{o}.

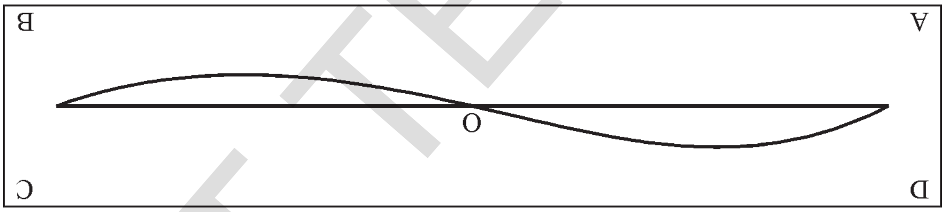


Figure 2

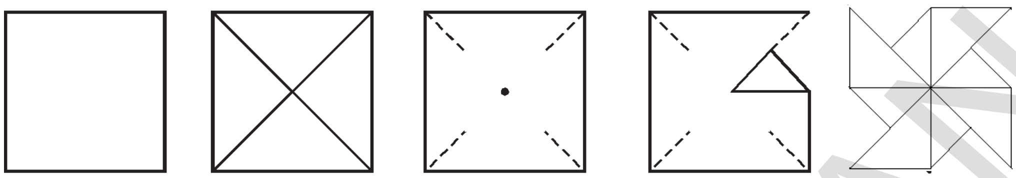
What do you notice? Does this diagram look different from the previous one?

Due to the rotation, the points A,B,C,D have changed position however the diagram seems to be unchanged. This is because the diagram has rotational symmetry.

Activity 2 : Lets make a wind wheel

- Take a paper and cut it into the shape of a square.
- Fold it along the diagonals.
- Starting from one corner, cut the paper along the diagonals towards the centre, up to one fourth of the length of the diagonal. Do the same from the remaining corners.
- Fold the alternate corners towards the centre.
- Fix the mid point to a stick with a pin so that the paper rotates freely.

- Face it in the opposite direction of the wind. You will find it rotates



Now, let us rotate the wind-wheel by 90°. After each rotation you will see that the wind-wheel looks exactly the same. The wind-wheel has rotational symmetry.

Thus, if we rotate a figure, about a fixed point by a certain angle and the figure looks exactly the same as before, we say that the figure has rotational symmetry.

15.2.1 Angle of Rotational Symmetry

We know that the square has line symmetry and 4 axes of symmetry. Now, let us see if the square has rotational symmetry.

Consider a square as in Figure (i) with P as one of its corners.

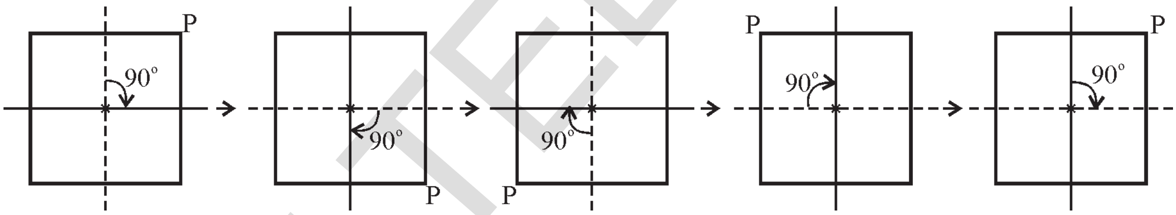


Figure 1 Figure 2 Figure 3 Figure 4 Figure 5

Figure 1 represent the initial position of square.

Rotate the square by 90 degrees about the centre. This quarter turn will lead to Figure 2. Note the position of P. In this way, rotate the square again through 90 degrees and you get Figure 3. When we complete four quarter turns, the square reaches its original position. After each turn, the square looks exactly like it did in its original position. This can also be seen with the help of the position taken by P.

In the above activity all the positions in figure 2, figure 3, figure 4 and figure 5 obtained by the rotation of the first figure through 90^0 , 180^0, 270^0 and 360^0 look exactly like the original figure 1. Minimum of these i.e., 90^0 is called the angle of rotational symmetry.

The minimum angle rotation of a figure to get exactly the same figure as original is called the "angle of rotational symmetry" or "angle of rotation".

Do This

1. What is the angle of rotational symmetry of a square?
2. What is the angle of rotational symmetry of a parallelogram?
3. What is the angle of rotational symmetry of a circle?

15.2.2 Order of rotational symmetry

In the above activity, the angle of rotational symmetry of square is 90^0 and the figure is turned through the angle of rotational symmetry for 4 times before it comes to original position. Now we say that the square has rotational symmetry of order 4.

Consider an equilateral triangle. Its angle of rotational symmetry is 120^0. That means it has to be rotated about its centre for 3 times to get exactly the same position as the original one. So the order of rotational symmetry of a equilateral triangle is 3.

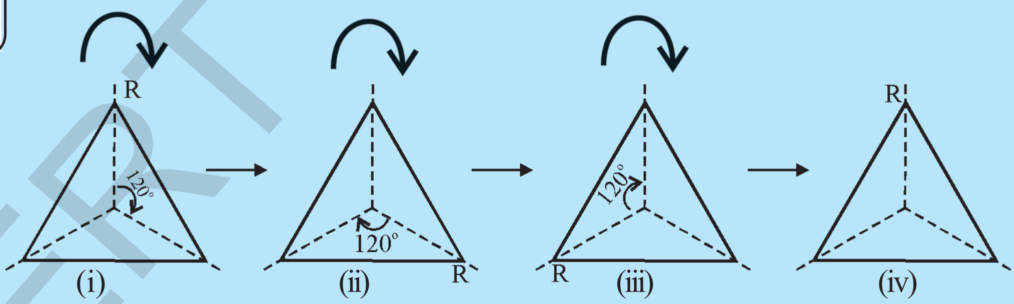
By these examples we conclude that the number of times a figure, rotated through its angle of rotational symmetry before it comes to original postion is called order of rotational symmetry.

Let us conclude from the above examples

- The centre of rotational symmetry of a square is its intersection point of its diagonals.
- The angle of rotational symmetry for a square is 90^0.
- The order of rotational symmetry for a sqaure is 4.

Try This

1. (i) Can you now tell the order of rotational symmetry for an equilateral triangle.



(ii) How many lines of semetry?

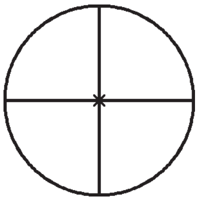
(iii) What is the angle between every adjacent axes?

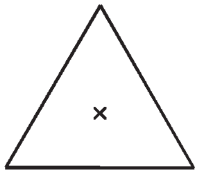
2. Look around you. Which objects have rotational symmetry (i.e. rotational symmetry of order more than 1).

Note: It is important to understand that all figures have rotational symmetry of order 1, as can be rotated completely through 360° to come back to its original position. So we say that an object has rotational symmetry, only when the order of symmetry is more than 1.

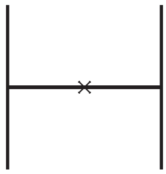
Exercise - 3

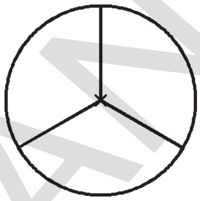
1. Which of the following figures have rotational symmetry of order more than 1?


(i)

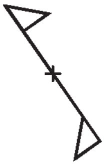

(ii)

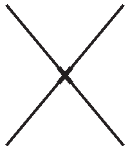

(iii)

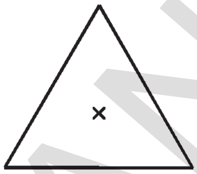

(iv)

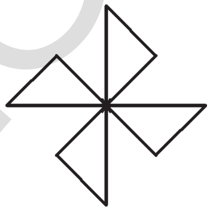

(v)

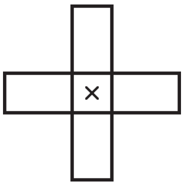
2. Give the order of rotational symmetry for each figure.

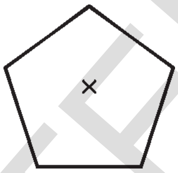

(i)

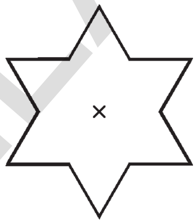

(ii)


(iii)


(iv)


(v)


(vi)


(vii)


(viii)

3. Draw each of the shapes given below and fill in the blanks.

Shape	Centre of Rotation (intersection of diagonals/ Intersection of axes of symmetry)	Angle of Rotation	Order of Rota -tion
Square			
Rectangle			
Rhombus			
Equilateral Triangle			
Regular Hexagon			
Circle			
Semi-circle			

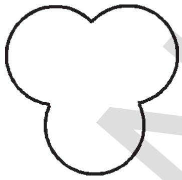
15.3 Line symmetry and rotational symmetry

By now you must have realised that some shapes only have line symmetry and some have only rotational symmetry (of order more than 1) and some have both. Squares and equilateral triangles have both line and rotational symmetry. The circle is the most perfect symmetrical figure, because it can be rotated about its centre through any angle and it will look the same. A circle also has unlimited lines of symmetry.

Example 1 : Which of the following shapes have line symmetry? Which have rotational symmetry?


(i)


(ii)


(iii)

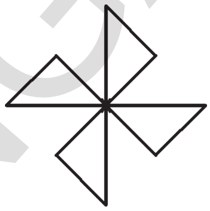

(iv)

Figure	Line symmetry	Rotational symmetry
1.	Yes	No
2.	No	Yes
3.	Yes	Yes
4.	No	Yes

Activity 3 :

- Take a square shaped paper.
- Fold it vertically first, then horizontally.
- Then fold along a diagnal such that the paper takes a triangular shape (Figure 4).
- Cut the folded edges as shown in the figure or as you wish (Figure 5).
- Now open the piece of paper.

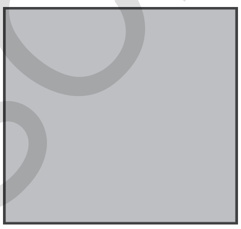


Figure 1

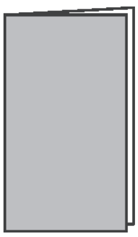


Figure 2

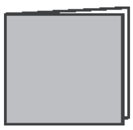


Figure 3

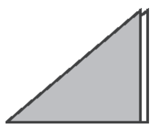


Figure 4


Figure 5

(i) Does the paper have line symmetry?

(ii) Does the paper have rotational symmetry?

Exercise - 4

1. Some english alphabets have fascinating symmetrical structures. Which capital letters have only one line of symmetry (like E))? Which capital letters have rotational symmetry of order 2 (like I)?

   Fill the following table, thinking along such lines.

Alphabets	Line symmetry	Number of lines symmetry	Rotational symmetry	Order of rotational symmetry
Z	No	0	Yes	2
S				
H				
O				
E	Yes	1	No	-
N				
C				

Home Project

Collect pictures of symmetrical figures from newspapers, magazines and advertisement pamphlets. Draw the axes of symmetry over them. Classify them.

Looking Back

- The line which divides a figure into two identical parts is called the line of symmetry or axis of symmetry.
- An object can have one or more than one lines of symmetry or axes of symmetry.
- If we rotate a figure, about a fixed point by a certain angle and the figure looks exactly the same as before, we say that the figure has rotational symmetry.
- The angle of turning during rotation is called the angle of rotation.
- All figures have rotational symmetry of order 1, as can be rotated completely through 360^{o} to come back to their original position. So we say that an object has rotational symmetry only when the order of symmetry is more than 1.
- Some shapes only have line symmetry and some have only rotational symmetry and some have both. Squares, equilateral triangles and circles have both line and rotational symmetry.

ANSWERS

01- Integers

Exercise - 1 (page - 2)

(1) Biggest number = 2, smallest number = -3

(2) (i) −9, −8, −7, −6 ; biggest number = −6 ; smallest number = −9

(ii) −1, 0 +1, +2, ; biggest number = +2 ; smallest number = −1

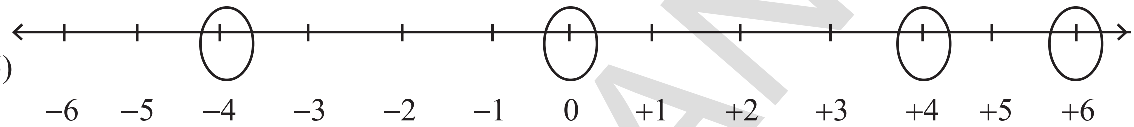
(iii) −7, −6, −5, −4, −3, −2, −1, 0, 1, 2, 3, 4

biggest number = +4 ; smallest number = −7

(3) (i) −8, −5, 1, 2 (ii) −5, −4, −3, 2 (iii) −15, −10, −7

(4) (i) −2, −3, −5 (ii) −1, −2, −8 (iii) 8, 5, −2

(5)



6. −8, −7, −6, −4, −3, −2, −1, 1, 2, 3, 5, 6, 7, 9

(7) i)

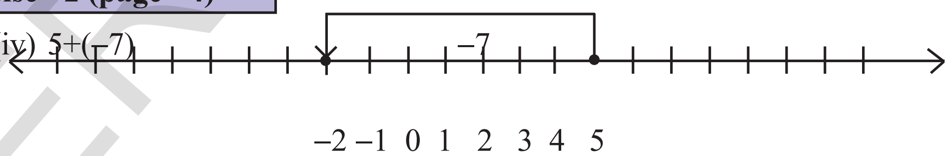
No.	Name of the City	Temperature
1	Bangalore	20°C
2	Ooty	15°C
3	Nainital	−3°C
4	Manali	−7°C
5	Kasauli	−9°C

(ii) Bangalore (20°C) (iii) Kasauli (−9°C)

(iv) Nainital (−3°C) Manali (−7°C) Kasauli (−9°C) (v) Ooty (15°C) Bangalore (20°C)

Exercise - 2 (page - 4)

(1) (iv) 5+(−7)



(2) (i) 11 (ii) 5 (iii) 14 (iv) 8 (v) 2 (vi) 4

(vii) −2 (viii) 0 (ix) 8 (x) 20 (xi) 80

Exercise - 3 (page - 6)

(1) (i) 5 (ii) 15 (iii) -4 (iv) 1 v) 13 vi) -1

(2) (i) 31 (ii) 21 (iii) 24 (iv) −13

(v) −8 (vi) 130 (vii) 75 (viii) 50

(3) Sl.No Negative integer + Whole No. = −6

Sl.No	Negative integer	+	Whole No.	=	−6
1	(−6)	+	0	=	−6
2	(−7)	+	1	=	−6
3	(−8)	+	2	=	−6
4	(−9)	+	3	=	−6 etc.,

Exercise - 4 (page - 11)

(1) (i) +600 (ii) −1 (iii) −600 (iv) +200 (v) −45

(2) (i) −3 (ii) −225 (iii) 630 (iv) 316 (v) 0
(vi) 1320 (vii) 162 (viii) −360 (ix) −24 (x) 36

(3) −10° (4) (i) 10 (ii) 18 (iii) 5 (5) (i) ₹ 5,000 profit (ii) 3200

(6) (i) −9 (ii) −7 (iii) +7 (iv) −11

Exercise - 5 (page - 19)

(1) (i) True (72 = 126 − 54 = 72) (ii) True (210 = 84 + 126 = 210) (2) (i) −a (ii) −5

(3) (i) 480 (ii) −53,000 (iii) −15000 (iv) −4182
(v) −62500 (vi) 336 (vii) 493 (viii) 1140

Exercise - 6 (page - 22)

(1) (i) −1 (ii) −49 (iii) not defined (iv) 0

Exercise - 7 (page - 23 & 24)

(1) (i) 24 (ii) 20 (2) (i) Profit 33,000 (ii) 3000

(3) 9 PM ; Temperature at Midnight = −14°C

(4) (i) 8 questions (ii) 13 question (5) 1 hour

02- Fractions, Decimals and Rational Numbers

Exercise - 1 (page - 29)

(1) (i) $2\frac{3}{4}$ (ii) $1\frac{1}{9}$ (iii) $\frac{3}{7}$ (iv) $3\frac{1}{6}$ (v) $\frac{11}{24}$ (vi) $6\frac{1}{6}$

(2) (i) $\frac{1}{2}, \frac{5}{8}, \frac{5}{6}$ (ii) $\frac{3}{10}, \frac{1}{3}, \frac{2}{5}$

(3) Sum in row = $\frac{21}{13}$, Sum in column = $\frac{21}{13}$, Sum in diagonal = $\frac{21}{13}$ All the sums are equal.

(4) $17\frac{11}{15}$ cm (5) $1\frac{7}{8}$ (6) $\frac{7}{12}$

(7) Perimeter of Δ ABE $= 10\frac{1}{5}$ cm; BCDE perimeter $= 7\frac{11}{15}$ cm ;

Δ ABE is greater ; difference $= 2\frac{7}{15}$

Exercise - 2 (page - 34)

(1) (i) $5\frac{0}{6}$ or 5 (ii) $1\frac{1}{3}$ (iii) $1\frac{5}{7}$ (iv) $1\frac{1}{9}$ (v) $6\frac{0}{5}$ or 6

(2) (i) 6 (ii) 6 (iii) 9 (iv) 15

(3) (i) 4 (ii) 6

Exercise - 3 (page - 37)

(1) (i) $\frac{35}{66}$ (ii) $1\frac{1}{5}$ (iii) $7\frac{7}{15}$ (2) (i) $3\frac{7}{15}$ (ii) $\frac{2}{21}$ (iii) 3

(3) (i) $\frac{3}{8} = \frac{3}{4}$ of $\frac{1}{2}$ (ii) Both are euqal (4) $17\frac{1}{2}$ hrs. (5) $85\frac{1}{3}$ km. (6) 1350 m.

(7) (i) $\frac{10}{7}$ (ii) $\frac{3}{5}$, 35 or 3,7

Exercise - 4 (page - 43)

(1) (i) $\frac{8}{5}$ (ii) $\frac{7}{8}$ (iii) $\frac{7}{13}$ (iv) $\frac{4}{3}$ (2) (i) 24 (ii) $3\frac{3}{7}$ (iii) $1\frac{2}{7}$ (iv) $\frac{7}{5}$

(3) (i) $\frac{2}{15}$ (ii) $\frac{7}{40}$ (iii) $\frac{5}{9}$ (4) $2\frac{1}{2}$ days

Exercise - 5 (pages- 45 & 46)

(1) (i) 0.7 (ii) 8.5 (iii) 1.51 (iv) 6 (2) (i) ₹. 0-09 (ii) ₹. 77-07 (iii) ₹. 2-35

(3) (i) 0.1 m, 0.0001 km (ii) 4.5 cm, 0.045 m, 0.000045 km.

(4) (i) 0.19 kg (ii) 0.247 kg (iii) 44.08 kg

(5) (i) $50+5+\frac{5}{10}$ (ii) $5+\frac{5}{10}+\frac{5}{100}$ (iii) $300+3+\frac{3}{100}$

(iv) $30+\frac{3}{10}+\frac{3}{1000}$ (v) $1000+200+30+4+\frac{5}{10}+\frac{6}{100}$

(6) (i) 3 (ii) 30 (iii) $\frac{3}{100}$ (iv) $\frac{3}{10}$ (v) $\frac{3}{100}$

(7) Radha walked 100 m. more than Aruna (8) 5.625 kg.

Exercise -6 (page- 50 & 51)

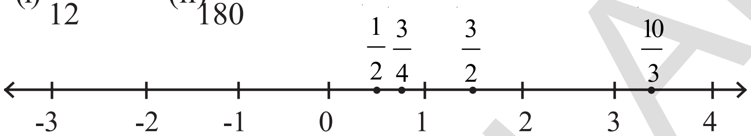
(1) (i) 1.8 (ii) 18.9 (iii) 13.55 (iv) 78.8 (v) 0.35
(vi) 1050.05 (vii) 1.72 (2) 24.8 cm^2

(3) (i) 213 (ii) 368 (iii) 537 (iv) 1680.7 (v) 13110
(vi) 15610 (vii) 362 (viii) 4307 (ix) 5 (x) 0.8
(xi) 90 (xii) 30 (4) 625 Km (5) (i) 0.45 (ii) 0.75
(iii) 42.16 (iv) 14.62 (v) 0.025 (vi) 1.12 (vii) 0.0214
(viii) 10.5525 (ix) 1.0101 (x) 77.011 (6) (i) 0.023 (ii) 0.09 (iii) 4.43
(iv) 0.1271 (v) 2 (vi) 590 (vii) 0.02 (7) 5 (8) 0.128 cm

Exercise -7 (page- 56)

(2) (i) $\frac{-5}{12}$ (ii) $\frac{-75}{180}$

(3)



(4) (i) false (ii) true (iii) false (iv) false (v) true

03 - Simple Equations

Exercise - 1 (page - 59)

(1) (i) L.H.S = 2x, R.H.S = 10 (ii) L.H.S = 2x–3, R.H.S = 9 (iii) L.H.S = 4z+1, R.H.S = 8 (iv) L.H.S = 5p+3, R.H.S = 2p+9

(v) L.H.S = 14, R.H.S = 27–y (vi) L.H.S = 2a–3, R.H.S = 5 (vii) L.H.S = 7m, R.H.S = 14 (viii) L.H.S = 8, R.H.S = q+5

(2) (i) y = 5 (ii) a = 8 (iii) m = 3 (iv) n = 7

Exercise - 2 (page - 63)

(1) (i) x = 4 (ii) y = 7 (iii) x = 5 (iv) z = 9 (v) x = 3 (vi) y = –20

(2) (i) y = 5 (ii) a = 4 (iii) q = 4 (iv) t = 4 (v) x = 13
(vi) x = 3 (vii) x = –5 (viii) x = –1 (ix) y = 4 (x) x = –2

Exercise - 3 (page - 67)

(1) 4 cm (2) 5 cm (3) 21 (4) 30 (5) 8 (6) 46, 49 (7) 7, 8, 9
(8) *l* = 34m, b = 2m (9) *l*=23m, b=19m (10) 5 years (11) 19, 44
(12) 40; 25, 15 (13) 2 (14) 40 (15) 30°, 60°, 90°
(16) 30

04 - Lines and Angles

Exercise - 1 (page - 69)

(1) (i) Line segment AB (ii) Ray CD (iii) Line XY (iv) Point ‘P’

(2) (i) 

(ii) 

(iii) 

(iv) 

(3) $\overline{AB}$, $\overline{AC}$, $\overline{AD}$, $\overline{BC}$, $\overline{BD}$, $\overline{CD}$

(5) (i) acute (ii) obtuse (iii) Right (iv) acute (v) obtuse

(6) ∠AOF, ∠FOE, ∠EOD, ∠DOC, ∠COB, ∠FOD, ∠EOC, ∠DOB - Acute angles.

∠AOE, ∠EOB, ∠FOC - Right angles ; ∠AOD, ∠AOC, ∠FOB - Obtuse angles.

∠AOB - straight angle (7) (i) and (iv) are parallel ; (ii) and (iii) non parallel

(8) (i),(ii) and (iv) are intersecting lines and (iii) non-intersecting lines.

Exercise - 2 (page - 71)

(1) iii (2) (i) 65° (ii) 50° (iii) 1° (iv) 35° (3) 45°, 45°

(4) Yes. Because the sum of the angles must be 90°

Exercise - 3 (page - 73)

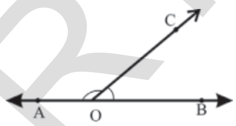
(1) (i), (ii) (2) (i) 75° (ii) 85° (iii) 30° (iv) 160°

(3) The sum of two acute angles is always less than 180° (4) 90°, 90°

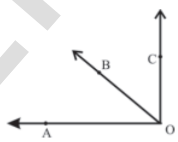
Exercise - 4 (page - 74)

(1) (i) a, b (ii) c, d (2) (i) ∠AOD, ∠DOB (ii) ∠DOB, ∠BOC

(iii) ∠BOC, ∠COA (iv) ∠COA, ∠AOD

(3) Yes. 

because ∠AOC + ∠COB = 180°

(4) Yes . 

because ∠AOB + ∠BOC = 90°

Exercise - 5 (page - 75)

(1) i, ii (2) No. Because there is no common arm.

Exercise - 6 (page - 76)

(1) (i) ∠AOD, ∠BOC (ii) ∠AOC, ∠BOD

(2) y = 160° (Vertically opposite angles) $x + 160° = 180°$ $\therefore x = 20°$

∠x = ∠z Vertically opposite angles $\therefore z = 20°$

Exercise - 7 (page - 85)

(1) (i) Transversal (ii) Parallel (iii) Parallel (iv) one
(2) (i) 100° (ii) 45° (iii) 90° (iv) 100°

(3) $\angle x = 180 - (75+45) = 60°$; $\angle y = 75°$; $z = 45°$

(4) $b + 50° = 180°$ $\therefore b = 130°$

$b + c = 180° \Rightarrow 130° + c = 180° \Rightarrow c = 50°$
$d + 50° = 180° \Rightarrow d = 130°$

(5) $l \parallel m$

(6) $\angle a = 50°$ (Alternate angles)

$\angle b = 50°$ (Alternate angles)

$\angle c = \angle d = \angle e = 50°$

(all are Alternate angles)

05 - Triangle and its Properties

Exercise - 1 (page - 93)

(1) (i) Possible (ii) Possible (iii) Not possible (iv) Possible (v) Not possible

Exercise - 2 (page - 94)

(1) (i) Median (ii) Altitude (Height) (2) Right angle triangle (3) Yes

(4) No, in some cases it lies in the exterior of the triangel (5) (i) XZ (ii) $\angle P$ (iii) B

Exercise - 3 (page - 100)

(1) (i) 70° (ii) 60° (iii) 40° (2) (i) x = 70° ; y = 60° (ii) x = 80° ; y = 50°
(iii) x = 110° ; y = 70° (iv) x = 60° ; y = 90° (v) x = 45° ; y = 90° (iv) x = 60°

(3) (i) 40° (ii) 34° (iii) 60° (4) 60° (5) (i) False (ii) True (iii) False (iv) False

(6) (i) 30° ; 60° ; 90° (7) x = 100° ; y = 50° ; z = 100° (8) 72°

(9) $\angle P = 80°$; $\angle Q = 40°$; $\angle R = 60°$ (10) 18° ; 72° ; 90° (11) 36°, 54°

(12) $\angle LPM = 40°$; $\angle PML = 50°$; $\angle PRQ = 50°$ (13) 540°

Exercise - 4 (page - 107)

(1) Interior angles : $\angle ABC, \angle ACB, \angle BAC$; Exterior angles : $\angle CBX, \angle ACZ, \angle BAY$

(2) $\angle ACD = 111°$ (3) $x = 115°$; $y = 35°$ (4) (i) $x = 50°$ (ii) $x = 33°$; $y = 82°$

(5) $\angle CDB = 76°$; $\angle DBC = 39°$; $\angle ABC = 58°$

(6) (i) $x = 55°$ (ii) $x = 100°$ (iii) $x = 120°, y = 30°$ (iv) $y = 70°$ (v) $x = 60°$; $y = 150°$;

(vi) $x = 50°$; $y = 130°$ (7) 50° ; 75° ; 55° (8) $\angle P = 35°$,yes (9) 70°

(10) 30° ; 75° ; 75° (11) $x = 135°$; $y = 80°$

06 - Ratio - Applications

Exercise - 1 (page - 111)

(1) 100 : 10 , 10:1 (2) ₹ 15 (i) 15 : 5 or 3 : 1 (Radha : Sudha)

(ii) 5 : 15 or 1 : 3 (Sudha : Radha) (3) Raju's Share = 40 ; Ravi's Share = 56

(4) $\overline{AX} = 18$ cm ; $\overline{XB} = 20$ cm. (5) ₹.60,000 (6) 8 liters

(7) 40 : 20 or 2: 1 (8) 1:2400 or 0.05 : 120

(9) (i) count No. of boys and girls in your class and write inthe form of ratio. If boys or girls will be zero, can you write it in the form of ratio? We can not compare such ratios.

(ii) Count of doors and number of windows of your classroom and number write in the form of ratios.

(iii) Count all textbooks and note books with you and write in ratio form.

Exercise - 2 (page - 114)

(1) (i) 8, 8 (ii) 450, 450 (iii) 96, 96 (iv) 6, 30 (v) 24, 72

(2) (i) False (ii) True (iii) True (iv) True (v) False

(3) ₹.90 (4) 10 kg (5) a) 45 b) 26 (6) i) 540° ii) 21°

Exercise - 3 (page - 120)

(1) 0.0001 cm ; 2cm (2) (i) Yes (ii) No (iii) No. (3) 4 cm

(4)
- Draw 5 different squares, measure their lengths and fill the table.
- 4 times of side will be perimeter of square find and fill the table.
- Square the side of each and fill the table.

(i) Yes, lenght of side is in direct proportion to perimeters of the squares.

(ii) No, length of side is not indirect proportion to area of the square.

Exercise - 4 (page - 125)

(1) School Y (2) 20% decrease (3) Mangoes = 35% (4) 16%

(5) Abscent = $16\frac{2}{3}\%$ or 16.66% Present = $83\frac{1}{3}\%$ or 83.33% (6) 7200

(7) 15 (8) gold 70% ; silver 25% ; Copper 5% (9) 2000

Exercise - 5 (page - 136)

(1) $12\frac{1}{2}\%$ or 12.5% (2) 6% (3) ₹. 2,00,000 (4) ₹. 875

(5) loss = 1200 (2.44%) (6) 561 (7) 202.5 (8) 800 (9) 1100

Exercise - 6 (page - 140)

(1) 2 years 8 months or $\frac{8}{3}$ years or $2\frac{2}{3}$ years (2) 12%

(3) ₹. 450 (4) ₹. 12958 (5) $1\frac{1}{2}$ years

07 - Data Handling

Exercise 1 (page - 147)

(1) (i) 33 ^{0}C (ii)30 ^{0}C (2) 15.9 kg

(3) (i) Ground nuts ₹:7500 ; Jawar ₹:4000 ; Millets ₹:5250 (ii) Ground nuts

(4) 42 (5) (i) 23 (ii) 21 (iii) 16.5 (iv) Lekhya

(6) (i) ₹ 18 (ii) ₹ 54 (iii) Proportional (7) 5.5 (8) 5.6 (9) 107

Exercise 2 (page - 152)

(1) 155 cm, 140cm (2) (i) Mean = 28, Mode = 27 (ii) 2 players of age 25 years each.

(3) 25 (4) (i) Mode (ii) Mean (iii) Mean (iv) Mode

Exercise 3 (page - 155)

(1) (i) F (ii) T (iii) F (iv) F (2) (i) ₹ 1400 (ii) ₹ 1450

(3) Mode is correct, but median is wrong. (4)

three 1, 7, 10 ; 2, 7, 9 ; 3, 7, 8 (5) 11

Exercise 4 (page - 160)

(5) (i) Education (ii) Food (iii)₹ 2250 (iv) ₹ 1500

08 - Congurencey of Triangles

Exercise - 1 (Page. 169)

(1) (i) True (ii) False

(2) (i) $\angle P = \angle R$ (ii) $\angle ROS = \angle POQ$

$\angle TQP = \angle SQR$ $\angle R = \angle Q$ or $\angle R = \angle P$

$\angle T = \angle S$ $\angle S = \angle P$ or $\angle S = \angle Q$

(3) (ii) Correct (4) Yes (S.S.S. Congruency)

Exercise - 2 (Page. 171)

(1) It is to be given that GH = TR and HJ = TS (2) AP = 4 km ($\therefore$ AP = BQ c.p.c.t.)

(3) (i) $\Delta ABC \cong \Delta STR$
AB = ST also BC = TR
$\angle A = \angle S$ $\angle B = \angle T$
AC = SR $\angle C = \angle R$

(ii) $\Delta POQ \cong \Delta ROS$
PO = RO also PQ = RS
OQ = OS $\angle P = \angle R$
$\angle POQ = \angle ROS$ $\angle Q = \angle S$

(iii) $\Delta DRO \cong \Delta OWD$, DR = OW also DO = OD
RO = WD $\angle ODR = \angle DOW$
$\angle R = \angle W$ $\angle DOR = \angle ODW$
in the fig $\square$ WORD
$\angle R = 90°$
WD = OR and WO = DR
$\therefore$ $\square$ WORD is a rectangle
$\therefore$ $\Delta WSD \cong \Delta RSO$
$\Delta WSO \cong \Delta RSD$
also $\Delta ORW \cong \Delta DWR$

(iv) ΔABC and ΔCDA not congruent

(4) (i) In ΔABC and ΔRQP we need to know that AB = RQ.
(ii) In ΔABC and ΔADC we need to know that AB = AD.

Exercise - 3 (Page. 175)

(1) (i) By A.A.S. $\Delta ABC \cong \Delta RPQ$ (ii) By A.S.A. or A.A.A. $\Delta ABD \cong \Delta CDB$

(iii) By A.A.A. or A.A.S. $\Delta AOB \cong \Delta DOC$ (iv) not congruent

(2) (i) $\Delta ABC \cong \Delta DCB$ (A.A.S)
(ii) from AB = CD (c.p.c.t.) (Corresponding Parts of Congruent Triangles)
$\therefore$ $\Delta AOB \cong \Delta DOC$
otherwise ΔAOB and ΔDOC are similar by A.A.A.
in congruent triangles corresponding parts are equal.

Exercise - 4 (Page. 178)

(1) (i) S.S.S. (ii) S.A.S. (iii) A.S.A. (iv) R.H.S. (2) (i) a) AR = PE b) RT = EN
c) AT = PN (ii) a) RT = EN b) PN = AT (iii) a) $\angle A = \angle P$ b) $\angle T = \angle N$

(3) (i) Side (ii) Angle (iii) Common side (iv) S.A.S.

(4) We can't say $\Delta ABC \cong \Delta PQR$ when the corresponding angles are equal, but can say that the triangles are simlar.

(5) $\Delta RAT \cong \Delta WON$ (6) $\Delta ABC \cong \Delta ABT$ and $\Delta QRS \cong \Delta TPQ$

(7) (i) Draw two triangles with same measures. (ii) Draw two triangles of different measures.
(8) BC = QR (A.S.A.) or AB = PQ (A.A.S.) or AC = PR (A.A.S.)
(9) $\angle B = \angle E$; $\angle A = \angle F$ by A.A.S. $\Delta ABC \cong \Delta FED$ are congruent ; BC = ED

10- Algebraic Expressions

Exercise - 1 (page - 192)

(1) (i) $3n$ (ii) $2n$

(2) (i) • In fig. 4 number of coloured tiles will be 4 on each side.
• In fig. 5 number of coloured tiles will be 5 on each side.
(ii) Algebraic expression for the pattern = 4n ; 4, 8, 12, 16, 20 . . . . . expression = 4n
(iii) Algebraic expression for the pattern = 4n + 1 ; 9, 13, 17, 21 . . .. expression = 4n + 1

(3) (i) $p + 6$ (ii) $x - 4$ (iii) $y - 8$ (iv) $-5q$ (v) $y \div 4$ or $\frac{y}{4}$

(vi) $\frac{1}{4}$ of pq or $\frac{pq}{4}$ (vii) $3z+5$ (viii) $10+5x$ (ix) $2y - 5$ (x) $10y + 13$

(4) (i) ' 3 more than x' or 3 is added to x (ii) 7 is substracted from 'y'
(iii) l is multiplied by 10. (iv) x is divided by 5
(v) m is multiplied by 3 and added to 11
(vi) y is multiplied by 2 and subtracted 5 or 5 is subtracted from 2 times of y.

(5) (i) Constant (ii) Variable (iii) Constant (iv) Variable

Exercise - 2 (page -199)

(1) (i) $(a^2, -2a^2)$ (ii) $(-yz, 2zy)$ (iii) $(-2xy^2, 5y^2x)$ (iv) $(7p, -2p, 3p)$ and $(8pq, -5pq)$
(2) Algebaric expression : Problem Numbers : i, ii, iv, vi, vii,ix, xi
Numerical expression : Problem Numbers : iii, v, viii, x
(3) Monomial i, iv, vi ; binomial : ii, v, vii ; trinomial : iii, viii, ix multinomial : x
(4) (i) 1 (ii) 3 (iii) 5 (iv) 4 (v) 2 (vi) 3 (5) (i) 1 (ii) 2 (iii) 4 (iv) 3
(v) 4 (vi) 2 (6) $xy + yz$ $2x^2 + 3x + 5$

Exercise - 3 (page - 204)

(1) $3a + 2a = 5a$ (2) (i) $13x$ (ii) $10x$(3) (i) $3x$ (ii) $-6p$ (iii) $11m^2$ (4) (i)-1

(ii) 4(iii) -2 (5) -9 (6) $2x^2 + 11x - 9, -23$ (7) (i) 3 (ii) 5 (iii) -1

(8) $54\text{ cm} \times \text{cm} = 54\text{ cm}^2$ (9) ₹. 90

(10) $s = \frac{d}{t} = \frac{135\,mt}{10\,\text{sec}} = \frac{27}{2}\,mt./Sec.,\ \text{or}\ 13\frac{1}{2}\,mt./Sec.,\ \text{or}\ 13.5\,mt./Sec.,$

Exercise - 4 (page - 209)

(1) (i) $-5x^2 + xy + 8y^2$ (ii) $10a^2 + 7b^2 + 4ab$ (iii) $7x + 8y - 7z$ (iv) $-4x^2 - 5x$

(2) $7x + 9$ (3) $18x - 2y$ (4) $5a + 2b$

(5) (i) $a+2b$ (ii) $(2x+3y+4z)$ (iii) $(-4ab - 8b^2)$ (iv) $4pq - 15p^2 - 2q^2$

(v) $-5x^2+3x+10$ (vi) $2x^2 - 2xy - 5y^2$ (vii) $3m^3 + 4m^2 + 7m - 7$

(6) $7x^2 + xy - 6y^2$ (7) $4x^2 - 3x - 2$ (8) $4x^2 - 3y^2 - xy$ (9) $2a^2 + 14a + 5$

(10) (i) $22x^2 + 12y^2 + 8xy$ (ii) $-14x^2 - 10y^2 - 20xy$ or $-(14x^2 + 10y^2 + 20xy)$

(iii) $20x^2 + 5y^2 - 4xy$ (iv) $-8y^2 - 32x^2 - 30xy$

11 - Exponents

Exercise - 1 (page - 214)

1. (i) Base = 3, exponent = 4, $3 \times 3 \times 3 \times 3$ (ii) Base = $7x$, exponent=2, $7 \times x \times 7 \times x$
   (iii) Base = 5ab, exponent = 3, $5 \times 5 \times 5 \times a \times a \times a \times b \times b \times b$
   (iv) Base = 4y, exponent = 5, $4 \times 4 \times 4 \times 4 \times 4 \times y \times y \times y \times y \times y$
2. (i) 7^5 (ii) $3^3 \times 5^4$ (iii) $2^3 \times 3^4 \times 5^3$
3. (i) $2^5 \times 3^2$ (ii) 2×5^4 (iii) $2 \times 3^2 \times 5^3$ (iv) $2^4 \times 3^2 \times 5^2$ (v) $2^5 \times 3 \times 5^2$
4. (i) 3^2 (ii) 3^5 (iii) 2^8 5. (1) 17 (ii) 31 (iii) 25 (iv) 1

Exercise - 2 (page - 225)

(1) (i) 2^{14} (ii) 3^{10} (iii) 5^5 (iv) 9^{30} (v) $\left(\frac{3}{5}\right)^{15}$ (vi) 3^{20}

(vii) 3^4 (viii) 6^4 (ix) 2^{9a} (x) 10^6 (xi) $\left(\frac{-5}{6}\right)^{10} = \frac{(-5)^{10}}{6^{10}} = \frac{5^{10}}{6^{10}}$ (xii) 2^{10a+10}

(xiii) $\frac{2^5}{3^5}$ (xiv) 15^3 (xv) -4^3 (xvi) $\frac{1}{9^8}$ (xvii) $\frac{1}{6^4}$ (xviii) -7^{15}

(xix) 6^{16} (xix) a^{x+y+z} (2) 3^{10} (3) 2 (4) 2 (5) 1

(6) (i) true (2+11=13) (ii) false (iii) true (iv) true (v) false (vi) fasle (vii) true

Exercise - 3 (page - 226)

(i) $3.84\times10^8 m$ (ii) 1.2×10^{10} (iii) $3\times10^{20} m$ (iv) $1.353\times10^9\ km^3$

12 - Quadrilaterlals

Exercise - 1 (page - 232)

(1) (i) Sides: $\overline{PQ}$, $\overline{QR}$, $\overline{RS}$, $\overline{SP}$ Angles: $\angle$QPS, $\angle$PSR, $\angle$SRQ, $\angle$RQP

Vertices: P, Q, R, S diagnals: $\overline{PR}$, $\overline{QS}$

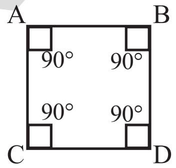
(ii) Pairs of adjacent sides $\overline{PQ}, \overline{QR}$; $\overline{QR}, \overline{RS}$; $\overline{RS}, \overline{SP}$ and $\overline{SP}, \overline{PQ}$

Pairs of adjacent angles: $\angle$QPS, $\angle$PSR; $\angle$PSR, $\angle$SRQ; $\angle$SRQ, $\angle$RQP and $\angle$RQP, $\angle$QPS

Pairs of opposite sides : $\overline{PS}, \overline{QR}$ and $\overline{QP}, \overline{RS}$

Pairs of opposite angles: $\angle$QPS, $\angle$SRQ and $\angle$PSR, $\angle$RQP

(2) 100° (3) 48°, 72°, 96°, 144° (4) 90°, 90°, 90°, 90°



(5) 75°, 85°, 95°, 105°

(6) Angle of the quadrilateral cannot be 180°

Exercise - 2 (page - 242)

(1) (i) false (ii) true (iii) true (iv) false (v) false (vi) true (vii) true (viii) true

(2) (i) Since it has 4 sides (ii) Since opposite sides in a square are parallel

(iii) Since diagonals of a square are perpendicular bisectors

(iv) Since opposite sides of a square are of equal length.

(3) $\angle$BAD = 140°, $\angle$DCB = 140°, $\angle$CDA = 40° (4) 50°, 130°, 50°, 130°

(5) It has 4 sides and one pair of parallel sides; $\overline{EA}$, $\overline{DR}$ (6) 1

(7) Opposite angles are not equal. (8) 15 cm, 9cm, 15cm, 9cm

(9) No, Rhomsun should have equal length of sides (10) $\angle C = 150^{\circ}$, $\angle D = 150^{\circ}$

(11) (i) Rhombus (ii) Squrare (iii) 180° - x°

(iv) equal/congruent (v) 10 (vi) 90°

(vii) 0 (viii) 10 (ix) 45

13 - Area and Perimeter

Exercise - 1 (page - 245)

(1) $2(l+b)$; a^2 (2) 60 cm; 22cm; 484 cm^2 (3) 280cm^2; 68cm; 18cm; 216cm^2; 10cm; 50cm

Exercise - 2 (page - 249)

(1) (i) 28cm^2 (ii) 15cm^2 (iii) 38.76cm^2 (iv) 24cm^2 (2) (i) 91.2cm^2 (ii) 11.4cm
(3) 42cm ; 30cm (4) 8cm ; 24cm (5) 30m, 12m (6) 80m

Exercise - 3 (page - 252)

(1) (i) 20cm^2 (ii) 12cm^2 (iii) 20.25cm^2 (iv) 12cm^2 (2) (i) 12cm^2 (ii) 3cm

(3) 30cm^2 ; 4.62 cm (4) 27cm^2 ; 7.2 cm

(5) 64cm^2 ; Yes ; ΔBEC, ΔBAE and ΔCDE are three triangles drawn between the two parallel lines BC and AD, BC = AE+ED

(6) Ramu in ΔPQR, PR is the base, because QS$\perp$PR. (7) 40 cm (8) 20 cm ; 40cm
(9) 20 cm (10) 800cm^2 (11) 160cm^2 (12) 192cm^2 (13) 18 cm ; 12cm

Exercise - 4 (page - 257)

(1) (i) 20cm^2 (ii) 24cm^2 (2) 96cm^2 ; 150 mm : 691.2m^2 (3) 18cm (4) ₹506.25

Exercise - 5 (page - 260)

(1) (i) 220cm (ii) 26.4cm (iii) 96.8 cm (2) (i) 55m (ii) 17.6 m (iii) 15.4m

(3) (i) (a) 50.24 cm (b) 94.2 cm (c) 125.6 cm (ii) 7 cm (4) 42 cm

(5) 10.5 cm (6) 3 times (7) 3 : 4 (8) 1.75cm (9) 94.20 cm (10) 39.25 cm

Exercise - 6 (page - 263)

(1) 475m^2 (2) 195.5m^2; 29.5m^2 (3) 624 m^2 (4) 68 m^2 (5) 9900 m^2 ; 200100m^2

14 - Understanding 3D and 2D Shapes

Exercise - 1 (Page - 265)

(1) Sphere: Foot ball, Cricket ball, Laddu
Cylinder: Drum, Biscuit pack, Log, Candle
Pyramid: Pyramid ; Cuboid: Match box, Brick, Biscuit pack
Cone : Ice-cream, Flower pot ; Cube: Dice, Carton

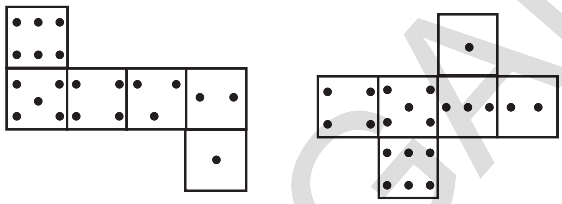
(2) (i) Cone: Ice-cream, upper part of a funnel (ii) Cube: Dice, Carton
(iii) Cuboid: Duster, Brick (iv) Sphere: Ball, Marble (v) Cylinder: Pencil, Pype.

(3)

	Cube	Cuboid	Pyramid
Faces	6	6	5
Edges	12	12	8
Vertices	8	8	5

Exercise - 2 (Page - 267)

(1) Do activity (2) i) C ii) a (3)



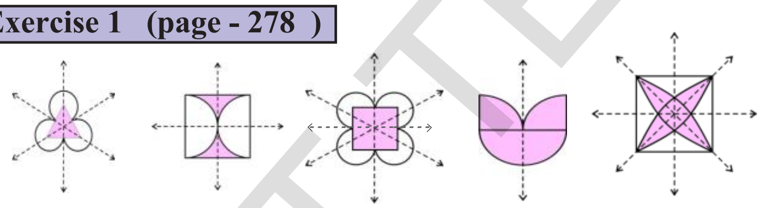
Exercise - 4 (Page - 276)

(1) A ball : a circle.
A Cylindrical pipe : a rectangle.
A book : a rectangle.

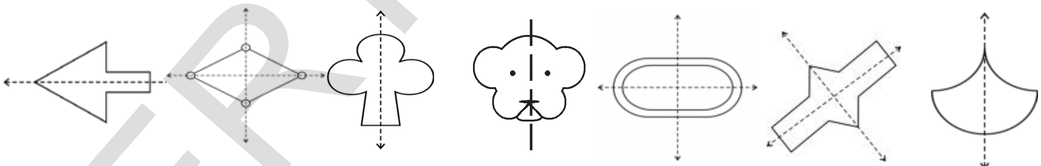
(2) (i) Spherical / Circular objects
(ii) Cube / Square sheets
(iii) Triangular shapes or Right prism with triangular base.
(iv) Cylinder / Rectangle sheets.

15 - Symmetry

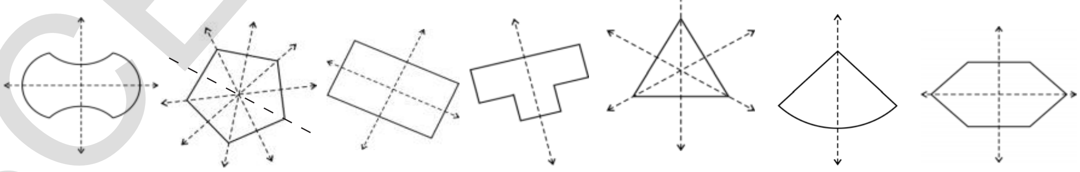
Exercise 1 (page - 278)



(i) (ii) (iii) (iv) (v) (vi) (vii)

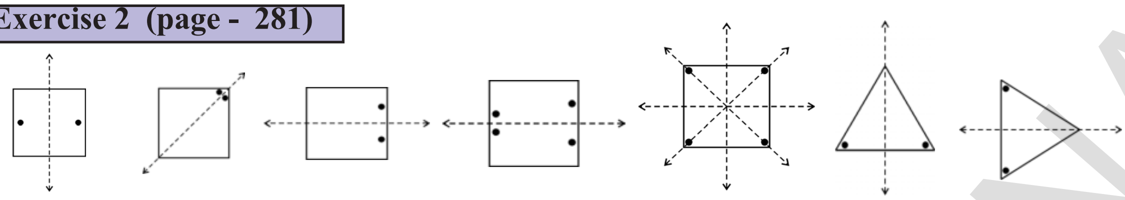


(viii) (ix) (x) (xii) (xiii) (xiv) (xv)

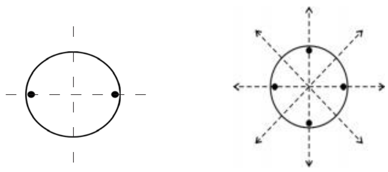


(xvi) (xvii) (xviii) (xix) (xx) (xxi) (xxiii)

Exercise 2 (page - 281)



(i) (ii) (iii) (iv) (v) (vi) (vii)



(viii) (ix)

(2)

(i) (ii) (iii) (ix)

(3)

(i) (ii) (iii) (iv) (v) (vi)

(4) (i) False (ii) True (iii) False

(5) Angle between successive axes = 360/2n =360/2x4 = 360/8 = 45^0

This is true for all regular polygons

Exercise 3 (page - 286)

1. Figures i, ii, iv and v have rotational symmetry.

2. (i) 2 (ii) 4 (iii) 3 (iv) 4 (v) 4 (vi) 5 (vii) 6 (viii) 3

3.

Square	yes	90^0	4
Rectangle	yes	180^0	2
Rhombus	yes	180^0	2
Equilateral Triangle	yes	120^0	3
Regular Hexagon	yes	60^0	6
Circle	yes	infinity	infinity
Semi-circle	No	-	-

Exercise 4 (page - 288)

1.

S	No	0	Yes	2
H	Yes	2	Yes	2
O	Yes	2	Yes	2
N	No	0	Yes	2
C	Yes	1	No	1

INSTRUCTIONS TO TEACHERS

Dear Teachers!!

Greetings and a hearty welcome to the newly developed textbook Mathematics for class VII.

- The present textbook is developed as per the syllabus and Academic standards conceived by the mathematics position paper prepared based on SCF – 2011 and RTE – 2009 for Upper Primary stage of education.

- The new textbook constitutes 15 chapters with concepts from the main branches of mathematics like Arithemetics, Algebra, Geometry, Mensuration and Statistics.

- These chapters emphasize the prescribed academic standards in achieving the skills like Problem Solving, Reasoning-proof, Communication, Connectivity and representation. The staratagies in building a chapter are observation of patterns, making generalization through deductive, inductive and logical thinking, exploring different methods for problem solving, questioning, interaction and the utilization of the same in daily life.

- The situations, examples and activities given in the textbook are based on the competencies acquired by the child at Primary Stage. So the child participates actively in all the classroom interactions and enjoys learning of Mathematics.

- Primary objective of a teacher is to achieve the "Academic standards" by involving students in the discussions and activities suggested in the textbook and making them to learn the concepts.

- Mere completion of a chapter by the teacher doesn't make any sense. The exhibition of prescribed academic standards by the student only ensures the completion of the chapter.

- Students are to be encouraged to answer the questions given in the chapters. These questions help to improve logical, inductive and deductive thinking of the child.

- Understanding and generalization of properties are essential. Student first finds the need and then proceeds to understand, followed by solving similar problems on his own and then generalises the facts. The strategy in the presentation of concepts followed.

- Clear illustrations and suitable pictures are given wherever it was found connection and corrects the misconnection necessary.

- Exercises of 'Do This' and 'Try This' are given extensively after completion of each concept. Exercises given under 'Do This' are based on the concept taught. After teaching of two or three concepts some exercises are given based on them. Questions given under 'Try This' are intended to test the skills of generalization of facts, ensuring correctness of statements, questioning etc., 'Do This' exercise and other exercises given are supposed to be done by students on their own. This process helps the teacher to know how far the students can fare with the concepts they have learnt. Teacher may assist in solving problem given in 'Try This' sections.

- Students should be made to digest the concepts given in "looking back" completely. The next chapter is to be taken up by the teacher only after satisfactory performance by the students in accordance with the academic standards designated for them (given at the end).

- Teacher may prepare his own problems related to the concepts besides solving the problems given in the exercises. Moreover students should be encouraged to identify problems from day- to-day life or create their own.

- Above all the teacher should first study the textbook completely thoroughly and critically. All the given problems should be solved by the teacher well before the classroom teaching.

- Teaching learning strategies and the expected learning outcomes, have been developed class wise and subject-wise based on the syllabus and compiled in the form of a Hand book to guide the teachers and were supplied to all the schools. With the help of this Hand book the teachers are expected to conduct effective teaching learning processes and ensure that all the students attain the expected learning outcomes.

Happy Teaching.

Syllabus

Number System: (50 hrs) **1. Integers** **2. Fractions, Decimals & Rational Numbers**	**(i) Integers** • Multiplication and division of integers (through patterns). • Properties of integers (including identities for addition & multiplication, (closure, commutative, associative, inverse, distributive) (through patterns). (examples from whole numbers as well). Expressing properties in a general form. Construction of counter examples, (eg. subtraction is not commutative). • Word problems involvingintegers (all operations)
	(ii) Fractions, Decimals and rationalnumbers: • Multiplication of fractions • Fraction as an operator "of" • Reciprocal of a fraction and its use • Division of fractions • Word problems involving mixed fractions (related to daily life) • Introduction to rational numbers (with representation on number line) • Difference between fraction and rational numbers. • Representation of rationalnumber as a decimal. • Word problems on rationalnumbers (all operations) • Multiplication and division of decimal fractions • Conversion of units (length & mass) • Word problems (including all operations)
Algebra (20 hrs) **11. Exponents** **10. Algebraic Expressions** **3. Simple Equations**	**Exponents and powersIntroduction** Meaning of x in a^x where a Ý Z • Laws of exponents (throughobserving patterns to arrive at5 generalization.)whereM, n $\in$N(i) a^m a^n = $a^{m?+n}$(ii)? $(a^m)^{?n}$ = a^{mn}(iii) a^m/a^n = a^{m-n}, where (m-n)$\in$ N(iv) $a^m.b^m$ = $(ab)^m$(v) number with exponent zerovi)Decimal number in exponential notation vii) Expressing large number in standard form (Scientific Notation)
	ALGEBRAIC EXPRESSIONS Introduction Generate algebraic expressions(simple) involving one or two variables • Identifying constants, coefficient, powers • Like and unlike terms, degree of expressions e.g., x^2y etc.(exponentd"?3, number of variables d"?2) • Addition, subtraction of algebraic expressions (coefficients should be integers).
	Simple equations • Simple linear equations in one variable (in contextual problems) with two operations (integers as coefficients)
6. Ratio - Applications (20 hrs)	• Ratio and proportion (revision) • Unitary method continued,consolidation, generalexpression. • Compound ratio : simple word problems • Percentage- an introduction • Understanding percentage as a fraction with denominator 100 • Converting fractions anddecimals into percentage andvice-versa. • Application to profit and loss (single transaction only) • Application to simple interest (time period in complete years).

<table>
<tr>
<td rowspan="6">Understanding shapes / Geometry

4. Lines and Angles
5. Triangle and Its Properties
8.Congurencey of Triangles
9.Construction of Triangles
12.Quadrilaterals
15. Symmetry
14.Understand-ing 3D and 2D Shapes</td>
<td>(i) Lines and Angles:
• Pairs of angles (linear,supplementary, complementary,adjacent, vertically opposite)(verification and simple proofof vertically opposite angles)
• Properties of parallel lines withtransversal (alternate,corresponding, interior, exteriorangles)</td>
</tr>
<tr>
<td>(ii) Triangles:
• Definition of triangle.
• Types of triangles acc. To sides and angles
• Properties of triangles
• Sum of the sides, difference of two sides.
• Angle sum property (with notion of proof and verification through paper folding, proofs , using property of parallel lines , difference between proof and verification
• Exterior angle property of triangle</td>
</tr>
<tr>
<td>(iii) Congruence:
• congruence through superposition ex. Blades, stamps etc..
• Extend congruence to simple geometrical shapes ex. Triange , circles,
• criteria of congruence (by verification only)
• property of congruencies of triangles SAS, SSS, ASA, RHS Properties with figures•</td>
</tr>
<tr>
<td>(iv) Construction of triangles (all models)
• Constructing a triangles when the lengths of its 3 sides are known (SSS criterion)
• Constructing a triangle when the lengths of 2 sides and the measure of the angle between them are known (SAS criterion)
• Constructing a triangle when the measures of 2 of its angles and length of the side included between them is given (ASA criterion)
• Constructing a right angled triangle when the length of one leg and its hypotenuse are given (RHS criterion)</td>
</tr>
<tr>
<td>(v) QuadrilateralsQuadrilateral-definition.
• Quadrilateral, sides, angles, diagonals.
• Interior, exterior of quadrilateral
• Convex, concave quadrilateral differences with diagrams
• Sum angles property (By verification) , problems
• Types of quadrilaterals
• Properties of parallelogram, trapezium, rhombus, rectangle, square and kite.</td>
</tr>
<tr>
<td>(vi) Symmetry
• Recalling reflection symmetry
• Idea of rotational symmetry,observations of rotationalsymmetry of 2-D objects. (900,1200, 1800)
• Operation of rotation through900 and 1800 of simple figures.
• Examples of figures with bothrotation and reflection symmetry(both operations)
• Examples of figures that havereflection and rotation symmetryand viceversa</td>
</tr>
</table>

	(vii) Understanding 3-D and 2-D Shapes: • Drawing 3-D figures in 2-Dshowing hidden faces. • Identification and counting ofvertices, edges, faces, nets (forcubes cuboids, and cylinders,cones). • Matching pictures with objects(Identifying names)
Mensuration *(15 hrs)* **13. Area and Perimeter**	**Area and Perimeter** • Revision of perimeter and Area of Rectangle, Square. • Idea of Circumference of Circle. • Area of a triangle, parallelogram, rhombus and rectangular paths.
7. Data Handling (15 hrs)	**Data Handling** • Collection and organisation ofdata • Mean, median and mode ofungrouped data – understandingwhat they represent.Reading bar-graphs • Constructing double bar graphs • Simple pie charts with reasonable data numbers

Academic Standards

CONTENT	ACADEMIC STANDARDS	
Number system 1. Integers	**Problem Solving:**	• Solves the problems involving four fundamental operations of integers • Solves the word problems involving the integers. • Used brackets for solving problems to simplify numerical statements.
	Reasoning Proof:	• Explains why the division by zero is meaning less. • Differentiates and compares the set of Natural numbers with integers. • Gives examples and counter examples to the number properties such as closure, Commutative, Associative etc.
	Communication:	• Expressing the number properties of integers in general form. • Uses the negative symbol in different contexts.
	Connections:	• Finds the usage of integers from their daily life situations • Understands the relation among N, W and Z.
	Representation:	• Represents the integers on number line. • Performs the operations of integers on the number line.
2. Fractions, Decimals and Rational numbers	**Problem Solving:**	• Solves the problems in all operation of fractions. • Solves the word problems of all operations of rational numbers. • Solves the problems of all operations of decimal fractions • Converts the small units into large units and vice versa.
	Reasoning : and Proof	• Differentiates rational numbers with fractions. • Justifies density property in rational numbers
	Communication:	• Expresses the need of set of rational numbers • Expresses the properties of rational numbers in general form
	Connections:	• Finds the usage of / inter relation among fractions, rational numbers, and decimal numbers.
	Representation:	• Represents rational numbers on the number line. • Represents the rational numbers in decimal form.
Algebra: 11. Exponents and powers	**Problem Solving:**	• Writes the large numbers in exponential form by using prime factorization
	Reasoning : and Proof	• Generalizes the exponential laws through the observation of patterns
	Communication:	• Understands the meaning of x in a^x where a € z. • Uses of exponential form when using large numbers

	Connections:	• Uses prime factorization in expression of large numbers in exponential form
	Representation:	• Expresses the large numbers in standard form
Algebra: 10. Algebraic Expression 3. Simple Equations	**Problem Solving**	• Finds the degree of algebraic expressions • Doing addition, subtraction of algebraic expressions (Co-efficient should be integers) • Solves the word problems involving two operations (Which can be expressed as simple equation and single variable)
	Reasoning and Proof	• Generates algebraic expressions involving one or two variables by using the patters
	Communication:	• Writes the standard form of first, second, third order expressions in one or two variables • Converts the daily life problems into simple equations. (Contains one variable only)
	Connections:	• Uses closure, commutative etc. properties in addition and subtraction of algebraic expressions. • Uses solving simple equations in daily life situations.
	Representation:	• Represents algebraic expressions in standard forms
6. Ratio - Applications	**Problem Solving**	• Finds the compound, inverse ratio of 2 ratios • Solves word problems involving unitary methods • Solves word problems involving percentage concept • Solves word problems to find simple interest (Time period in complete years)
	Reasoning and Proof	• Compares the decimals, converting into percentages and vice versa. • Formulates the general principles of ratios and proportions
	Communication:	• Expresses the fractions into percentages and decimal forms and their usage.
	Connections:	• Uses profit and loss concepts in daily life situations (Single transactions only) • Understands and uses the solutions for percentage problems in daily life.
	Representation:	• Converts fractions and decimals into percentage form and vice versa.

Understanding Shapes / Geometry **4. Lines and Angles**	**Problem Solving**	• Solves problems on angles made by transversal intersecting parallel line
	Reasoning and proof	• Differentiates the types of pair of angles from given angles • Verifies the parallel ness of the given lines with the use of properties of parallel lines. • Proofs and verifies the angle sum property through paper folding and using property of parallel lines.
	Communication:	• Gives examples of pairs of angles.
	Connections:	• Observes the parallelness in surroundings.
	Representation:	• Represents the notation of angle.
5. Triangle and Its Properties	**Problem Solving**	• Determines whether the given lengths of sides are shapes suitable to make triangle. • Finds the angle which is not given from exterior and other angles of triangle.
	Reasoning and proof	• Makes relationship between exterior angle to its opposite. • Classifies the given triangles on the basis of sides and angles. • Estimates the kind of triangle by observing the given triangle.
	Communication:	• Explains the different types of triangles according to sides and angles. • Explains the property of exterior angle of triangle.
	Connections:	• Uses the concept of triangle.
	Representation:	• ______________________
8.Congruency of Triangles	**Problem Solving**	• Identifies the congruent triangles from given triangles suitable to make triangle.
	Reasoning and proof	• _________________________
	Communication:	• Appreciates the congruency in 2-D figures.
	Connections:	• ______________________
	Representation:	• Represents the congruent triangles using symbols, notation.

9. Construction of Triangles	**Problem Solving**	• Construct triangles using given measurements.
	Reasoning and proof	• ______________________________
	Communication:	• ______________________________
	Connections:	• ______________________________
	Representation:	• ______________________________
12.Quardilateral	**Problem Solving**	• ______________________________
	Reasoning and proof	• Differentiates the convex, concave quadrilaterals. • Verifies and justifies the sum angle property of quadrilaterals.
	Communication:	• Explains the inter relationship between triangle and quadrilateral. • Explains the different types quadrilaterals based on their properties.
	Connections:	• Tries to define the quadrilateral. • Classifies the given quadrilaterals using their properties and their inter relationship.
	Representation:	• ______________________________
15.Symmetry	**Problem Solving**	• Rotate the figure and find its angular symmetry.
	Reasoning and proof	• Can differentiate linear and reflection symmetry using objectives or figures.
	Communication:	• Gives examples that have reflection symmetry.
	Connections:	• ______________________________
	Representation:	• ______________________________

14.Unders-tanding 3-D and 2-D shapes	Problem Solving	• Identifying and counting of faces, Edges, Vertices, nets for 3D Fig (Cube, Cuboid, Cone, Cylender).
	Reasoning and proof	• Matches picture with 3-D objects and visualize fells the Faces, Edges, Vertices etc.
	Communication:	• ____________________
	Connections:	• ____________________
	Representation:	• Can draw simple 3-D shapes in to 2-D figures.
Mensuration 13. Area and Perimeter	Problem Solving	• Solves the problem of Area and perimeter for square, rectangle, parallelogram, triangle and Rhombus shapes of things.
	Reasoning and Proof	• Understands the relationship between square, Rectangle, Parallelogram with triangle shapes for finding the area of triangle. • Understands the Area of Rhombus by using area of triangles.
	Communication:	• Explains the concept of Measurement using a basic unit.
	Connections:	• Applies the concept of Area perimeter to find the daily life situation problems (Square, Rectangle, Parallelogram, Triangle, Rhombus and Circle) • Applies the concept of area of Rectangle, Circle. • Finds the area of the rectangular paths, Circular paths.
	Representation:	• Represent word problems as figures.
7. Data Handling	Problem Solving	• Organization of raw data into classified data. • Solves the problems for finding the Mean, Medium, Mode of ungrouped data
	Reasoning	• Understands the Mean, Mode and Medium of ungrouped data and what they represent.
	Communication:	• Explains the Mean, Mode and Medium for ungrouped data.
	Connections:	• Understands the usage of Mean, Mode and Medium in daily life situation problems. • Understands the usage of double graphs and pie graphs in daily life situation (Year wise population, Budget, Production of crops etc.)
	Representation:	• Representation of Mean, Medium and Mode for ungrouped data. • Representation of the data in to double bar graphs and pie graphs.